Does a Dual de Broglie Wave Exist ?

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ABSTRACT. Lorentz transformed electromagnetic fields trapped inside a rectangular cavity are considered in the framework of the two-wave hypothesis of matter and the photon concept of elementary particle structure. It is demonstrated that TE or TM mode excited inside a cavity may be treated as a system of the time-like de Broglie wave and the space-like matter wave of second kind (dual D-wave or transformed Compton wave) which *lock* to form a photon-like wave propagating as luminal-type excitation.

RÉSUMÉ. Le champs éléctromagnétique soumis á la transformation de Lorentz et enfermé dans la cavité rectangulaire, est considérée dans le cadre d'hypothése bi-ondulaire de matiére et la conception photonique de structure des particules élementaires. On montre que les ondes éléctriques et magnétiques excitées dans la cavité, peuvent être traitées comme le systéme d'onde de Broglie et d'onde de matiére de deuxiéme type (l'onde duale ou l'onde de Compton transformée), formantes l'onde qui se propage comme l'excitation de type lumineux.

Introduction

In recent work [1], space and time cavities with internally trapped electromagnetic fields have been considered as wave-corpuscular models of extended time- and space-like particles. The proposed approach permits derivation of the Klein-Gordon equation for outer fields associated with a moving cavity, by making use of the Lorentz transformation of the field equation for trapped radiation. In this paper we propose to extend the research area to include the Lorentz transformed inner fields associated with TM or TE mode excited inside cavity. The aforementioned problem has not been investigated in details in our previous work, however, it seems to be of great importance for full comprehension of the wave structure of the cavity interior. In particular it will be demonstrated that the radiation trapped inside a cavity undergoes conversion into a system of the time-like de Broglie (B-wave) wave and the space-like matter wave of the second kind so-called D-wave which *lock* to the form a photon-like wave propagating as luminal-type excitation.

Theoretical framework

Let us consider a rectangular electromagnetic space cavity (S^3-cavity) of the size a_α ($\alpha = 1, 2, 3$) with perfectly conducting walls and charge-free interior. Under the above assumptions the imprisoned electromagnetic fields satisfies the luminal Maxwell equation [1]

$$\Box\phi(x^\alpha)_{n_\alpha}\exp[i(m_{n_\alpha}x^0)] = 0, \tag{1}$$

$$m_{n_\alpha} = m^0_{n_\alpha}c\hbar^{-1} = \pi\left(\sum_{\alpha=1}^{3}\frac{n_\alpha^2}{a_\alpha^2}\right)^{1/2}, \tag{2}$$

where $\Box = \partial_0^2 - \Delta$ denotes d'Alembert's operator, $m^0_{n_\alpha}$ is a rest mass attributed to trapped radiation [1], $x^\mu \equiv \{ct, \mathbf{r}(x, y, z)\}$, and $\phi(x^\alpha)_{n_\alpha}$ are solutions of the Helmholtz equation

$$\left(\Delta + m^2_{n_\alpha}\right)\phi(x^\alpha)_{n_\alpha} = 0 \qquad n_\alpha = 0, 1, 2\ldots.. \tag{3}$$

If we assume that the TE mode endowed with a longitudinal x^3-component of the magnetic $\mathbf{H}$ field is excited in a S^3-cavity, then the solutions of (3) satisfying the suitable boundary conditions may be given in the form [2]

$$\phi(x^\alpha)_{n_\alpha} = H_0\sin\left(\frac{\pi n_3 x^3}{a_3}\right)\prod_{\alpha=1}^{2}\cos\left(\frac{\pi n_\alpha x^\alpha}{a_\alpha}\right). \tag{4}$$

The function (4) slightly differs from that considered previously [1], however, such a form ensures that the boundary conditions are satisfied also for the transversal field components $\mathbf{E}_\perp$ and $\mathbf{H}_\perp$ of the TE mode, to be obtained from (4) and the Maxwell equations [2]

$$\nabla_\perp\cdot\mathbf{H}_\perp = -\partial_3 H_3, \qquad \nabla\times\mathbf{E}_\perp = -\partial_0(\mathbf{H}_\perp + \mathbf{e}_3 H_3), \tag{5}$$

where $\nabla_\perp = \nabla - \mathbf{e}_3\partial_3$ and $\mathbf{e}_3$ is the unit vector along the x^3-direction.

Now, let us assume that the S^3-cavity moves in the $+x^3$-direction at a velocity $v = c\beta = cdx^3/dx^0$ relative to the laboratory frame. Then, the wave equation governing the propagation of associated inner fields my be derived by applying the Lorentz transformation

$$x^0 \longrightarrow \frac{x^0 - x^3\beta}{\sqrt{1-\beta^2}}, \quad x^1 \longrightarrow x^1, \quad x^2 \longrightarrow x^3, \quad x^3 \longrightarrow \frac{x^3 - \beta x^0}{\sqrt{1-\beta^2}}, \tag{6}$$

to equation (1) yielding the result

$$\Box\,\phi(x^\mu)_{n_\alpha}\psi(x^\mu) = 0, \tag{7}$$

$$\phi(x^\mu)_{n_\alpha} = H_0 \sin\left[\frac{\pi n_3(x^3 - \beta x^0)}{a_3\sqrt{1-\beta^2}}\right]\prod_{\alpha=1}^{2}\cos\left(\frac{\pi n_\alpha x^\alpha}{a_\alpha}\right), \tag{8}$$

$$\psi(x^\mu) = \exp[\frac{i m_{n_\alpha}(x^0 - \beta x^3)}{\sqrt{1-\beta^2}}]. \tag{9}$$

Wave structure of the cavity interior

It is easy to see that functions (8) and (9) satisfy the space- and time-like Klein-Gordon equations

$$\left(\Box - m_{n_\alpha}^2\right)\phi(x^\mu)_{n_\alpha} = 0, \qquad \left(\Box + m_{n_\alpha}^2\right)\psi(x^\mu) = 0, \tag{10}$$

consequently, from (7) one obtains an invariant interaction condition

$$\partial_\mu\phi(x^\mu)_{n_\alpha}\partial^\mu\psi(x^\mu) = 0. \tag{11}$$

Additionally one may note that (8) and (9) can be given in covariant forms

$$\phi(x^\mu)_{n_\alpha} = H_0 \sin[\hbar^{-1}p'_\mu x^\mu]\cos[\hbar^{-1}p''_\mu x^\mu]\cos[\hbar^{-1}p'''_\mu x^\mu], \tag{12}$$

$$\psi(x^\mu) = \exp[i\hbar^{-1}p_\mu x^\mu], \tag{13}$$

$$p^\mu \equiv \{m_{n_\alpha}^0 c/(1-\beta^2), 0, 0, m_{n_\alpha}^0 c\beta/(1-\beta^2)\}, \tag{14}$$

$$p'^\mu \equiv \{m_{n_3}^0 c/(\beta'^2 - 1), 0, 0, m_{n_3}^0 c\beta'/(\beta'^2 - 1)\}, \tag{15}$$

$$p^{''\mu} \equiv \{0, m^0_{n_1}c, 0, 0\}, \qquad p^{'''\mu} \equiv \{0, 0, m^0_{n_2}c, 0\}, \tag{16}$$

$$c\beta' = v' = c\beta^{-1} = c^2/v > c, \tag{17}$$

in which v' is the so-called pseudovelocity [3,4], whereas p^μ, $p^{'\mu}$, $p^{''\mu}$ and $p^{'''\mu}$ denote the time- and space-like 4-momenta of the particle constituents associated with trapped radiation [1].

Conclusions

The obtained equations (10) - (17) are fully compatible with the extended *two-wave* description of matter [3-10], the tachyonic theory of elementary particle structure [11-15], as well as with the photon concept of internal particle structure [1,16].

In particular $\phi(x^\mu)_{n_\alpha}$ may be interpreted [1] as a superimposition of the three independent space-like states

$$\phi(x^\mu)_{n_\alpha} = \phi(x^1)_{n_1}\phi(x^2)_{n_2}\phi(x^0, x^3)_{n_3}, \tag{18}$$

identified with the three super-luminal matter waves of second kind (dual D-wave or transformed Compton wave) associated with the three space-like particle constituents endowed with the 4-momenta $p^{''\mu}$, $p^{'''\mu}$ and $p^{'\mu}$, and related generally to the three different mass-states $m^0_{n_\alpha} = \pi\hbar c^{-1} n_\alpha a_\alpha^{-1}$. On the contrary, $\psi(x^\mu)$ is the *ordinary* de Broglie wave (B-wave) connected with a time-like particle constituent described by the 4-momentum p^μ. Consequently, TE mode excited inside S^3-cavity may be treated as a system of B- and D-waves which *lock* to form a photon-like wave propagating as luminal-type excitation. In the particle picture it may be interpreted as a photon conversion into bradyon-tachyon bounded system of particles characterized by the 4-momenta p^μ, $p^{'\mu}$, $p^{''\mu}$, $p^{'''\mu}$, which trap each other in the relativistically invariant way [1,16].

In view of the above, the concept under consideration predicts appearance of the D-wave in the spectrum states associated with the cavity interior, contrary to the external cavity domain where only *ordinary* time-like outer fields propagate [1].

The presented approach can be applied also to the TM mode for which the Lorentz transformed x^3-component of the electric field $\mathbf{E}$ satisfying the suitable boundary conditions takes the form

$$\Psi(x^\mu) = E_0 \cos\left[\frac{\pi n_3 (x^3 - \beta x^0)}{a_3\sqrt{1-\beta^2}}\right] \times \prod_{\alpha=1}^{2} \sin\left(\frac{\pi n_\alpha x^\alpha}{a_\alpha}\right) \exp[\frac{i m_{n_\alpha}(x^0 - \beta x^3)}{\sqrt{1-\beta^2}}]. \tag{19}$$

Consequently, the presented considerations also hold true for TM mode excited inside the rectangular S^3-cavity.

It is interesting to note that the derived equations (7), (10) and (11), are identical to those describing propagation of a non-dispersive soliton-like wave packet constructed by Mackinnon [17-19]. It suggests that Lorentz transformed inner fields associated with the rectangular S^3-cavity, may be viewed as a special case of the solitary wave considered by Mackinnon. Consequently, the S^3-cavity with internally trapped electromagnetic fields may be treated as the simplest model of extended particles endowed with rectangular geometry [1], competing with the Mackinnon's construction playing the important role in the different wave-corpuscular models of particles [3,4,20-23].

Finally, let us recall a Mackinnon's [19] opinion that de Broglie wave may prove to be as real as a classical electromagnetic wave, and d'Alembert's equation (7) may prove to be of more importance to quantum mechanics than has hitherto been supposed. The results obtained in this paper as well as in our previous work [1] fully confirm this supposition.

References

[1] M. Molski, J. Phys. A: Math. Gen. **26**, 1765 (1993).
[2] J.D. Jackson, *Classical Electrodynamics* John Willey&Sons Inc. New York (1975) 2nd. ed. ch. 8 (Polish edition).
[3] S.N. Das, Phys. Lett. **129A**, 281 (1988).
[4] S.N. Das, Nuovo Cimento B. **107**, 1185 (1992).
[5] C. Elbaz, Phys. Lett. **109A**, 7 (1985).
[6] C. Elbaz, Phys. Lett. **114A**, 445 (1986).
[7] C. Elbaz, Phys. Lett. **123A**, 205 (1987).
[8] R. Horodecki, Phys. Lett. **133A**, 179 (1988).
[9] R. Horodecki, Nuovo Cimento B. **102**, 27 (1988).
[10] R. Horodecki, Ann. der Phys. **48**, 479 (1991).
[11] H.C. Corben, Lett. Nuovo Cimento. **20**, 645 (1977).
[12] H.C. Corben, Lett. Nuovo Cimento. **22**, 166 (1978).

[13] H.C. Corben, *Tachyons, Monopoles and Related Topics* ed. E. Recami, North-Holand Amsterdam (1978) pp. 31-41 .
[14] P. Castorina and E. Recami, Lett. Nuovo Cimento, **22**, 195 (1978).
[15] E. Recami, Riv. Nuovo Cimento. **9**, 6, 1-178 (1986).
[16] M. Molski, J. Phys. A: Math. Gen. **24**, 5063 (1991).
[17] L. Mackinnon, Found. Phys. **8**, 157 (1978).
[18] L. Mackinnon, Lett. Nuovo Cimento. **31**, 37 (1981).
[19] L. Mackinnon, Lett. Nuovo Cimento. **32**, 311 (1981).
[20] L. Mackinnon, *The non-dispersive wave-packet and its significance for quantum mechanics*, in: Problems in quantum physics, World Scientific, Singapore (1988).
[21] R.C. Jennison, J. Phys. A: Math. Gen. A **16**, 3635 (1983).
[22] L. Kostro, Phys. Lett. **107A** 429, **112A**, 283 (1985).
[23] L. Kostro, *Mackinnon's soliton on top of Einstein's covariant ether*, in: Problems in quantum physics, World Scientific, Singapore (1988).

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