A micro-realistic explanation of the $|\psi|^2$ probability rule

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ABSTRACT. We start from our theory explaining waves (as a phenomenon "dual" to particles) by the joint effects of action metric ("distorting" distances) and the encoding of physical data in a non-corpuscular way in the basic material of the universe: action. Then, "monochromatic" matter waves correspond to realistic harmonic oscillators carrying energy in a way comparable to how classical waves do so. If we have a "complex" Fourier component of a wave packet, $F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})} = F(\mathbf{p})[\cos\frac{1}{\hbar}(Et-\mathbf{p}.\mathbf{r}) + i\sin\frac{1}{\hbar}(Et-\mathbf{p}.\mathbf{r})]$, its energy is proportional to $F(\mathbf{p})^2[\cos\frac{S}{\hbar} + i\sin\frac{S}{\hbar}][\cos\frac{S}{\hbar} - i\sin\frac{S}{\hbar}] = F(\mathbf{p})^2[\cos^2\frac{S}{\hbar} + \sin^2\frac{S}{\hbar}]$ (if we write S for the action $Et - \mathbf{p}.\mathbf{r}$). That is, the "complex" notation $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})} = e^{\frac{i}{\hbar}S}$ for finding relevant energies (and probabilities) is a mere mathematically expedient way to find the sum of the energies of the real waves $F(\mathbf{p})\sin\frac{S}{\hbar}$ and $F(\mathbf{p})\cos\frac{S}{\hbar}$, whose integration by the complex factor $e^{\frac{i}{\hbar}S}$, in turn, is the mathematical mode of expressing that such real waves are physically coupled in a specific way. Subsequently, we argue that the probability of finding a particle is proportional to the local energy density.

RÉSUMÉ. Dans cette étude, le point de départ est notre théorie décrivant les ondes comme phénomène dual des corpuscules, à l'aide de la métrique d'action et du codage des quantités physiques sous forme non corpusculaire dans l'élément de base de l'univers: l'action.

Dès lors, les ondes de matière monochromatiques correspondent à des oscillateurs harmoniques réels, propageant l'énergie tout comme les ondes classiques.

Les notations complexes utilisées pour trouver les énergies ne sont qu'un moyen mathématique d'exprimer la somme des énergies d'ondes réelles, ondes qui doivent donc être couplées d'une manière spécifique.

On démontre ensuite que la probabilité de trouver une particule est proportionnelle à la densité locale d'énergie.

1. Introduction

In an article [1] that, though largely neglected, will probably appear to be of major importance, Battey-Pratt and Racey (BR) found some results summarised below:

(1) If you lay an apple on your hand, a continuously proceeding rotation through $2\pi, 4\pi, 6\pi, \ldots$ of the apple effected by the hand is possible without your wrist, arm or shoulder getting progressively twisted.

Analogously, you can connect a sphere or core S by some strings (compare the arm) with a stable environment and make S rotate without the strings getting progressively twisted or mutually entangled, on the conditions that the strings have leeway and you do it in a special way, e.g., S dipping under the strings at the right moment. (Your hand with the apple does the same thing with respect to your arm.) BR call the combined movements of S and strings *spherical rotation* and consider the successive *configurations* of it, phases of the periodical movement. Each time S rotates through 4π , the strings witness periodical phases adding up to 2π . (See figure 1 for some indication.)

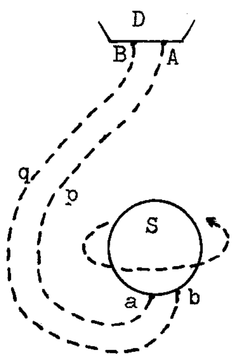

Figure 1. Core S, strings p and q and stable environment D in a spherically rotating configuration

(2) BR relate spherical rotation to both the *Zitterbewegung* and the de Broglie waves. They emphasize that such rotation is the simplest mode of spinning that does not disrupt or endlessly wind up connections with the environment, e.g., the surrounding continuum conceived as a "gel".

The spinning *Zitterbewegungen* a particle's existence in time can be a succession of are considered to be physically realistic manifestations of what mathematically are spherical rotation processes. Essentially, BR see matter waves as periodic movements of the relevant "strings", the waves' phases corresponding to those of the string configurations that somehow manifest themselves physically over the spatial domains where the waves appear.

(3) They prove that the successive configurations in space, or phases, of a set of strings participating in a spherical rotation that (as "de Broglie clock ticks") defines a momentum carrier (particle) M

(a) are isomorphic with the set R of radius vectors $(\alpha,\ \beta,\ \gamma,\ \delta)$ pointing to the three-dimensional surface of a four-dimensional Euclidean sphere whereas, subsequently,

(b) R is isomorphic with a set of spinors $\begin{bmatrix} \alpha + i\delta \\ \gamma + i\beta \end{bmatrix}$ that, in turn,

(c) can generally be written in the form $\begin{bmatrix} \phi_1 e^{\frac{i}{\hbar}S} \\ \phi_2 e^{-\frac{i}{\hbar}S} \end{bmatrix}$ (S being the action $Et - \mathbf{p}.\mathbf{r}$) for an observer with respect to which M is moving. Finally, they prove that

(d) the spinor waves $\begin{bmatrix} \phi_1 e^{\frac{i}{\hbar}S} \\ \phi_2 e^{-\frac{i}{\hbar}S} \end{bmatrix}$ so derived from the spherical-rotation phases *satisfy the Dirac equation* after some mathematical operations making 4-spinor waves of them ([1], pp. 452-5). That is, they demonstrated that spherical-rotation configuration phases have something very direct to do with Dirac matter waves, i.e., are in any way isomorphic with them "via a few steps".

Now it is the physical interpretation of such isomorphism in which we differ from BR, as earlier described in Refs. [2] and [3]. We state the differences briefly:

1. We use the concept of action metric to explain the appearance of matter waves, in particular using the discrepancy between such metric and Minkowski metric [2, 3, 4, 5]. Essentially, the action-metrical "stretching" of particles to waves - their "spreading over space" -, as discussed in the references, is implied by *action* being the core variable of physical processes, and particularly of spherical rotation as alternatively manifesting itself in waves which are co-defined by the factor $e^{\frac{i}{\hbar}(Et - \mathbf{p}.\mathbf{r})}$. For, this primacy of action causes two point-events corresponding to the same action $Et - \mathbf{p}.\mathbf{r}$ to make no difference as regards the wave phenomenon.

That is, a certain wave field strength extends itself "automatically" to all point-events $(\mathbf{r}, t)$ for which $Et - \mathbf{p}.\mathbf{r}$ is the same, because *such point-events are essentially physically mutually contiguous, or even equivalent in the internal action metric of the process in question.* As is explained in the Refs., the action-metrical "stretching" of physical phenomena can be somewhat compared with the extension of a relativistic trajectory $s = 0$ to a finite Euclidean one via $c^2t^2 - x^2 = s^2 = 0$, in which $x \neq 0$.

2. We interpret matter waves as physical manifestations of the same (or very similar) information about the relevant momentum carrier as in other circumstances is "encoded" in the *Zitterbewegung* - that is, corpuscular - mode of existence of such carrier, calling this conception the *coded-information theory.* I.e., if a particle P becomes wavelike, the action quanta, as the four-dimensional building blocks or "atoms" P's existence in time consists of, start carrying or encoding the information P's existence and properties are characterized by, in another information code. The relevant information is *translated into such new physical code.* E.g., energy is encoded by $\nu = \frac{E}{h}$, momentum by $\lambda = \frac{h}{p}$ and spin state by (the phase difference of) certain wave components. The waves do neither carry some hidden rotating top, nor are they of a purely mathematical (statistical) nature, nor do they "guide a corpuscule".

Summarizing: Waves are equally real as corpuscules; both encode in the basic stuff of the universe - action - the physical "information chunks" existing in time that we call momentum carriers, though waves and particles do so in two different but mutually isomorphic physical languages that are translated into one another, e.g., at emissions and impacts of the relevant carriers. That is, both corpuscules and waves encode the properties of a physical system in series of realistic action quanta, doing so in two different information codes.

Our above interpretation of matter waves indeed differs much from the one of Ref. [1], where they are seen as vibrations of the strings going with spherical rotation. Vibrations that, largely at macroscopic distances from the core, satisfy the Dirac equation. Alternatively, BR see the waves as a spinning or vibration of the vacuum, conceived as a gel surrounding the spherically rotating core. Again, a problem is constituted here by the question how such vibrations can be important at macroscopic distances from the core, without our calling on the "distances-distorting" action metric as referred to. Or, we see it as a drawback of the interpretation of [1] as compared with ours that, in the

BR one, matter wave "string or vacuum vibrations" should be expected to become weaker and weaker as we recede from the "proper location" of the particle, that is, from the rotating core. Actually, there is no basis for this in the quantum formalism.

In contradistinction to that of Ref. [1], our conception takes the four-dimensional wave slices themselves to be action quanta that in "corpuscular" physical circumstances embody a *particle's* existence in time. Wave-like quanta are merely stretched because of the discrepancy between action metric and the Euclidean one [4, 5], and also carry the momentum carrier's characteristics (information) in another data code [2, 3, 6].

2. The $|\psi|^2$ probability rule derived from the coded-information theory of matter waves

(1) Our theory stating that matter waves differ from the corpuscular manifestation of a particle P they are associated with by the mere *information code* in which they carry (or "formulate") P's characteristics (data), it is obvious that such waves, *inter alia*, carry (free) P's constant energy and momentum *in a realistic way* and, generally, satisfy the conservation laws just as P does if being in the corpuscular state.

(2) Within this context of the physically realistic translation, under certain circumstances, of corpuscules into waves we can interpret the derivation of the Schrödinger equation on p. 64 of Ref. [7] as follows:

(a) Because

$$i\hbar\frac{\partial}{\partial t}\int F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}d\mathbf{p} = E\int F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}d\mathbf{p} = E\psi(\mathbf{r},t) \tag{1}$$

and

$$\frac{\hbar}{i}\frac{\partial}{\partial x}\int F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}d\mathbf{p} = p_x\int F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}d\mathbf{p} \tag{2}$$

and

$$E = \frac{p^2}{2m} = \frac{p_x^2+p_y^2+p_z^2}{2m} \tag{3}$$

we see that

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) = -\frac{\hbar^2}{2m}\Delta\psi(\mathbf{r},t) \tag{4}$$

In the realistic interpretation, the left-hand side of Eq. (4), as well as Eq. (1), physically reflects *E's being spread out over the* $(\mathbf{r},t)$ *space*

$\psi(\mathbf{r},t)$ refers to, just as the r.h.s. of Eq. (4) expresses $\frac{p^2}{2m}$ being distributed analogously. This means that the Schrödinger equation, *inter alia*, embodies a spread-out version of $E = \frac{p^2}{2m}$. [Note that $i\hbar\frac{\partial}{\partial t}$ is "the operator for E" simply because

$$i\hbar\frac{\partial}{\partial t}\int F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}d\mathbf{p} = E\int F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}d\mathbf{p}$$

Similarly for $\frac{\hbar}{i}\frac{\partial}{\partial x}$ and p_x.]

Physically more precisely, but otherwise analogously, the Dirac equation implies a spreading out over relevant domains of space of a linearized form of $E^2 = p^2c^2 + m^2c^4$:

$$\{p_0 - \alpha_1 p_1 - \alpha_2 p_2 - \alpha_3 p_3 - \alpha_4 mc\}\psi = 0 \tag{5}$$

As we see in Ref. [8], pp. 255-6, Eq. (5) only translates into

$$\left(\frac{E^2}{c^2} - p^2 - m^2c^2\right)\psi = 0 \tag{6}$$

by its multiplication by $p_0 + \alpha_1 p_1 + \alpha_2 p_2 + \alpha_3 p_3 + \alpha_4 mc$. The α's are 4×4 matrices here.

Further,

$$\psi(t) = \psi(0)e^{-\frac{i}{\hbar}Ht} \tag{7}$$

is the solution of Eq. (5), in which H is the Hamiltonian operator, whereas we have four components of ψ.

The gist of the above is that, in order to "distribute over space" and translate (into the wave-like data code) the structured four-vector $(i\frac{E}{c}, \mathbf{p})$ - structured by the condition $E^2 = p^2c^2 + m^2c^4$ -, *we need the essential factor* $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})} = e^{\frac{i}{\hbar}S}$, at least if we want to do everything - such as constructing a (linear!) wave equation - as simply as possible (e.g., see Ref. [9], pp. 107-11). Such factor is essential because of its role above and especially because of *its normally inevitably appearing in the solutions of the Dirac and Schrödinger equations* [e.g., compare Eq. (7)], *such equations being "inevitable" in turn* on account of considerations of simplicity and coherence. [Note that, in the case of a free particle, the complex factor $e^{\frac{i}{\hbar}S}$ in (all Fourier components of) the solution of the wave equation cannot possibly be eliminated or made real by superposition; as a rule, non-complex wave functions do not satisfy the equation.]

The above is also relevant to some role $E^2 = p^2c^2 + m^2c^4$ plays in the *Zitterbewegung*, co-defining its structure. For not only can the Dirac equation and, therefore, waves of the structure $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}$, be derived from both spherical rotation [1] and (a linearized form of) $E^2 = p^2c^2 + m^2c^4$ (as we saw above), but it also appears that the *Zitterbewegung* (spherical rotation) can be derived from $E^2 = p^2c^2 + m^2c^4$, viz. via the Dirac equation! (Compare [8], pp. 261-3.) *Our conclusion is that both* $E^2 = p^2c^2 + m^2c^4$ *and* $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}$ *have something essential to do with spherical rotation, that is, with the structure of action quanta.*

This point (2) mainly refers to the algebraic side of the relation between particles and waves.

(3) The geometric side of such relation is partly covered by points 1. and 2. of Section 1 above. Its essence amounts to the combination of

(a) an action-metrical stretching of local spherical rotation processes to action-quantal slices as discussed in Refs. [4, 5];

(b) the isomorphism "in steps": phases of spherical-rotation configurations $\rightarrow$ vectors $(\alpha,\ \beta,\ \gamma,\ \delta)$ $\rightarrow$ spinors $\begin{bmatrix} \alpha + i\delta \\ \gamma + i\beta \end{bmatrix} = \begin{bmatrix} \phi_1 e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})} \\ \phi_2 e^{-\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})} \end{bmatrix}$ $\rightarrow$ four-component (Dirac) matter waves in which $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})} = e^{\frac{i}{\hbar}S} = \cos\frac{S}{\hbar} + i\sin\frac{S}{\hbar}$ figures prominently. It is a core feature of our theory that *such mathematical isomorphism corresponds to a real physical "translation" of characteristics of a momentum carrier P from corpuscular into wave-like "language", according to experimental circumstances.* I.e., a translation of so physically real a proces as *Zitterbewegung* - each period of which constitutes a realistic four-dimensional quantum of action - into the equally real process of matter waves, each four-dimensional wave slice again embodying a quantum of action. Such wave slice corresponds to one period of $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}$.

Joining with (2) above, we see a translation into variables characterizing waves of those defining corpuscules *and* a translation of the relation $E^2 = p^2c^2 + m^2c^4$ between E, m and p into the one they obey in a wave function satisfying the Dirac equation and in which $e^{\frac{i}{\hbar}S}$ is indeed essential. As already observed, conservation of E and $\mathbf{p}$ is continued by the translations.

(4) Coming to an explanation of the $P = |\psi|^2$ probability rule, we first emphasize that wave functions satisfying wave equations can all be decomposed into Fourier components

$$F(\mathbf{p})e^{\frac{i}{\hbar}S} = F(\mathbf{p})\cos\frac{S}{\hbar} + iF(\mathbf{p})\sin\frac{S}{\hbar} \quad (8)$$

whose superposition then "constructs" the proper wave function. Eq. (8) will appear to be crucial with respect to the relevant explanation. I.e., it fits in our theory to conceive $F(\mathbf{p})\cos\frac{S}{\hbar}$ as well as $F(\mathbf{p})\sin\frac{S}{\hbar}$ *as realistic waves that are mutually coupled in a specific way by* $F(\mathbf{p})e^{\frac{i}{\hbar}S}$ *and with which, therefore, the factor* i *and the complex notation play an essential part. This being so, such realistic waves jointly carry the conserved energy (and momentum) of what in other circumstances is a corpuscule. Their being mutually coupled is irrelevant as to this.* (For simplicity, we now only consider a monochromatic wave packet.) It is a mere question of mathematics that such coupling, and calculations in which they (the $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$ waves) play a part, can be formulated in the complex language in which $F(\mathbf{p})e^{\frac{i}{\hbar}S}$ is central.

The above means that the coupled waves $F(\mathbf{p})\cos\frac{S}{\hbar}$ and $F(\mathbf{p})\sin\frac{S}{\hbar}$ "merely" distribute the energy of a spherically rotating "corpuscular" entity over space.

The waves being realistic, it is now obvious to consider their energy dependence to be the normal one for harmonic waves: $E = \frac{1}{2}Cu^2$, u being their amplitude and C a constant. The constant energy of a "monochromatic" particle in the wave state then is the sum of the energies of the cos and the sin wave, i.e., proportional to

$$F(\mathbf{p})^2(\cos^2\frac{S}{\hbar} + \sin^2\frac{S}{\hbar}) = F(\mathbf{p})^2 \quad (9).$$

That is, it is an aspect of the realistic coded-information translation that the *Zitterbewegung* transfers its energy to waves that are also normal - "classical" - with respect to their energy-related behaviour.

E.g., as to interference and its relation to energy (density), the cos and sin matter waves get comparable to water waves or the vibrations of strings. Note, however, that the i factor causes *that "cos" and "sin" waves are never mutually superposed.*

For the rest, the mutual coupling of the $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$ waves by $e^{\frac{i}{\hbar}S}$ is to be expected physically because each 2π period of such waves (say, between 0 and 2π) originates from and is a mere aspect ("projection") *of* *one indivisible, integral, action-quantal process (one period of spherical rotation)*, as appears from the derivation in [1] of the isomorphy between spherical rotation and waves as summarized above. For this reason, the Dirac and "approximating" Schrödinger equations for the free particle do not allow solutions of the $\cos\frac{S}{\hbar}$ or $\sin\frac{S}{\hbar}$ type. For separately these would "only half" represent the spherical rotation process in the relevant isomorphism, as actually appears from correct solutions' containing the factor $e^{\frac{i}{\hbar}S}$.

In such isomorphism, the coupling of the $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$ waves by the coefficient i more specifically reflects the earlier-mentioned step $(\alpha,\ \beta,\ \gamma,\ \delta) \rightarrow \begin{bmatrix} \alpha + i\delta \\ \gamma + i\beta \end{bmatrix}$ in it, where - as we saw in (3) (b) and (c) of Section *1* - α and δ correspond to the cos and sin waves, respectively, since $\alpha + i\delta$ is actually $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}$, as derived in [1], pp. 444, 446 and 447. (If it would later appear that the cos and sin "components" of $e^{\frac{i}{\hbar}S}$ are *very direct* reflections of the *Zitterbewegung*, they probably would be found to correspond to two orthogonal projections into which such process would turn out to be decomposable.)

(5) In fact, it is irrelevant to our intended explanation of the $|\psi|^2$ probability rule whether the isomorphic translation of corpuscular data into wave-like ones figuring above is direct and simple - say, a virtual spatial stretching of the *Zitterbewegung* - or, on the contrary, unrecognizably distorting and radical, such as, e.g., the way in which our optic nerve translates the image of a horse we see into electric pulses. It is irrelevant because however complicated and distorting the translation is, some things are certain:

(a) The Dirac (or Schrödinger) equation is the simplest *linear wave equation* describing the waves' evolution in time and relating E, p and m correctly ([9] and [8], pp. 255-6).

(b) Its solution for the free fermion (and most other systems) contains the factor $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})} = e^{\frac{i}{\hbar}S} = \cos\frac{S}{\hbar} + i\sin\frac{S}{\hbar}$ rather than separate waves characterized by $\cos\frac{S}{\hbar}$ or $\sin\frac{S}{\hbar}$ ([9], pp. 107-11).

(c) If – in accordance with the above – we consider the wave-like manifestation of a momentum carrier P as equally realistic as the corpuscular one – as a physically realistic isomorphic translation of corpuscular P as an information chunk –, one conclusion was the waves' carrying energy in the normal $E = \frac{1}{2}Cu^2$ way, which has also the well-known "classical" consequences for the energy at the superposition of waves. Assuming such realistic separate existence of the $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$ waves - though they are coupled, mutually integrated, by the factor $e^{\frac{i}{\hbar}S}$ *constitutes the first main step in our explanation of the* $|\psi|^2$ *rule.* Remind that the foregoing implies that only in such coupled way the $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$ waves - consisting of equally real action stuff as a *Zitterbewegung* - can figure in the isomorphic translation of spherical rotation into a wave-like data code. We can also say that the coupled appearance of the $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$ waves, as a realistic physical phenomenon, ensues from the fact that the Dirac and Schrödinger equations (for the free particle) only allow solutions of the complex $e^{\frac{i}{\hbar}S}$ type rather than solutions of a purely $\cos\frac{S}{\hbar}$ or $\sin\frac{S}{\hbar}$ character. (Mathematically equivalently, the physical coupling between the "cos" and "sin" waves is formulated by the coefficient i of the latter.) Actually, the coupling continues the integral nature of the *Zitterbewegung* process - of action quanta - into the wave state. Also remind that in Minkowski space the 2π periods of the integrated waves $e^{\frac{i}{\hbar}S}$ appear as four-dimensional slices, as is fully discussed in [3, 4, 5].

Now the second main step in explaining the $|\psi|^2$ rule *refers to how the total energy the realistic* $\cos\frac{S}{\hbar}$ *and* $\sin\frac{S}{\hbar}$ *waves jointly carry can be found via the normal complex quantum formalism.* It is simply implied by the fact that such energy (in the "monochromatic" case) is proportional to

$$\begin{aligned} &F(\mathbf{p})^2\cos^2\frac{S}{\hbar} + F(\mathbf{p})^2\sin^2\frac{S}{\hbar} \\ &= F(\mathbf{p})\left(\cos\frac{S}{\hbar} + i\sin\frac{S}{\hbar}\right)F(\mathbf{p})\left(\cos\frac{S}{\hbar} - i\sin\frac{S}{\hbar}\right) \\ &= \psi\psi^* = |\psi|^2 \end{aligned} \qquad (10).$$

Note that the first formulation, with $\cos^2\frac{S}{\hbar}$ and $\sin^2\frac{S}{\hbar}$, *is physically the most direct one – immediately joining with* $E = \frac{1}{2}Cu^2$ *for both waves –, whereas the complex alternative* $e^{\frac{i}{\hbar}S} = \cos\frac{S}{\hbar} + i\sin\frac{S}{\hbar}$ *as the basis of calculations has indeed the advantage of mathematical simplicity and expediency, though it obscured the physical essence for decades.*

Also note that it is already implied by the coded-information theory that the sin and cos waves carry all energy of an "erstwhile" corpuscule P; they are realistic energetic phenomena because of mere conservation, for the two coupled waves simply *are* P in a differently encoded form. They carry P's energy in the shape of two harmonic vibrations of mutual phase difference $\frac{1}{2}\pi$, which guarantees P's constant energy to be spread over a domain of space, in proportion to the local energies of the coupled cos and sin waves. That is, as we see from Eq. (10), *in proportion to* $|\psi|^2$. The constancy in time of a free P's energy is actually reflected here by $\sin^2 \frac{S}{\hbar} + \cos^2 \frac{S}{\hbar} = 1 =$ constant.

Realize within the above scope that the complex notation $e^{\frac{i}{\hbar}S}$ by no means implies the relevant waves to be of a "merely mathematical", non-realistic nature. It does not less refer to realistic physical processes and situations than, e.g., the four-vector $(i\frac{E}{c}, \mathbf{p})$ or $(i\phi, \mathbf{A})$, the four-potential.

(d) Within the constraints embodied by their coupling by $e^{\frac{i}{\hbar}S}$,the $\cos \frac{S}{\hbar}$ and $\sin \frac{S}{\hbar}$ wave components are normal waves that, *just like the waves in strings or water*, obey both the rule $E = \frac{1}{2}Cu^2$ and the superposition principle. [It is such very coupling that makes possible the "simplified calculation" of some quantity proportional to the waves' combined local energies which is implied by Eq. (10).] Also note that *it is implied by this obeyance that it suffices for us to explain the* $|\psi|^2$ *probability rule as far as we did - that is, up to showing that the* local energy *is proportional to* $|\psi|^2$ *- for the "monochromatic" case (for Fourier components). For, such components carrying energies proportional to* $\frac{1}{2}Cu^2$ *, and also satisfying the superposition principle like classical water waves, the "normal" waves resulting from the superposition of Fourier components will also obey the* $E = \frac{1}{2}Cu^2$ *rule and the superposition principle.*

(e) As to addition in case of superposition, the $\cos \frac{S}{\hbar}$ and $\sin \frac{S}{\hbar}$ (real and "imaginary") components act as independent waves, as can be seen from figure 2. I.e., the "superposition" of the complex vectors $v_1 = Ae^{i\omega t}$ and $v_2 = Be^{i(\omega t+\delta)}$ occurs by adding their real and imaginary components separately. See also Ref. [10], pp. 46-7.

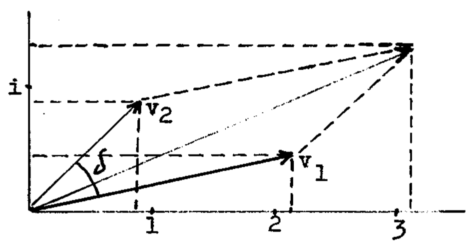

Figure 2. Addition of complex quantities

(6) Summarizing, we can say that the coupled waves $F(\mathbf{p}) \cos \frac{S}{\hbar}$ and $F(\mathbf{p}) \sin \frac{S}{\hbar}$ are action-metrically stretched, and also – possibly radically

– geometrically distorted, components of corpuscules (or: of the *Zitterbewegung* as an action-quantal periodic process) in the shape of realistic harmonic oscillators or vibrations, that carry their energy. Then, it is obvious that matter waves will act as classical harmonic ones as to both the relation of amplitude and energy and superposition. The coupling of the sin and cos waves by $e^{\frac{i}{\hbar}S}$, subsequently, allows the usual complex quantum algorithm "symbolized" by Eq. (10), coefficients i *merely mathematically reflecting the special mutual relation of the sin and cos waves.* Such coupling, *inter alia*, prevents the $\sin\frac{S}{\hbar}$ and $\cos\frac{S}{\hbar}$ waves from "freely running mutually apart", without this, in the first instance, being relevant with respect to questions of energy.

(7) It tells for the realistic coded-information theory, in coherence with the concept of action metric - that sometimes causes action quanta to "spread over space" -, that it rather simply conduces to an explanation of the $P = |\psi|^2$ rule, which remained elusive to other theories for decades. (Think of the idea of waves as mere mathematical entities referring to probabilities, of waves as guiding particles, or of them as unspecified "dual" manifestations of corpuscules.)

(8) We now proceed from the dependence of energy on $|\psi|^2$ to the dependence of the probability to find a particle on it. Imagine we have many (N) momentum carriers in, say, an interference experiment. On account of what precedes we can say then that the *energy densities* at various locations are proportional to the local squared wave amplitudes $|\psi|^2$. Then, it is obvious to conclude that the number of particles found at such locations is also proportional to $|\psi|^2$. For, if many particles are there *the particle distribution will be proportional to the energy distribution.* Now consider the case in which there is one rather than N particles. Then, because of the mutual independence of particles in the relevant kind of experiments, the probability to find the particle at some location $\mathbf{x}$ at time t will be divided by N as compared with the many-particles case, as will be the energy density everywhere. This implies that the probability, still proportional to the local energy density, will be proportional to $|\psi|^2$, too.

(9) If a relevant particle P is not free, nothing fundamental changes as to our problem; we still have Fourier components whose real and "imaginary" parts separately are superposed like water waves, such parts behaving accordingly with respect to energy and, therefore, probability.

E.g., in the square-well case of Ref. [9], pp. 115-6 the Fourier components add up to

$$\psi(x,t) = A_n \sin\frac{n\pi x}{L} e^{-\frac{i}{\hbar}E_n t} \qquad (11),$$

L being the distance between the well's "walls" and n the number of half wavelengths fitting between them. We see here that the interference producing standing waves can indeed result in a virtually real wave function, because the complex e factor is irrelevant as to the particle distribution in the waves in question. We also get an uneven particle distribution here, in contradistinction to the case of our earlier discussion about separate Fourier components.

We see the coupled "$\cos\frac{S}{\hbar}$" and "$\sin\frac{S}{\hbar}$" waves still act independently insofar that, in producing standing waves, we have

$$\begin{aligned} Ae^{i(\omega t-kx)} + Ae^{i(\omega t+kx)} &= 2A\cos kx[\cos\omega t + i\sin\omega t] \\ &= 2A\cos kx e^{i\omega t} \end{aligned} \qquad (12),$$

the $e^{i\omega t}$ factor not influencing $|\psi|^2$ and the spatial distribution of particles. Hence the "real" matter wave terms survive in the superposition whereas the "imaginary" ones virtually do not.

It is discussed in Ref. [5] how a micro-particle P's movement may indeed correspond to many Fourier components originating from both space-time and energy-momentum shifts *that do not change the action* (action-metrically infinitesimal shifts). Therefore, many point-events as well as energy-momentum situations that differ in the Minkowski scheme are demonstrated in [5] to be equivalent as to the internal action physics of the process (i.e., the moving P), this resulting in some spread - "uncertainty" - as regards P's location and momentum in such scheme.

(10) Finally, electromagnetic waves very directly visibly fit into the $E \sim P = |\psi|^2$ probability rule. For it has been theoretically demonstrated that in both electric and magnetic fields the energy is proportional to the squared field strength. Therefore, in electromagnetic waves the energies of both the electric and the magnetic "oscillations" are so, too, which may be compared with the $\frac{1}{2}Cu^2$ dependence of the energies of the matter wave components $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$. (E.g., see [10] pp. 269-71, 278-81 and 310-1.) Such analogy is made rigorous by the circumstance that the electric and magnetic field vectors in the waves linearly depend on the four-potential $(i\phi, \mathbf{A})$ - that in electromagnetic waves functions as the spinor field strength in matter wave equations and waves -, they being proportional to it. (E.g., see Ref. [11], pp. 141-4.)

3. Various additional aspects of the $|\psi|^2$ rule

(1) It is the very physical coupling of the realistic $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$ waves – of the "real" and the "imaginary" parts of the true $F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}$-like

Fourier components – that is responsible for the fact that in all calculations on wave functions we can best operate the complex "integrated" waves, the results always being correct even if we "lose sight" of the separate behaviour of the realistic cos and sin wave "parts".

(2) Continuing the realistic model of the coded-informational translation of spherical rotation into matter waves, it would be well conceivable that "the arm that moves the apple", or the "strings fixed to the core" (the fields by which the *Zitterbeweging* interacts with the environment; compare figure 1), attain their maximum "torsion" or potential energy at the 2π rotation phase of the "apple". [Compare the detailed discussion of spherical rotation in Ref. [1], and also (1) of Section *1*.] Assuming an harmonic-oscillator-like variation of such energy, it might produce the $\sin^2 \frac{S}{\hbar}$ term in the total energy of the *Zitterbewegung* and in $|\psi|^2$. (See figure 3.) The latter energy being constant in time, a $\cos^2 \frac{S}{\hbar}$ term should complete it. If (more generally) we could associate the $\sin \frac{S}{\hbar}$ and $\cos \frac{S}{\hbar}$ terms in $e^{\frac{i}{\hbar}S}$ with the potential and kinetic energies of the action-quantal process represented here by the *Zitterbewegung*, respectively, we would in any case see a very straightforward and direct physical connection between the corpuscular and wave-like data codes in which momentum carriers will manifest themselves. Also note that this model would simply explain why the "coupling" of $\sin \frac{S}{\hbar}$ and $\cos \frac{S}{\hbar}$ waves appears.

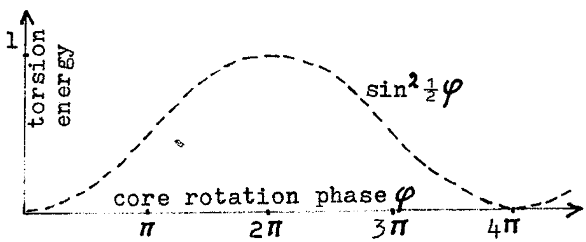

Figure 3. The torsion of the "twisted strings" as an harmonic oscillator; maximum torsion if the core rotation phase $\phi = 2\pi$ and the string phase is $\frac{1}{2}\phi = \pi$

Even more generally, our comprehensive theory suggests that it is essential in the translation corpuscle $\rightarrow$ waves that a *Zitterbewegung*-like process is decomposed into two harmonic-oscillator-like ones. *Again, this would amount to taking the mathematical formalism very seriously as to its corresponding to physical processes.* [Also compare (4) and figure 4 below, where a possible model of the decomposition in question is discussed.]

(3) In conjunction with (2) of Section *2* we see an elementary logic and simplicity in the Schrödinger and Dirac "translation" of $E = \frac{p^2}{2m}$ and $E^2 = p^2c^2 + m^2c^4$ into waves. Viz. $i\hbar\frac{\partial}{\partial t}$ directly refers to E, whereas $-\frac{\hbar^2}{2m}\Delta\psi$ logically represents $\frac{p^2}{2m}$, on the understanding that $F(\mathbf{p})e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}$ is the basic - and wavelike - solution of an optimally simple linear wave equation correctly interrelating E, p and m, which actually happens to be the case. (Compare Refs. [9], pp.107-12, [10], pp. 63-5, [12], pp. 418-20 and [8], pp. 254-7.)

Within this context, note the difference with "mechanical" wave equations - such as for vibrating strings -

$$\frac{\partial^2 y}{\partial t^2} = w^2\frac{\partial^2 y}{\partial x^2},$$

where w is the velocity of propagation and $F = ma$ is at the basis of the derivation ([9], pp. 107-8). In the one case, $\frac{\partial\psi}{\partial t}$ is based on the structure of the action-quantal process as an element of the system – which element is closely connected with both (a) $E^2 = p^2c^2 + m^2c^4$ and (b) $e^{\frac{i}{\hbar}(Et-\mathbf{p}.\mathbf{r})}$ –, in the other case $\frac{\partial^2 y}{\partial t^2}$ refers to $F = ma$ applied to quite another kind of element (viz. a string section) of the system. $e^{\frac{i}{\hbar}S}$-like and $\cos(\omega t - kx)$-like solutions are the result, in the respective situations. That is, the coupled sin and cos waves in the quantum-mechanical one, implied by the complex solution of the wave equation, in the last resort ensue from the structure of the quantum of action as a process, associated with formulas (a) and (b). Linearized (a) makes the $\frac{\partial\psi}{\partial t}$ term necessary as coupled with $\frac{\partial^2\psi}{\partial x^2}$, thus enforcing (b)-like waves.

Note that not only spherical rotation (that essentially describes mathematically such quanta of action as embody corpuscular momentum carriers) leads to fermion matter waves (Dirac spinor waves), but that also conversely the latter - via the Dirac Hamiltonian which defines his equation - lead to the *Zitterbewegung* and spin that, in turn, embody spherical rotation physically realistically. (See Ref. [8], pp. 261-3.)

Again realize that our above discussion highly amounts to taking the quantum formalism so much serious that we conceive it as a description of realistic physical phenomena.

(4) A special case of the spherical rotation Ref. [1] considers - and whose phases, as we saw, correspond to vectors $\mathbf{V} = (\alpha, \beta, \gamma, \delta)$ satisfying $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = 1$ - is the one reflected by $\alpha^2 + \delta^2 = 1$, $\beta = \gamma = 0$.

Varying α and δ, $\mathbf{V}$'s endpoint describes a big circle dividing the hypersurface of the earlier-mentioned four-dimensional Euclidean sphere into equal halves. Such variation now corresponds to the pure state

$$\begin{bmatrix} \alpha + i\delta \\ 0 \end{bmatrix} = \begin{bmatrix} e^{\frac{i}{\hbar}(Et - \mathbf{p}.\mathbf{r})} \\ 0 \end{bmatrix}$$

of spin $s_z = \frac{1}{2}\hbar$, or $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$. In the complete isomorphism (between spherical rotation and matter waves) at stake, α and δ, as projections of $\mathbf{V}$ on the two axes of figure 4, correspond to $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$, respectively. This makes the latter more imaginable in their playing a part in the isomorphism as a whole, which much clarifies such isomorphism. [We saw already in (5), (d) of Section *2* that generalizing pure states to the case of superposed ones makes no difference as regards energy and, therefore, the explanation of the $|\psi|^2$ rule. See Ref. [5] for a micro-realistic explanation of the appearance of mixed states at all.]

For the rest, we saw earlier that it makes no difference for our explanation of the $|\psi|^2$ probability rule whether the isomorphism is either simple and imaginable or not, e.g., whether the $\cos\frac{S}{\hbar}$ and $\sin\frac{S}{\hbar}$ waves correspond to recognizable sub-processes or energies of the spherical rotation. Note that in particular the essence of our explanation - viz. that the cos and sin waves carry the energy corresponding to such rotational process in a "normal" harmonic-oscillator way - amounts to taking the formalism and the waves figuring in it very seriously. (Remind that, in our general theory, they simply "organize" the elements of the universe - action quanta - in a non-corpuscular way, the quanta being stretched by the discrepancy between Minkowski and action metric, and physical data being encoded in a wave-like mode, but still coherently and efficiently.)

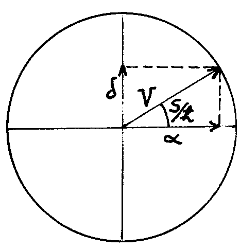

Figure 4. Big circle going with $s_z = \frac{1}{2}\hbar$ in the coded-information isomorphism; as soon as $S = h$, $S/\hbar = 2\pi$

Actually, we can see the degree of mathematical complication of the isomorphism between nature's two main data codes - corpuscular and wave-like - from Ref. [1], pp. 441-2, 444 and 452-5. It indeed appears to be rather complicated; still, physical simplicity and directness might hide behind the relevant mathematics.

(5) It might be objected to our explanation of the $|\psi|^2$ rule that it implies super-luminal velocities for energy at the contraction or collapse of realistic, energy-carrying wave packets as conceived above. In order to obviate such criticism, we have to go somewhat further into the nature of action metric as a means of coordinating four-dimensional reality. Our general theory [2, 3, 4, 5] contains here two particularly relevant points:

(a) The four-dimensional world is real, including its future parts; (micro-)physical processes such as interactions proceed - better: exist - as more-than-locally integrated wholes, final situations pre-existing and even partly "casting their shadows before" in what physically are called retroactive effects;

(b) Action metric is the proper, physically most relevant one. E.g., if a momentum carrier, amount of energy or field strength E manifests itself at both point-event A and point-event B, A and B having an action distance 0, *it might very well be that, speaking in the Minkowskian framework or ordering scheme S, E is "properly" at A rather than B. That is, if we, say, after E's absorption at point-event C by an instrument, construct E's world-line to C, such line appears to have passed through A rather than B. Still, A and B having an action distance zero, they are properly physically contiguous. This means that E manifests itself at B as well as A.*

Well, (a) and (b) conduce to the conception that in a real physical sense the appearance of matter (or electromagnetic) waves, and that of "nonlocal" phenomena in general, *is merely due to a discrepancy between Minkowskian and action metric.* Hence, though speaking in Minkowskian terms a wave-like momentum carrier M and its "widely spread" energy may *manifest* themselves at mutually distant points P and Q on a same wave front – viz. because P and Q have an action distance zero –, such energy is actually (that is, action-physically) "compact", concentrated. If, at M's subsequent impact at some point O, its "reconstructed" world-line happens to pass through P rather than through Q, *we may very well assume that, in the four-dimensional pre-existing world, the energy "on its way" to O passed P rather than Q,* *though, for all physical purposes (such as interference), it also* manifested *itself at Q because of the latter's action-physical contiguity to P.* Hence, more generally, what we call the ("instantaneous") collapse of a wave packet is a mere question of "optical illusion": what actually occurs is that, at and after the impact, *distant points (distant in the Minkowskian sense) no longer have action-metrical distances zero to the "proper" location of the momentum carrier and its energy* (as explained in the above References, where we treated such collapse in micro-realistic terms).

The above elucidates that neither our explanation of the $|\psi|^2$ rule, nor the "collapse" of wave packets, nor "nonlocal" phenomena in a more general sense, ever imply the instantaneous transmission of energy, mass or momentum (which would also violate special relativity).

(6) Finally mind that, in our argument, we do not consider a momentum carrier P's energy density H (as distributed over space) *in general* to be proportional to $|\psi|^2$, but *only the specific part of H stemming from P's invariant rest mass* (from the *Zitterbewegung* in its rest system). That is, arguing about P,

(a) in $H = \sum P_i \dot{\psi}_i - L$ the term $\sum P_i \dot{\psi}_i$ is now irrelevant, whereas

(b) in the Dirac Lagrangian

$$L = -\hbar c \psi^* \left(\gamma_\mu \delta_\mu + \frac{m_0 c}{\hbar} \right) \psi$$

only the term $-\hbar c \psi^* \frac{m_0 c}{\hbar} \psi$ is considered. Then we see only $m_0 c^2 \psi^* \psi$ remain as mattering to our problem, $\psi^* \psi$ "distributing" P's rest energy $m_0 c^2$ over space.

Note that $|\psi|^2$ is separately relativistically invariant, so that our argument, thought referring to P's rest system, also holds in other inertial systems.

In the foregoing, we actually discussed an imaginable model of the wave function ψ, and of $|\psi|^2$. Now remind that the Lagrangian density L, too, represents a realistic physical quantity, viz. a *"now section" (of dimension energy density) of the four-dimensional action density.* This corresponds to $W = \frac{1}{ic} \int L dx\, dy\, dz\, d(ict)$, W being the action. As a consequence, we may strongly suspect that, in the expressions L and the Noether quantity

$$\frac{1}{ic} F_4 = \frac{1}{ic} \int \left[\left(L \delta_{4\nu} - \frac{\partial L}{\partial \partial_4 \psi_\alpha} \partial_\nu \psi_\alpha \right) \delta x_\nu + \frac{\partial L}{\partial \partial_4 \psi_\alpha} \delta \psi_\alpha \right] \alpha V$$

{see Ref. [12], p. 220, Eq. (1.11)}, terms other than the one proportional to $|\psi|^2$ *will also appear to represent imaginable realistic physical quantities.*

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