On Quantum Mechanics from General Relativity: A Dialogue With J.S. Bell

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ABSTRACT. After a review of some of the personal correspondence between the author and John Bell on the foundations of the quantum theory, this paper discusses Bell's theorem and its usefulness in an asymptotic form of general relativity, applied to the microdomain. It is argued that, within the context of a generally relativistic theory of matter in all domains, and under the physical conditions where interacting bodies are separated to spacelike distances in four-space, Bell's inequalities should apply; when they are separated to timelike distances, the statistical analysis according to Bohr's interpretation of quantum mechanics should apply (as the low energy experiments indeed confirm). The role of the Pauli exclusion principle is found to be crucial in coming to these conclusions. The analysis applies to correlated inertial matter, where nonrelativistic quantum mechanics applies, as an asymptotic limit of a generally covariant field theory of matter, such as correlated protons in low energy scattering experiments. It does not apply to manifestly covariant field correlations, such as the photon experiments that have been used in the past to test Bell's inequalities.

RÉSUMÉ. Après une revue de certaines des correspondances personnelles entre l'auteur et John Bell au sujet des fondements de la mécanique quantique, cet article traite du théorème de Bell et de son utilité dans une forme asymptotique de la théorie de la relativité générale appliquée au micro-domaine. Dans le contexte de la théorie de la relativité générale appliquée à la matière quelque soit le domaine et pour certaines conditions physiques sous lesquelles les objets en interactions sont séparés par des distances spatiales, l'auteur argumente la validité des inégalités de Bell. En revanche, quand ces objets sont séparés par des distances temporelles, l'analyse statistique suivant l'interprétation de Bohr devrait être valable comme le montrent les expériences à basse énergie. Le rôle du principe d'exclusion *de Pauli s'avère être crucial pour permettre ces conclusions. Cette analyse s'applique à la matière inertielle corrélée pour laquelle la mécanique quantique non-relativiste est valable comme limite asymptotique de la théorie générale de champ covariant de la matière, comme c'est le cas pour les protons corrélés dans les expériences de diffraction à basse énergie. Ceci ne s'applique cependant pas aux corrélations de champ covariant comme dans les expériences photoniques qui ont été menées dans le passé pour tester les inégalités de Bell.*

I. Bell vis à vis Bohr

Among John Bell's outstanding contributions to contemporary physics was his remarkable analysis –leading to "Bell's Theorem"– that would allow a clear experimental comparison between the statistical implications of the Copenhagen interpretation of quantum mechanics and a statistical theory of correlations of localized 'classical-like' particles [1].

The Copenhagen view is based on the type of ontology proposed by Niels Bohr in his interpretation of the quantum formalism as a fundamental theory of elementary matter that entails the elementarity of the measurements of the physical properties of micromatter by a macroapparatus. The localized particle view, on the other hand, that Bell has referred to as "Einstein locality", entails the type of ontology considered in the *"Einstein-Podolsky-Rosen* analysis, leading them to the conclusion that quantum mechanics, as it stands, is an incomplete theory of the localized elements of matter [2]. Particles also appear to spontaneously interact at a distance without time delay –contrary to the requirement of relative simultaneity of special relativity theory. A possible resolution that has been proposed by some physicists is that of a hidden variable theory, wherein extra independent parameters are introduced to complete the description of the elements of matter, in the context of a quantum mechanical formalism [3].

It is well known that most of the experimental investigations of the consequences of Bell's theorem have ruled out the inequalities that are predicted by his analysis. The physics community has taken this negative result to be a substantiation of the Copenhagen ontology, thereby ruling out the hidden variable resolution. But does this truly imply that the Copenhagen view and quantum mechanics –a probability calculus in terms of a Hilbert space formalism– is an established law of micromatter, once and for all?

The answer is clearly: NO. For neither of the statistical interpretations of quantum mechanics –that of Bohr or that of a set of "Einstein localized particles", according to the EPR argument– are indeed foundational in regard to the laws of elementary matter. That is to say, the statistical analyses are in a different context than dynamical theories of elementary matter in the same sense that Boltzmann's statistical analysis of gases (or that of Bose-Einstein or Fermi-Dirac quantum gases) ask different sorts of questions than the questions asked about the underlying dynamics of the constituent elements of a gas. Nevertheless, it is still important that the correct statistics to use in analyzing a closed system, in the limit where it appears to be an open system of interacting things, depends on the nature of the original closed system, and its limit where the coupling is sufficiently weak to allow the use of a many-body description with its proper statistics. Still, it is the dynamics of the system that is foundational, rather than the statistics, useful as the latter may be to describe (rather than explain) a system of things. The foundational aspect relates to the ontological basis of a theory of matter; the statistical aspect relates to its epistemological basis, in this view.

With the Copenhagen positivistic view, the epistemological aspects are identical with the ontological aspects of a law for a system of matter. However, in the view of realism, this is not the case, as in the theory of matter anticipated by Einstein (and, to some degree, by the respective versions of Schrö dinger and de Broglie)[4].

The theory of matter that is entailed in Einstein's general relativity is that of a nonsingular field theory –where truly localized particles play no role nor where the linear (Hilbert space) formal structure of quantum mechanics –compatible with the requirements of a probability calculus– is valid as a general law of elementary matter. Rather, the closed field theory, in this view, is indeed 'nonlocal' in the strictest sense, since its explanatory aspects rely on an underlying field, distributed in spacetime continuously and nonsingularly, *everywhere*.

In a letter that Einstein wrote to David Bohm in 1953, which I discussed with Bell in our correspondence, Einstein said:[5] "When one is not starting from the correct elementary concepts, if, for example, it is not correct that reality can be described as a continuous field, then all of my efforts are futile, ..."

Shortly before the untimely end of John Bell's life, I had some correspondence with him on Einstein's attitude.

In a letter to me, dated 13 February, 1990, he said: "I myself was once named as misleading the public about Einstein. I wrote a reply. It has gone out into the big world without echo, alas like almost all that I have written."

In a letter, dated 26 February, 1990, I replied: "To show that your reply 'has gone out into the world without echo' is false, I take the liberty of replying here. Of course, your statement 'alas like almost all that I have written' is certainly not true! I have learned a great deal from things you have written over the years. Perhaps if you haven't had as much echo as you wished it was because you are so clear that it does not require as much questioning".

In a letter that he addressed to me, dated 6 March, 1990, Bell asked: "It is possible for you 'hidden variables' mean particle positions? ... And in the discussion of the locality difficulty, a formidable obstacle in my opinion to any classical theory, there is certainly no assumption that hidden variables are particle positions rather than field variables or anything else. All that is essential is that the experimental results and the experimental settings *are well localized*".

In my reply to Bell, on 15 March, 1990, I said the following: "To me, 'hidden variable' does not necessarily refer to 'particle position'. (Although this is the view of some who take a deterministic view to quantum mechanics). "What the 'hidden variables' do mean to me (and I believe that this was Einstein's view) is an extension of the space of *independent variables*, from the four space and time variables (or, equivalently, their Fourier transformed momentum and energy variables) to the addition of extra independent variables. The enlarged space of independent variables is then used to map the *dependent variables* –that is, the solutions of field equations, such as the wave function solutions of the Schrö dinger wave equation".

"The meaning of field theory, in Einstein's view, is entirely different. First, it is not a 'quantum field theory' –the latter is, in fact, a scattering theory of quantum particles. It is not even, in principle, Fourier transformable, because it is not a linear field theory. Here, the dependent variables are, instead, field solutions of nonhomogeneous, nonlinear equations, mapped in a single four-dimensional spacetime (of independent variables). There are, in the theory, asymptotic conservation laws in the local limit, where numbers are yielded that are to be associated with observables, such as energy, momentum and angular momentum. These numbers are determined from integrations of particular functions of the field solutions *over all of space*. Thus localization per se is not involved in the definitions of the observables in such a field theory. Space and time are not, in themselves, observables here."

"Of course, (as you say) one may do an experiment 'over here' and then look for a correlated experimental result 'over there'. But the interpretation of these correlations is not necessarily in terms of spatial localizations of separable particles of matter. Particularly, in a nonsingular field theory of the type encountered in general relativity, as a general theory of matter, spacetime is only a language of independent variables, in principle translatable into other languages of independent variables, so long as the dependent (field) variables maintain their relational structure."

Unfortunately this was the end of our dialogue. The main point that I was trying to emphasise to Bell was that the "distinguishable things", according to the empirical data, are in a field theory (as viewed by Einstein or Schrö dinger) the distinguishable modes of a single, in principle nonseparable (holistically understood) field of matter. This is in contrast with all of the particularistic theories of matter as a collection of free things, later to be coupled, but each of them separable from the rest without altering the physical characteristics of the whole system of matter.

A close analogy for the field view of distinguishable modes of a closed system is the set of distinguishable, though not separable ripples of a pond. Indeed, an individual ripple cannot be removed from the pond, and then studied on its own in regard to its intrinsic properties, such as size and weight. Rather than being a 'thing-in-itself', the ripple is a distinguishable manifestation of the whole pond, just as a note sounded by a violin string is a manifestation of the whole violin. In some approximation it might be possible to treat the collection of ripples of the pond as though they were separate entities, interacting with each other to cause mutual scattering into particular directions, etc. But we know all the while that this is only a mathematical artifice! At the elementary level, we know that the ripples are not truly separable, individuated things in the pond. This is the idea of holism that is implicit in Einstein's field theory of matter, as I understand it.

In answer to Bell's comment to me, it is not meaningful, in my view, to say, as many have interpreted the experimental violations of Bell's inequalities in favor of the Copenhagen view, that one measures a 'spin', for example, "over here", and one then correlates this with a measure of a 'spin' "over there", *without referring to the noun that the quality 'spin' modifies*! That is to say, one may meaningfully ask: "the spin of what?" Otherwise, it would be like saying that one has detected "blue" without attaching this to what it is that has this color. Certainly "blueness" is a noun that one may discuss objectively –its frequency in the visible spectrum, its effect on young lovers, etc. But the adjective "blue" must modify the real thing that has this quality! Thus there is a serious question here on the logical interpretation of the experimental results that are alleged to confirm the Copenhagen interpretation of quantum mechanics, *because of the violation of Bell's inequalities.*

One possibility for a resolution is to recognize that the formal structure of the matter field that we associate with the quantum mechanical wave function for the correlated system may only be an asymptotic approximation for a field solution that follows from an entirely different theory of matter, such as the implications of a field theory of matter rooted in general relativity. I believe that this is the idea that Einstein would have considered, had he lived to witness the development of physics inspired by Bell's analysis and the associated experimental studies.

2. The state vector and Bell's inequalities [6]

In a more explicit discussion of correlated systems it is convenient to consider two uncoupled spin one-half particles, that were initially coupled in an S-state, as an example of the thought-experiment considered by Einstein, Podolsky and Rosen (referred to hereafter as EPR). The following analysis applies only to nonrelativistic particles, such as correlated scattered protons. It does not apply to the experiments, such as that of Aspect, Dalibard and Roger[7], that entail photon correlations, since the latter are manifestly relativistically covariant entities. In the present analysis the nonrelativistic representation of the matter fields, in terms of the quantum mechanical formalism, is an asymptotic, low energy limit of a generally covariant expression.

The corresponding state function for the previously bound system, according to linear quantum mechanics, or the linear limit of a generally relativistic theory of matter, is the antisymmetrized state function of a Hilbert space:

$$|\Psi\rangle = 2^{-1/2}(|\boldsymbol{r}_1+\rangle|\boldsymbol{r}_2-\rangle - |\boldsymbol{r}_1-\rangle|\boldsymbol{r}_2+\rangle) \tag{1}$$

If the particles are removed from each other to a *spacelike* separation, $|\boldsymbol{r}_2 - \boldsymbol{r}_1| > c(t_2 - t_1)$, so that, according to special relativity theory, they no longer interact during the times that they are observed, the EPR conclusion implies that the new state functions in the separated condition (where we refer to Bell's concept of "Einstein locality") must have a form that depends on the spins for each of the particles in such a way that the mutual orientations of their spins are arbitrary. But if these particles were initially bound in an S-state, then if their separation did not entail any spin-dependent force, the total angular momentum of the separated system must still be an S-state, because of the conservation of angular momentum. That is, the spin correlation must persist, even though these particles no longer interact.

Thus, with "Einstein locality", as one proceeds from the bound system at timelike separations, as described by the pure state (1), to the unbound state of spacelike separation, there is nothing to break the antisymmetric feature of this matter field. Thus, even at spacelike separations, the state function that represents the matter field is still a "pure state", with the antisymmetric form:

$$|\Phi\rangle = 2^{-1/2}(|\boldsymbol{r}_1, \boldsymbol{\sigma}_1^{\pm}\rangle|\boldsymbol{r}_2, \boldsymbol{\sigma}_2^{\mp}\rangle - |\boldsymbol{r}_1, \boldsymbol{\sigma}_2^{\mp}\rangle|\boldsymbol{r}_2, \boldsymbol{\sigma}_1^{\pm}\rangle) \tag{2}$$

where $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2$ are the spin vectors of particles 1 and 2 at either of the spatial locations $(\boldsymbol{r}_1, \boldsymbol{r}_2)$ and $\pm$ refer to the axes of quantization –that are not generally parallel for the respective separated particles, though the total spin for the correlated system must remain zero.

A special case of the matter field $|\Phi\rangle$ is the state $|\Psi\rangle$, when the spin vectors for the individual matter fields at $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ would be parallel. But this need not be the case when the particle fields are at spacelike separations.

Explicitly, when the separated particle fields are correlated in a total S-state, $|\Phi\rangle$ is an eigenstate of $(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)^2$ and $(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)_3$, with eigenvalues equal to zero –according to the conservation of angular momentum. But $|\Phi\rangle$ is not an eigenstate of $(\boldsymbol{\sigma}_1^2, \boldsymbol{\sigma}_{13})$ and $(\boldsymbol{\sigma}_2^2, \boldsymbol{\sigma}_{23})$ separately. That is,

$$\boldsymbol{\sigma}_1|\Phi\rangle \neq -\boldsymbol{\sigma}_2|\Phi\rangle$$

With this consequence of "Einstein locality", the predetermined spin vectors $\boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2$ at either of the locations $\boldsymbol{r}_1$ or $\boldsymbol{r}_2$, may be oriented relative to each other in any predetermined direction. Thus, the expectation value of the angle $\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$, within the context of this probability calculus, is adjustable in coincidence experiments on correlated spin one-half objects, separated by spacelike distances.

Bell exploited this feature in deriving the inequalities for correlation probabilities[1]:

$$P(\boldsymbol{a}^+, \boldsymbol{b}^-) \leq P(\boldsymbol{a}^+, \boldsymbol{c}^-) + P(\boldsymbol{b}^-, \boldsymbol{c}^+) \tag{3}$$

where $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c})$ are any three localizing spatial vectors, oriented relative to each other according to particular choices of experimental arrangements for measuring spin orientations of (previously bound) spin one-half particle fields, in coincidence, when they have been separated to spacelike distances. The variable $P(\boldsymbol{a}^+, \boldsymbol{b}^-)$ is the probability for the correlation that an experimental determination of one of the particles is at $\boldsymbol{a}$ with spin "up" relative to the direction of $\boldsymbol{a}$, coincident with the experimental determination that the other particle (previously bound to the first one) is at $\boldsymbol{b}$ with spin "down" relative to the spatial direction of $\boldsymbol{b}$.

In contrast with this description, according to the Copenhagen interpretation as expressed by Bohr (in his response to the Einstein-Podolsky-Rosen paper[8], quantum mechanics relates to the outcome of a measurement, *when the measurement is carried out.* With this interpretation, then, quantum mechanics does not deal with the history of localized entities, say from an earlier time when the particles were bound to the later time when they are unbound. That is to say, if one should observe the constituents in a bound state, or when they would be observed separately in ubound states, these would be two different sorts of measurements, thus they must be represented by different sorts of state functions, where the latter do not entail a history of the former. Bohr then argued that the EPR *gedankenexperiment* that led to the concept of "Einstein locality" is a false interpretation of quantum mechanics because it is out of context.

With this view, Bohr would have predicted that the separated, non-interacting system, that was initially in an S-state, must have the following possible state functions:

$$|\boldsymbol{r}_1, +\rangle |\boldsymbol{r}_2, -\rangle \quad \text{or} \quad |\boldsymbol{r}_1, -\rangle |\boldsymbol{r}_2, +\rangle \tag{4}$$

The state function for the separated system is then a "mixed state", rather than a "pure state", since the functions above are each combinations of singlet and triplet states, with no predetermination for the amount of admixture.

On the other hand, it is the "pure" correlated state that characterizes the wave function in Bell's analysis for the spacelike separated components. We see, then, that an empirically verified violation of Bell's inequalities (3) indicates that the model he started with (where the state functions relate directly to measurements on localized particles, as in the EPR analysis) is not valid *under the experimental conditions that were investigated.*

This negative empirical result has been interpreted to mean that the Copenhagen interpretation is *necessarily true.* But this is a false conclusion because it is possible that neither the statistical analysis of Bell nor that of the Copenhagen school is a fundamental law of micromatter! For if a different theory should be more foundational in nature, such as the nonlinear field approach of general relativity, whose asymptotic limit only, in a linear approximation, is to be used to describe certain experiments that have been done to study Bell's inequalities, then we must see these limits are for the general form of the matter fields. Indeed, it is possible that under some physical conditions the asymptotic limit of a general relativistic field theory would correspond with those that lead to the statistical analysis of correlations expressed with Bell's inequalities, while under other conditions, the asymptotic limit may lead to the statistics implied by the Copenhagen school.

In the generally covariant, nonlinear field theory that I have been investigating, there is a prediction of a total connective field, depending on the coupled spinor solutions for a closed system of matter fields, in which this field entails the *Pauli exclusion principle, as an exact, derived feature*[9]. With this feature in mind, consider the physical conditions that underlie the tests of Bell's inequalities.

Within the context of this theory, that incorporates the exclusion principle, if a pair of (asymptotic) spinor fields, that were at an earlier time correlated with a total spin $S = 0$, but at the time of measurement are separated at a *spacelike* distance, in the asymptotic approximation used, (in principle there is never a real separation in this closed system field theory) then since there can be no coupling between these fields at the respective locations $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ [without approximation there is only the single spacetime $(\boldsymbol{r}, t)$] then there can be no correlation in regard to these spatial locations. In the limit of no coupling, then, the 'two-body' matter field must be symmetric with respect to the interchange, $\boldsymbol{r}_1 \leftrightarrow \boldsymbol{r}_2$. However, the totally antisymmetrized form for the matter field that is *generally* predicted (as a theoretical consequence of the closed and nonlinear features of this field theory –features that are absent in principle in the quantum mechanical formalism!) implies that this solution must then be antisymmetric with respect to the interchange of the spin variables, $\boldsymbol{\sigma}_1 \leftrightarrow \boldsymbol{\sigma}_2$. This result then implies the (asymptotic) Hilbert space function of the type (1) –thus indicating that there should be consistency with Bell's inequalities (3).

On the other hand, if the correlated, asymptotic matter fields should be considered to be separated by *timelike* intervals, then the spatial, coordinate-dependent interaction is transmitted between the locations $\boldsymbol{r}_1$ and $\boldsymbol{r}_2$ during the time of the measurement of matter at both locations, in coincidence. In this case, the spatial coordinates would be correlated variables. This implies that, in this case, the spatial part of the two-body matter field could be either symmetric or antisymmetric with respect to the interchange $\boldsymbol{r}_1 \leftrightarrow \boldsymbol{r}_2$, thence corresponding to two possibilities for the matter field solutions for the interacting system. The requirement of this field theory, that the asymptotic limit of the matter fields must be totally antisymmetric (under all conditions) then implies that the latter two possibilities would correspond, in terms of the total spin, to the singlet and triplet states. This result would then imply, in turn, that the matter field must be in the form of a "mixed state" –formally in agreement with the Bohr prediction (4), in the context of his interpretation of quantum mechanics.

Summing up, the theory of elementary matter in general relativity predicts that there would be agreement with a) the statistics that leads to Bell's inequalities when the correlated measurements (in the asymptotic "particle" approximation) correspond with the formal description of two entities that were previously bound and later separated to spacelike distances, and b) the statistics of Bohr's interpretation, when they are related to measurements of timelike separated entities.

In principle, however, both of the latter views of elementary matter, if considered as exact theories, are out of context in regard to the advocated theory of matter in general relativity. For in the theory in general relativity, statistics and indeterminacy play no basic role, since, fundamentally, there are no individual, separable particles with this field model –there is only the single, continuous matter field.

Nevertheless, these asymptotic limits of the general matter field play important roles in terms of the boundary conditions to specify the empirically required limits of the exact matter field solutions, thus serving as a test of the general theory of matter in general relativity. The most important general feature of this theory in general relativity that led to our conclusions was the *necessary* incorporation of the equivalent of the Pauli exclusion principle in its exact, nonlinear form.

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