Electromagnetic Model of Extended Spin-0 Particles

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ABSTRACT. Spherical cavities with internally trapped electromagnetic fields are considered as wave-corpuscular models of extended spin-0 particles. It is shown that radiation imprisoned inside a uniformly moving cavity undergoes conversion into a non-linear system of time-like and space-like waves which form a non-dispersive wave packet travelling at a group velocity equal to that of the moving cavity. The Mackinnon and Klein-Gordon wave equations for inner and outer fields associated with trapped radiation have been derived.

RÉSUMÉ. Des cavité sphériques avec champ électromagnétique piégé sont considérées comme des modéles onde-corpuscle de particules étendues avec spin-0. On montre que la radiation enfermée dans la cavité en mouvement subit une conversion en un systéme non linéaire d'ondes de type spatial et de type temporel, qui forment un paquet d'ondes non-dispersif, se mouvant avec une vitesse égale á celle de la cavité. Les équations de Mackinnon et de Klein-Gordon pour les champs associés avec la radiation piégée sont déduites de ces considérations.

1 Introduction

The fundamental equations of classical and quantum mechanics such as the Hamilton-Jacobi equation or the Klein-Gordon and Dirac equations are based on the Newton's concept of the mass point, and in the quantum domain, on the point particle notion. This useful approximation leads however to the unphysical effects of infinite self-energy and self-field, so the Dirac distribution or the renormalization procedure must be used to obtain the equations correctly describng the observed properties of material objects.

In order to avoid these complicated and abstract operations several models have been proposed [1-10] treating elementary particles as extended structures and not point-like objects. In particular, an electromagnetic approach has been developed [11-13] in which rectangular cavities with internally trapped radiation are considered as wave-corpuscular models of massive particles, as they very well reproduce fundamental properties of the *ordinary* matter, *e.g.* inertiality, mass quantization, non-point character, three-dimensional spatial extension and time-like kinematical characteristics [12].

The above concept assumes a rectangular geometry of extended particles, however, this stands in distinct contradiction to our intuitive expectations and standard conceptualization attributing to all particles a spherical geometry rather than a rectangular one. In view of the above, it would be desirable to construct in the electromagnetic framework, a more *real* and *adequate* model of extended particles, which would take into account not only their wave-corpuscular character but also spherical geometry.

The first suggestion in this respect comes from de Broglie [14], who considered electron as a superposition of two spherically symmetrical waves, a converging and a diverging one, both having phase velocity equal to the velocity of light. The de Broglie's concept was developed by Mackinnon [15-18], who, on the basis of the *phase connection principle*, has constructed a non-dispersive wave packet which does not spread with time and constitutes the particle-like solitary wave. Jennison [9] has shown that the Mackinnon's soliton represents a massive particle of a sharp and finite boundary, and is entirely consistent with the model of the electromagnetic phase-locked cavity [6,7]. According to this concept the origin of inertia of all finitely bounded material particles lies in the echo effect for feedback process occuring for c-velocity waves, that is intrinsic to phase-locked particles [9]. Considering the wave-corpuscular phenomena occurring in phase-locked cavities, it has been demonstrated [11,12] that trapped radiation undergoes conversion into a system of time-like and space-like waves which *lock* to form the non-linear wave propagating as a *luminal*-type excitation. In the corpuscular picture it may be interpreted as a photon conversion into a bradyon-tachyon compound whose particle constituents trap each other in a relativistically invariant way yielding a photon-type particle.

This short review of the most important concepts in the field suggests that the best theoretical framework for construction of extended particle models is an electromagnetic approach [6-10] enriched by the tachyonic theory of elementary particle structure [19-23]. In this paper we propose to consider in the aforementioned framework a relativistic wave-corpuscular model of extended spin-0 particles treated as spherical cavities with internally trapped electromagnetic fields. It will be shown that such a model is consistent with the well-known theoretical results in the field and predicts for example, (i) the presence of the matter waves of second kind (D-waves) in the spectrum waves associated with the cavity (particle) interior, (ii) the formation of a non-linear non-dispersive wave packet (C-wave) by the fields trapped inside a moving cavity, and (iii) the appearance of the bradyonic and tachyonic constituents in the mass spectrum associated with imprisoned radiation.

2 General formulation

In order to realize the above program, let us consider a spherical cavity of an internal radius a, which has perfectly conducting walls and a charge-free interior. If we assume that TE wave is excited inside cavity, its field components $\{E_\phi, H_\theta, H_r\}$ propagate according to the Maxwell equations [24]

$$c^{-2}\partial_t^2\mathbf{E} + \nabla\times\nabla\times\mathbf{E} = 0, \tag{1}$$

$$c^{-1}\partial_t\mathbf{H} + \nabla\times\mathbf{E} = 0. \tag{2}$$

The first of the mentioned equations and its solutions can be given [24] in the explicit form

$$\left\{c^{-2}\partial_t^2 - \left[\partial_r^2 + r^{-2}\left(\sin\theta^{-1}\partial_\theta\sin\theta\partial_\theta - \sin\theta^{-2}\right)\right]\right\} rE_\phi = 0, \tag{3}$$

$$E_\phi = E_0(\mu_{nl}r)^{-1}R_{nl}(\mu_{nl}r)\Theta(\theta)_l^1\exp[i\mu_{nl}ct], \tag{4}$$

where $n, l = 0, 1, 2...$, $\mu_{nl}c$ is the resonator frequency of the $\mathrm{TE}_{n,l,m}$ wave, $\Theta_l^m(\theta)$ denotes the associated Legendre polynomials with $m = \pm 1$, whereas $R_{nl}(\mu_{nl}r)$ are the spherical Bessel functions of the order $\frac{1}{2}$ satisfying

$$\left[\partial_r^2 + \mu_{nl}^2 - l(l+1)r^{-2}\right]R_{nl}(\mu_{nl}r) = 0, \qquad R_{nl}(0) = 0. \tag{5}$$

Having the solutions for E_ϕ, the remaining field components can be easily calculated by making use of the equation (2)

$$H_r = iE_0l(l+1)(\mu_{nl}r)^{-2}R_{nl}(\mu_{nl}r)\Theta(\theta)_l\exp[i\mu_{nl}ct], \tag{6}$$

$$H_\theta = -iE_0(\mu_{nl})^{-2}r^{-1}\partial_rR_{nl}(\mu_{nl}r)\Theta(\theta)_l^1\exp[i\mu_{nl}ct]. \tag{7}$$

The explicit values of the resonator frequency can be determined by imposing on electric field the boundary condition $E_\phi(a) = 0$ leading to

$$\mu_{nl} = \frac{x_{nl}}{a}, \tag{8}$$

where x_{nl} denotes the n-th root of the spherical Bessel function $R_{nl}(\mu_{nl}a) = 0$. In the simplest case of the $\text{TE}_{n,0,0}$ wave we have

$$R_{n0}(\mu_{n0}r) = \sin(\mu_{n0}r), \tag{9}$$

so from (8) one obtains

$$\mu_{n0} = \frac{n\pi}{a}, \tag{10}$$

and the electric field E_ϕ can be given as

$$\Psi_{n00}(t,r) = E_0 \frac{\sin(\mu_{n0}r)}{\mu_{n0}r} \exp[i\mu_{n0}ct]. \tag{11}$$

It is easy to verify that the magnetic components H_r and H_θ associated with $\text{TE}_{n,0,0}$ wave take the form

$$H_r = 0, \qquad H_\theta = -iE_0 \frac{\cos(\mu_{n0}r)}{\mu_{n0}r} \exp[i\mu_{n0}ct]. \tag{12}$$

Inspection into (12b) reveals that H_θ, having normal orientation relative to the cavity surface, satisfies the boundary condition $\partial_r r H_\theta(a) = 0$ giving the same values of the resonator frequency as (10).

3 Lorentz transformed electromagnetic fields

So far our considerations have concerned the electromagnetic structure of the cavity at rest characterized by the solutions (11). It is interesting to note that wavefunction (11) is not only solution of (1), but also satisfies the wave equation

$$\Box \frac{\sin(\mu_{n0}r)}{\mu_{n0}r} \exp[i\mu_{n0}ct] = 0, \tag{13}$$

$$r = \sqrt{x^2 + y^2 + z^2}, \tag{14}$$

with d'alembertian

$$\Box \equiv c^{-2}\partial_t^2 - \Delta = \partial_\mu \partial^\mu, \tag{15}$$

expressed in terms of the Cartesian coordinates. So in order to obtain the suitable solutions and the corresponding wave equation describing fields associated with the cavity moving in the $+x$-direction at a velocity $v = c\beta = dx/dt$ (relative to the labolatory frame), one may apply to (13) the Lorentz transformation

$$ct \longrightarrow \frac{ct - x\beta}{\sqrt{1-\beta^2}}, \quad x \longrightarrow \frac{x - \beta ct}{\sqrt{1-\beta^2}} \quad y \longrightarrow y, \quad z \longrightarrow z, \tag{16}$$

yielding the result

$$\Box \frac{\sin[\mu_{n0} r(v)]}{\mu_{n0} r(v)} \exp[\frac{i\mu_{n0}(ct - \beta x)}{\sqrt{1-\beta^2}}] = 0, \tag{17}$$

$$r(v) = \sqrt{(x - \beta ct)^2/(1-\beta^2) + y^2 + z^2}. \tag{18}$$

A detailed analysis of (17) reveals that the Lorentz transformed fields associated with the moving cavity satisfy the following equations

$$\Box \Psi_{n00}(t, \mathbf{r}) = 0, \qquad \Psi_{n00}(t, \mathbf{r}) = \chi_{n0}(t, \mathbf{r}) \psi_{n00}(t, \mathbf{r}), \tag{19}$$

$$\left(\Box + \mu_{n0}^2\right) \chi_{n0}(t, \mathbf{r}) = 0, \qquad \chi_{n0}(t, \mathbf{r}) = \exp[i(k_\mu x^\mu)], \tag{20}$$

$$\left(\Box - \mu_{n0}^2\right) \psi_{n00}(t, \mathbf{r}) = 0, \qquad \psi_{n00}(t, \mathbf{r}) = \frac{\sin[\mu_{n0} r(v)]}{\mu_{n0} r(v)}, \tag{21}$$

$$\partial_\mu \chi_{n0}(t, \mathbf{r}) \partial^\mu \psi_{n00}(t, \mathbf{r}) = 0, \tag{22}$$

$$k^\mu \equiv \{\mu_{n0}(1-\beta^2)^{-1/2}, 0, 0, \mu_{n0}\beta(1-\beta^2)^{-1/2}\}, \tag{23}$$

from which (20a) and (21a) have identical form as the time-like and space-like Klein-Gordon equation for a particle endowed with rest mass $\mu_{nl}^0 = \mu_{nl}\hbar c^{-1}$.

The obtained above results may be accounted for by assuming that a non-zero rest mass

$$\mu_{nl}^0 = \frac{\hbar x_{nl}}{ca}, \tag{24}$$

may be attributed to trapped radiation [12], which depends on the mode characteristics n, l, and the cavity dimension a. Needless to mention that the mass μ^0_{nl} associated with the imprisoned fields is quantized, in full accordance with the results of Jennison and Drinkwater [6] and Jennison [7].

A look into (19)-(22) additionally reveals that the derived equations are consistent with the tachyonic theory of elementary particle structure [19-23] and the two-wave particle model [25-35], so χ_{n0} and ψ_{n00} waves may be identified with the *ordinary* subluminal de Broglie wave (B-wave) and the superluminal matter wave of the second kind (D-wave), whose superimposition yields a Ψ_{n00} wave propagating as *luminal*-type excitation.

In the particle picture it may be interpreted as a photon conversion into a bradyon-tachyon system whose particle constituents trap each other in the relativistically invariant way yielding a photon-type particle. As the tachyonic component has the *co-latitudal* (θ), *azimuthal* (ϕ) as well as *radial* (r) degrees of freedom, it can move on the surface of a sphere whose centre coincides with the bradyonic constituent, however, the radial degree of freedom enables the tachyon to travel also between infinitesimally close spherical surfaces. In view of the above, such a bradyon-tachyon system created in the spherical cavity, structurally resembles a hydrogen-type (H-type) compound.

It is interesting to note that equations indentical to the (19)-(22) ones have been obtained by Mackinnon [15-18], who has constructed, with a particular superimposition of de Broglie waves, a wave packet which does not spread with time and constitutes a particle-like solitary wave. Gueret and Vigier [37] have shown that such a soliton wave follows the geodetics in the external gravitational field, so it behaves as a singularity of the gravitational field, *i.e.* as a test particle. Because, the Mackinnon soliton is endowed with a 3-dimensional spatial extension and inertia property [16,17], it has been proposed as a wave-corpuscular model of extended massive particles [18,35,36]. Let us recall additionally that in the $\mathbf{M}(1,1)$ space, the propagation of Ψ_{n00} wave is governed by the non-linear propagation law [26,37], so it may be interpreted in the framework of the *non-linear wave hypothesis* [31,32,34] as C-wave involving B- and D-waves as the internal spectrum waves associated with the cavity interior.

From the context of the presented considerations a clear, and consistent with our knowledge in the field, wave picture of the spherical cavity interior emerges. Namely, the radiation trapped inside the cavity undergoes a conversion into the plane time-like wave χ_{n0} and the space-like ψ_{n00} one which *lock* to form a solitary photon-like wave Ψ_{n00}. Such imprisoned inner fields form a non-dispersive wave packet which does not spread with time and travels at a group velocity equal to that of the moving cavity.

4 Particles as spherical cavities

The results obtained in the previous section indicate that electromagnetic spherical cavities may be employed as the extended wave-corpuscular models of massive particles. Exploiting this idea the trapped fields may be identified with inner (self) fields propagating in the cavity (*i.e.* particle) interior ($r \in < 0, a >$) as *luminal*-type excitation. In the cavity exterior ($r \in (a, \infty)$) the spatial propagation of the fields vanishes and the Maxwell equation (1) reduces to the purely time-like equation

$$\left(\partial_0^2 + \mu_{n0}^2\right) \exp[i\mu_{n0}x^0] = 0, \tag{25}$$

describing the propagation outer fields in the external cavity domain along the x^0-axis being incidentally the cavity worldline. So, in order to obtain the wave equation governing the propagation of outer fields associated with the cavity moving at the velocity v, one may apply to (25) the time Lorentz transformation yielding [12,29]

$$\left(\,\Box + \mu_{n0}^2\right) \exp[ik_\mu x^\mu] = 0, \tag{26}$$

$$k^\mu \equiv \{\mu_{n0}(1 - \mathbf{v}^2/c^2)^{-1/2}, \mu_{n0}c^{-1}\mathbf{v}(1 - \mathbf{v}^2/c^2)^{-1/2}\}, \tag{27}$$

which is nothing else but the Klein-Gordon wave equation describing propagation of time-like fields associated with a spin-0 free moving particle.

4.1 Geometrical relationships

Now, let us focus our attention on the problem of a correlation between the geometrical characteristics of spherical cavities and extended particles. For this purpose, on the basis of the rest-mass formula (24), one may derive the relationship

$$d = 2a = \left(\frac{h}{\mu_{n0}c}\right)\frac{x_{n0}}{\pi}, \tag{28}$$

which under the substitution $x_{n0} = n\pi$ can be given as

$$d = n\lambda_C, \tag{29}$$

where d is a cavity diameter whereas $\lambda_C = h/\mu_{n0}c$ denotes a Compton wavelength characterizing the massive cavity at rest.

A detailed analysis of equation (29) indicates that for the fundamental TE_{100} mode, a diameter of the spherical cavity at rest is equal to the Compton wavelength of the associated mass, and corresponds to the width of the wave packet measured at the first zero-point of the function $\Psi_{100}(t, a)$. This implies a strong correlation between the standard parameters characterizing extended particles and the geometry of spherical cavities.

4.2 Relativistic mass problem

It is obvious, that geometry of the cavity as well as the associated mass change in the moving frame. So in order to find the suitable relations let us consider for the sake of interpretative simplicity, the propagation of inner fields (17) projected onto $\mathbf{M}(1,1)$ space, which is governed by the wave equation

$$\Box \frac{\sin\left[\frac{x_{10}(x-vt)}{a\sqrt{1-v^2/c^2}}\right]}{\frac{x_{10}(x-vt)}{a\sqrt{1-v^2/c^2}}} \exp\left[i\frac{x_{10}(ct-vx)}{a\sqrt{1-v^2/c^2}}\right] = 0. \tag{30}$$

Inspection into (30) reveals that the radius of the moving cavity undergoes relativistic deformation and grows shorter in the direction of motion

$$a \longmapsto a\sqrt{1-v^2/c^2}, \tag{31}$$

in strict connection to the transformation of the Compton wavelength

$$\lambda_C \longmapsto \lambda_C\sqrt{1-v^2/c^2}. \tag{32}$$

Simultaneously, the mass associated with the cavity changes according to the equation

$$\mu_{10}^0 \longmapsto \frac{\mu_{10}^0}{\sqrt{1-v^2/c^2}}, \tag{33}$$

which indicates that, on the one hand, radiation trapped in the spherical cavity behaves as *ordinary* ponderable matter, and on the other, that the relativistic mass problem may be considered in the purely geometrical framework [12].

In the case of the de Broglie wavelength associated with the moving cavity $\lambda_B = h/p_x$, where p_x is the x-component of the cavity 3-momentum, one gets

$$\lambda_B = \lambda_C\sqrt{c^2/v^2 - 1} = d\sqrt{c^2/v^2 - 1}, \tag{34}$$

so, it is apparent, that de Broglie wavelength measured in diffraction experiments can give important information on the geometry of particles considered as spherical cavities.

4.3 Geometry of a bradyon-tachyon system

Finally, let us investigate the geometrical characteristics of the bradyon-tachyon H-type compound created inside the cavity. Because the space-like fields associated with the tachyonic component are delocalized in space [23,32], one may obtain the useful information on the internal structure of the particle system calculating the radial distribution of the tachyonic field in the space surrouding a bradyonic constituent. Taking advantage of the well-known formula

$$d\rho(r) = 4\pi r^2 R_{n0}(r)^2 dr, \tag{35}$$

and introducing into (35) the radial function (21b), for $v = 0$ we arrive at the equation

$$\frac{d\rho(r)}{dr} = 4\pi r^2[R_{n0}(r)]^2 = 4\pi\left(\frac{a}{n\pi}\right)^2 \sin\left(\frac{n\pi r}{a}\right)^2, \tag{36}$$

giving the probability of finding the tachyonic constituent within a spherical shell of a radius r and thickness dr; that is, within a volume $4\pi r^2 dr$ at the radius r. Function (36) sinks to zero at the bradyonic constituent as well as on the cavity wall, and has n-maxima at the points

$$r_k = \left(\frac{2k+1}{2n}\right)a, \qquad k = 0, 1, 2 ... n-1. \tag{37}$$

It is easy to check that for the fundamental mode ($n = 1$) the radial distribution (36) attains the maximum for $r_0 = \frac{1}{2}a$, so the greatest probability of finding a tachyon moving in the space surrounding the bradyon is at a half of the cavity radius.

5 Conclusions

The spherical cavities with internally trapped electromagnetic fields very well reproduce the fundamental properties of *ordinary* particles, *e.g.* inertiality, mass quantization, three-dimensional spatial extension, spherical symmetry and subluminal kinematical characteristics. Because outer fields associated with such imprisoned radiation propagate according to the Klein-Gordon equation, the spherical cavity with an excited TE_{100} wave may be considered as the relativistic wave-corpuscular model of time-like spinless particles.

The proposed approach is consistent in many points with the well-known theoretical results in the field, and predicts for example: (i) the presence of B- and D-waves in the spectrum states associated with the cavity (particle) interior which is compatible with the two-wave particle model [25-35], (ii) the formation of a non-linear non-dispersive wave packet (C-wave) by the fields trapped inside a moving cavity, in full accordance with *the non-linear wave hypothesis* [30-32,34], (iii) the appearance of the bradyonic and tachyonic constituents in the mass spectrum associated with imprisoned radiation, which is consistent with the extended wave-particle description of matter [30-34] and the tachyonic theory of elementary particle structure [19-23]. Moreover, the last point suggests the composite internal bradyonic-tachyonic structure of photons in agreement with the model proposed by Dutheil [38-43].

In particular the proposed approach permits explaining why the particle-like solitary wave obtained by Mackinnon, satisfies the *luminal*-type wave equation, and does not satisfy the *ordinary* time-like Klein-Gordon one [26,37]. The solution of the problem is simple, namely, the Mackinnon equation governs the propagation of inner fields associated with the cavity interior. Because the spectrum of inner fields includes both time-like and space-like waves in the form of the non-linear superposition its propagation is governed by the *luminal* Mackinnon equation. On the other hand outer fields associated with the cavity exterior are endowed with the time-like characteristics, so they propagate according to the Klein-Gordon equation for massive subluminal particles. Both of the equations are correct, however, in the two different cavity (*i.e.* particle) domains.

The presented considerations can be extended onto the TM waves endowed with the field components $\{H_\phi, E_\theta, E_r\}$. However, in this case the boundary conditions [24]

$$E_\theta(a) = 0, \qquad \partial_r r H_\phi(a) = 0, \tag{38}$$

imposed onto TM_{n00} wave lead to the resonator frequency

$$\mu_{nl} c = \frac{c(n + 1/2)\pi}{a}, \tag{39}$$

and then the geometrical correlation given by (29) is not found.

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