Extended Wave-Particle Description of Luminal-Type Objects

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ABSTRACT. Luxons, *i.e.* massless particles moving at the light velocity are considered as 2-dimensional space-time cavities with internally trapped electromagnetic fields. Such imprisoned inner fields are endowed with zero rest mass, and the associated outer fields propagate as luminal-type excitations. In the particle picture, luxons may be treated as bradyon-tachyon compounds whose particle constituents endowed with the same rest mass are coupled to each other in the relativisticaly invariant way. The obtained results are applied in a geometrical interpretation of the Bragg's law for photons interacting with a periodic crystal-like structure.

RÉSUMÉ.Les luxons, c'est-à-dire les particules sans masse, se déplaçant à la vitesse de la lumiére, sont traitées comme des cavités spatio-temporelles bidimensionnelles, avec un champ électromagnétique enfermé. Les champs intérieurs ont une masse propre égale á zéro, et les champs associés, extérieurs, se propagent comme des excitations de type lumineux. Dans l'image corpusculaire, les luxons peuvent être traités comme des systémes se composant d'un bradyon et d'un tachyon de mêmes masses propres, se conjugant d'une façon invariante relativiste. Les résultats obtenus sont appliqués dans l'interprétation géométrique de la loi de Bragg pour les photons en interaction avec la structure périodique de type cristallin.

1 Introduction

The wave-particle duality is one of the fundamental properties of matter including both massive and zero rest mass objects called luxons as they move at the velocity of light. Investigation of the problem, initiated by de Broglie in 1923, has been focused first of all on explaining the internal structure of microobjects, and especially, the way matter wave and its associated particle are related to each other giving one and the same physical reality. In order to answer this question, a few interesting concepts have been developed in which a massive particle is treated as a relativistic phase-locked cavity with internally trapped electromagnetic fields [1-5], or as associated with a system of c-velocity standing waves [6-12]. In particular, the N-dimensional (N=1,2,3) electromagnetic space (S^N) and time (T^1) cavities have been considered [13,14] as wave-corpuscular models of time- and space-like objects, permitting reproduction of their fundamental properties, as ponderability, inertness, extended structure and time(space)-like kinematical characteristics.

Because the wave-particle duality appears as a genuine property of matter including both massive and zero rest mass particles it would be interesting to consider in the aforementioned framework an extended wave-corpuscular model of massless luminal objects of photon, neutrino or graviton type. The starting point for our discussion is the fact that luxons may be treated [13-17] as a special case of bradyon-tachyon compounds whose constituents endowed with the same rest mass are coupled to each other in the relativistically invariant way [18-22]. Such a compound particle has luxon-like characteristics, *e.g.* it moves at the light velocity, has zero rest mass, and is associated with a system of nonlinear waves propagating as luminal-type excitation. On the other hand, considering time- and space-like objects as electromagnetic S^N- and T^1-cavities respectively [14], it is expected that a combination of both approaches will permit constructing a wave-corpuscular model of luminal-type particles treated as extended objects and not point-like structures.

However, as photons (spin $s = 1$) and gravitons ($s = 2$) belong to the class of intiger spin particles *i.e.* bosons, in contradiction to neutrinos ($s = 1/2$) which are fermions, their formulation in the terms of bradyonic-tachyonic fields should be based on the second and the first order differential equations respectively, *i.e.* Klein-Gordon and Dirac equations. As the second of the problems mentioned has already been considered [16-19], we restrict our considerations only to the particles of integer spin.

2 Luxons as bradyon-tachyon compounds

In order to realize the above program let us recall the most important results obtained in our previous works [13,14], which are indispensable in a consistent formulation of the luxon theory.

It is well-known that bradyons (time-like, slower-than-light particles) and tachyons (hypothetical space-like, faster-than-light objects) can trap each other in a relativistically invariant way yielding a compound particle endowed with an effective rest mass which depends on the masses of the interacting particles [20-24]. If bradyonic and tachyonic constituents have identical rest mass m_0, a system of particles is endowed with zero rest mass, *i.e.* it behaves as a luxon-type object [13,14, 16-19]. Such a bradyon-tachyon compound may be considered as a coupled pair of particles associated with the time-like ψ wave and the space-like ψ' one, which interact and *lock* to form the plane luxon-like wave χ. The interacting fields obey the following equations

$$[\partial_\mu \partial^\mu + (m_0 c/\hbar)^2]\psi(x^\mu) = 0, \qquad \psi(x^\mu) = \exp[ik_\mu x^\mu], \tag{1}$$

$$[\partial_\mu \partial^\mu - (m_0 c/\hbar)^2]\psi(x^\mu)' = 0, \qquad \psi(x^\mu)' = \exp[ik'_\mu x^\mu], \tag{2}$$

$$\partial_\mu \psi(x^\mu)\partial^\mu \psi(x^\mu)' = 0, \tag{3}$$

$$\partial_\mu \partial^\mu \chi(x^\mu) = 0, \qquad \chi(x^\mu) = \psi(x^\mu)\psi(x^\mu)', \tag{4}$$

$$k^\mu \equiv (\omega/c, \mathbf{k}), \qquad k^{'\mu} \equiv (\omega'/c, \mathbf{k}'), \qquad x^\mu \equiv (ct, \mathbf{r}), \tag{5}$$

$$k_\mu k^\mu = (m_0 c/\hbar)^2, \qquad k'_\mu k^{'\mu} = -(m_0 c/\hbar)^2, \tag{6}$$

in which k^μ and $k^{'\mu}$ are the wave 4-vectors of the time-like and space-like waves, respectively, whereas x^μ is the position 4-vector. If we assume that a bradyonic constituent is at rest, whereas a tachyonic one attains the limiting infinite velocity, the associated field ψ is periodic in time and independent of position, in contrast to the field ψ' which is static in time and periodic in space. In this case equations (1-6) reduce to the simple form

$$[\partial_0^2 + (m_0 c/\hbar)^2]\psi(x^0) = 0, \qquad \psi(x^0) = \exp[i(m_0 c/\hbar)x^0], \tag{7}$$

$$[\Delta + (m_0 c/\hbar)^2]\psi(\mathbf{r})' = 0, \qquad \psi(\mathbf{r})' = \exp[-i(m_0 c/\hbar)\mathbf{r}], \tag{8}$$

$$\partial_\mu \psi(x^0)\partial^\mu \psi(\mathbf{r})' = 0, \tag{9}$$

$$\partial_\mu \partial^\mu \chi(x^\mu) = 0, \qquad \chi(x^\mu) = \exp[if_\mu x^\mu], \tag{10}$$

$$f^\mu \equiv \{w/c, \mathbf{f}\} = \{m_0 c\hbar^{-1}, m_0 c\hbar^{-1}\mathbf{s}\}, \tag{11}$$

$$f_\mu f^\mu = (w/c)^2 - \mathbf{f}^2 = 0, \tag{12}$$

where f^μ is the wave 4-vector of the superimposed waves, $\mathbf{s} = \mathbf{c}/\mathrm{c}$ denotes the unit vector along the direction of the wave propagation, and c is the velocity of light.

A detailed analysis of the obtained equations indicates that a non-linear superposition of time-like and space-like waves χ propagates at the phase (group) velocity equal to the velocity of light, whereas the associated compound of a bradyon at rest and an infinite speed tachyon (termed as *transcendent* tachyon [24,p.23], is endowed with zero rest mass. In view of this, such a bradyon-tachyon compound may be formally treated as a luminal-type object, and *vice versa* luxons may be considered as composite objects built up of both bradyonic and tachyonic components endowed with the same rest mass [13,14,16-19]. This result plays a very important role in our considerations presented in the following parts of the paper.

3 Extended model of tachyons and bradyons

In our previous work [14] it was shown that electromagnetic S^N and T^1 cavities may be treated as extended wave-corpuscular models of time- and space-like objects. In this paper we propose to consider the simplest model in which bradyons and tachyons are considered as S^1 and T^1 cavities with charge-free interior and perfectly conducting walls. If we assume that electromagnetic wave characterized by the 4-potential $A^\mu = \{0, A^x, A^y, 0\}$ propagates in the $\pm$z-direction, then the non-zero components of the 4-potential satisfy the luminal Maxwell equation

$$\partial_\mu \partial^\mu A^\alpha = 0, \qquad A^\alpha = A_0^\alpha \exp[i(f_3 x^3 \mp f_0 x^0)], \qquad \alpha = x, y \quad (13)$$

where $f^\mu = \{w/c, 0, 0, f^3\}$ denotes the wave 4-vector of electromagnetic field. After trapping such a wave in a S^1 cavity of dimension a_3, placed on the z-axis, the imprisoned fields obey the equation

$$\partial_\mu \partial^\mu A_0^\alpha \phi_{n_3}(x^3) \exp[\mp i f_0 x^0] = 0, \quad (14)$$

in which $\phi_{n_3}(x^3)$ are solutions of the Helmholtz equation

$$(\Delta + m_{n_3}^2)\phi_{n_3}(x^3) = 0, \qquad n_3 = 0, 1, 2.... \quad (15)$$

$$\phi_{n_3}(x^3) = \sin\left(\frac{\pi n_3 x^3}{a_3}\right), \qquad m_{n_3} = \pi \frac{n_3}{a_3} = f_0, \quad (16)$$

whereas $m^0_{n_3} = \hbar m_{n_3} c^{-1}$ is a rest mass associated with imprisoned electromagnetic fields [14].

In the case of radiation trapped in the one-dimensional T^1-cavity of dimension a_0, the field equation takes the form [14]

$$\partial_\mu \partial^\mu A_0^\alpha \phi_{n_0}(x^0) \exp[i f_3 x^3] = 0, \tag{17}$$

where the functions $\phi_{n_0}(x^0)$ obey the time-dependent Helmholtz equation

$$(\partial_0^2 + m_{n_0}^2)\phi_{n_0}(x^0) = 0, \qquad n_0 = 0, 1, 2\ldots.. \tag{18}$$

$$\phi_{n_0}(x^0) = \sin\left(\frac{\pi n_0 x^0}{a_0}\right), \qquad m_{n_0} = \pi \frac{n_0}{a_0} = f_3, \tag{19}$$

including rest mass $m^0_{n_0} = \hbar m_{n_0} c^{-1}$ attributed to time-trapped radiation.

4 Extended model of luxons

A detailed analysis of the obtained results indicates that solutions of the Maxwell equation for the fields associated with space- and time-trapped radiation take the form of a superposition of the amplitudinal term ϕ (so-called C-wave [12,15]) and the exponential time-like ψ wave, or space-like ψ' one termed as B- and D-wave [25,26], respectively. For S^1 and T^1 cavities the associated fields may be generally given as

$$\psi(x^\mu) = \phi(x^3)\psi(x^0), \qquad \psi(x^\mu)' = \phi(x^0)\psi(x^3)', \tag{20}$$

so we may build up from them a nonlinear superposition

$$\Psi(x^\mu) = \psi(x^\mu)\psi(x^\mu)' = \phi(x^\mu)\chi(x^\mu), \tag{21}$$

$$\phi(x^\mu) = \phi(x^0)\phi(x^3), \qquad \chi(x^\mu) = \psi(x^0)\psi(x^3)', \tag{22}$$

or in the explicit form

$$\Psi(x^\mu) = A_0^{\alpha 2} \sin\left(\frac{\pi n_0 x^0}{a_0}\right) \sin\left(\frac{\pi n_3 x^3}{a_3}\right) \exp[i(f_3 x^3 \mp f_0 x^0)], \tag{23}$$

associated with the bradyon-tachyon system of particles treated as S^1 and T^1 cavities. It is easy to prove that the superposition (23) as well as its field components ϕ and χ satisfy the luminal wave equations

$$\partial_\mu \partial^\mu \phi(x^\mu)\chi(x^\mu) = 0, \tag{24}$$

$$\partial_\mu \partial^\mu \chi(x^\mu) = 0, \qquad \partial_\mu \partial^\mu \phi(x^\mu) = 0. \tag{25}$$

provided that the associated bradyonic and tachyonic constituents are endowed with the same rest mass $m^0_{n_0} = m^0_{n_3}$, and the following interaction condition is fulfilled

$$\partial_\mu \phi(x^\mu) \partial^\mu \chi(x^\mu) = 0. \tag{26}$$

Equality of the rest masses of the interacting particles implies the correspondences

$$a_3 = a_0 = a \qquad n_3 = n_0 = n, \tag{27}$$

which mean that dimensions of the S^1 and T^1 cavities must be equal to each other, and the trapped radiations have the same mode characteristics.

Introducing into (26) functions χ and ϕ given in the explicit form, and then employing the relations (16b), (19b) one gets

$$\left[\cos\left(\frac{\pi n_0 x^0}{a_0}\right)\sin\left(\frac{\pi n_3 x^3}{a_3}\right) \pm \sin\left(\frac{\pi n_0 x^0}{a_0}\right)\cos\left(\frac{\pi n_3 x^3}{a_3}\right)\right]\chi(x^\mu) = 0, \tag{28}$$

or in equivalent form

$$\sin\left[\frac{\pi n_0}{a_0}(x^3 + x^0)\right] = \sin\left[\frac{\pi n_3}{a_3}(x^3 \pm x^0)\right] = 0 \tag{29}$$

Equation (29) is satisfied for

$$x^3 \pm ct = a_3 = a_0, \tag{30}$$

which determines [27] the initial position of the wave $\chi = \exp[i(k_3 x^3 \mp k_0 x^0)]$ propagating in the $\pm$z-direction at the constant phase velocity equal to the velocity of light. This indicates that χ wave propagates in the external domain of the system of S^1 and T^1 cavities, consequently it may be called the outer or radiating wave [11], in contradiction to ϕ wave representing inner fields associated with trapped radiation [14,15].

5 Results and discussion

A detailed analysis of the obtained results shows that luxons in our formalism appear as 2-dimensional space-time (ST) cavities of equal space and time dimensions being the consequence of identical mass of bradyonic and tachyonic constituents. Such imprisoned inner fields are endowed with effective zero rest mass (see eq.(25b)) so the outer ones obey the luminal wave equation (25a) in full agreement with our knowledge in the field of luxons. In view of the above, the electromagnetic ST-cavities may be considered as wave-corpuscular models of luminal-type objects, as they reproduce very well their fundamental properties, *e.g.* massless, non-point extended structure and luminal kinematics.

If we take into consideration de Broglie relations which are valid for massive and zero rest mass particles

$$p^{\mu} = \hbar f^{\mu}, \qquad p^{\mu} \equiv \{E/c, \mathbf{p}\}, \tag{31}$$

where p^{μ} is the luxon 4-momentum, from (16b) and (19b) one obtains

$$E = \hbar \pi n a^{-1} c, \qquad p^3 = \hbar \pi n a^{-1}, \tag{32}$$

or in the equivalent form, relevant for further disscusion

$$E = \hbar w = m_n^0 c^2 = n\hbar w_0, \qquad w_0 = \pi a^{-1} c, \tag{33}$$

$$p^3 = \hbar f^3 = m_n^0 c = n\hbar f_0^3, \qquad f_0^3 = \pi a^{-1}. \tag{34}$$

The above obtained relations indicate that extended model of luminal particles provides not only a value of the elementary energy-momentum quantum (for $n = 1$), but also predicts the possibility of existence of their integral multiple (multiphotons) for $n = 2, 3.....$ In the case of photons, it is manifested, for example in: Planck's emission, photoelectric effect or the Compton scattering. It is also worth noticing that the aforementioned equations for $a \in < 0; \infty >$, reproduce the full spectrum of waves associated with luxons, suggesting a strict connection between the wave and geometrical characteristics of the associated luminal particles viewed as ST-cavity; *i.e.* the values of such parameters as frequency or wavelength are strictly related to the ST-cavity dimensions.

A look into (19b) reveals that tachyonic component of a luxon, endowed with infinite momentum $p^3 = \hbar f^3 = m_n^0 c$ and zero energy is responsible for the momentum carried by the luminal wave. On the contrary, relation (16b) indicates that energy of the c-velocity wave is strictly connected to bradyonic components endowed with zero momentum end energy $E = m_n^0 c^2$. It is noteworthy that equation (33a) was first considered for photons by Einstein, and next extended to include the class of massive particles by de Broglie giving the conceptual background for the fundamental hypothesis of wave-particle duality and formulation of the quantum mechanics. The results obtained indicate that wave-particle duality for luxons may be accounted for by de Broglie postulate $w = \hbar^{-1} m_n^0 c^2$ for massive particles.

Now let us focus our attention on the transformational properties of the fields associated with extended luxons under the Lorentz transformation. To this purpose let us assume that the luxon treated as ST-cavity is observed in the reference frame moving in the $-x^3$-direction at a velocity $v = c\beta$. Then, the Lorentz transformed fields and the corresponding wave equation are to be obtained by applying the following transformations to equations (23) and (24)

$$x^0 \longrightarrow \frac{x^0 - \beta x^3}{\sqrt{1-\beta^2}}, \qquad x^3 \longrightarrow \frac{x^3 - \beta x^0}{\sqrt{1-\beta^2}}, \tag{eq38}$$

yielding the results

$$\Psi(x^\mu) = A_0^{\alpha 2} \sin\left[\frac{\pi n_0 (x^0 - \beta x^3)}{a_0\sqrt{1-\beta^2}}\right] \sin\left[\frac{\pi n_3 (x^3 - \beta x^0)}{a_3\sqrt{1-\beta^2}}\right] \times \exp[i f(v)(x^3 \mp x^0)], \tag{36}$$

$$\partial_\mu \partial^\mu \Psi(\mu) = 0, \tag{37}$$

$$f(v) = f_0 \frac{\sqrt{1 \pm \beta}}{\sqrt{1 \mp \beta}}, \tag{38}$$

where $f(v)$ denotes the Doppler shifted frequency of the luminal wave associated with the luxon. Inspection in (36) and (37) reveals that in the moving frame both the time and space ST-cavity dimensions undergo the relativistic compression

$$a \rightarrow a\sqrt{1-\beta^2}, \tag{39}$$

so the masses of the associated bradyonic and tachyonic components increase, however they still remain equal to each other. This explains why luxons and associated luminal waves observed in the moving frame always travel at the light velocity. In the limiting case ($\beta \rightarrow 1$) luxons become point-like objects of infinite energy and momentum.

6 Final remarks

The presented approach permits constructing a relativistic wave-corpuscular model of luxons which may be treated as 2-dimensional electromagnetic ST-cavities. Such imprisoned inner fields have luminal-type characteristics, *i.e.* zero rest mass may be attributed to them, and the associated outer fields propagate as c-velocity excitations.

In the particle interpretation luxons may be considered as composite objects built up of both bradyonic and tachyonic components trapping each other in the relativistically invariant way. Such a compound particle is endowed with a luminal-type kinematical characteristics, *i.e.* it behaves as massless object moving at the velocity of light.

In view of the above, luxons may be viewed as objects of time-like and space-like *faces*, which *live* on the border line of two kinematically unpenetrable worlds of sub- and super-luminal particles. It is consistent with the well known fact that the luxon wordlines seperate the Minkowski cone into the domains of time- and space-like kinematical characteristics. It also suggests that luxons seem to be universal objects interacting with both bradyonic and tachyonic type of particles.

Summarizing our considerations the fundamental question arises: may the extended model of luxons be experimentaly verified ? In order to answer this question let us consider a beam of luxons (treated as ST-cavities) incident on a periodic crystal-like stucture, and next reflected from the planes of the crystal. In such circumstances it becomes obvious that

(i) Such an interaction may be considered as an elastic collision between a non-point luminal particle and the periodic obstacle, so the angle of reflection will be always equal to the angle of incidence.

(ii) The cavity dimensions are invariant under reflection, as an elastic collision excludes the dissipative processes.

(iii) The values of the angle θ leading to a large reflected intensity of luxon beams can be determined by making use of the Bragg's law $2d\sin(\theta) = n\lambda$ [28], where d is a distance between the crystal planes whereas θ is the angle measured from the crystal planes to the incident and reflected beams.

Now if we take into considerations the relation (19b) given in the form

$$f^3 = 2\pi/\lambda = \pi n a^{-1}, \tag{40}$$

where λ is the wavelength of the luminal wave, after simple mathematical operations one gets

$$2a = n\lambda, \tag{41}$$

so the Bragg's formula can be given in the form

$$a = d\sin(\theta), \tag{42}$$

relating only geometrical parameters of a periodic structure and interacting luxons. The relation (42) indicates that the Bragg's reflection may appear only for luxons of internal dimensions $a \leq d$. This is a geometrical version of the well known wave condition $\lambda/2 \leq d$ obtained for $n = 1$ [28], which clearly show that Bragg's reflection may be interpreted in the purely geometrical framework considering luxons as extended non-point objects of the internal dimension a corresponding to the ST-cavity dimensions.

The most important conclusion which can be draw from the presented considerations is that a point-like character attributed usually to material particles seems to be only a rough mathematical approximation having nothing to do with the real properties of the microobjects occurring in nature. In view of the above, the wave-particle dualism of matter including both massive particles and radiation should be enriched by the geometrical aspect of their internal structure, for a simple reason that wave, corpuscular and geometrical picture are merely three different aspects of one and the same physical reality.

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