Does The Non-Locality in Discrete-Time Quantum Theory Violate the Principle of Equivalence

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ABSTRACT. By considering a free-relativistic particle and calculating the quantum mechanical free particle energy according to discrete time quantum theory, we demonstrate that due to the non-locality of the theory the principle of equivalence is violated because the inertial mass and passive gravitational mass receive different corrections due to the non-locality.

RÉSUMÉ. En considérant une particule libre au sens de la relativité et en calculant l'énergie d'une particule libre selon la mécanique quantique en temps discret, nous montrons qu'en vertu de la non-localité de la théorie, le principe d'équivalence est violé parce que la masse inerte et la masse gravitationnelle passive sont soumises à des corrections différentes dues à la non-localité.

1 Introduction

A very important question that is oft forgotten in this age of quantum field theory and string theory is whether or not the "principle of equivalence" which is at the basis of general relativity survives corrections induced by radiative effects. In its original form the weak equivalence principle stated that the inertial mass should be equal to the passive gravitational mass for all particles thus allowing us to view physics in a locally Lorentz frame of reference [1]. The equivalence of inertial and gravitational mass has been demonstrated within one part in 10^{12} by the famous Eötvos experiment [2] [3]. In its original form, no account has been made for the interaction of the particle with the surrounding space-time or its own radiative cloud of virtual particles. It is well known that certain torsion theories violate the principle of equivalence and so does a class of non-symmetric gravitational theories [4] [5]. After its original formulation the weak equivalence principle became embedded in the Einstein equivalence principle which states that all bodies accelerate at the same rate in a gravitational field and local Lorentz invariance and local position invariance are also respected. Local Lorentz invariance insists that all locally Lorentz frames cannot make any distinction in non-gravitational experiments, the "Hughes-Denver" experiment tests for the anisotropy of space by looking for a difference in the frequency for transitions between the $Li(J = 3/2)$ angular momentum states, and has tested local Lorentz invariance to a great degree of accuracy [6]. Local position invariance has been tested by looking for any spatial or temporal variation of the fundamental constants with negative results to date. When quantum field theory is taken into account, Adler et. al. [7] have demonstrated that at zero temperature the inertial and gravitational masses receive equal corrections from radiative effects thus establishing the correctness of the equivalence principle in zero temperature quantum field theory.

However, at finite temperatures, Donoghue et. al. [8] have demonstrated that the inertial and gravitational masses receive different corrections thus leading to a violation of equivalence principle. This violation can be traced to the non-Lorentz invarient vacuum. In previous studies [9] [10] [11] [12] we have discussed a non-local discrete time difference quantum theory essentially expressing the uncertainty of tuning between the particles sense of space-time and a surrounding frame of synchronous observers. In this note, by calculating the quantum corrected free energy of a particle we demonstrate that the effective passive gravitational mass and inertial mass receives different corrections due to the non-locality of the theory. Such a result may have profound consequences in formulating fundamental geometric theories of space-time as well as effecting Hawking radiative process for black holes. After a brief calculation we discuss the implications of these equivalence principle violations.

2 Violations of Equivalence Principle by Discrete Time Quantum Theory

In (Ref. 9, 10, 11, 12) we have discussed the fundamental generic motivation for a discrete time difference quantum mechanics based on a microscopic uncertainty principle allowing for an uncertain tuning between the particle's frame and the frame of synchronous observers, the generalized Schrödinger equation reads

$$\mathbf{H}\Psi = i\hbar\left(\frac{\Psi(t+\frac{\tau}{2}) - \Psi(t-\frac{\tau}{2})}{\tau}\right)\Psi \tag{2.1}$$

$\tau =$ discrete time interval.

We now consider the bare relativistic free particle hamiltonian,

$$\mathbf{H} = \sqrt{P^2c^2 + m^2c^4}$$

Eq. (2.1) gives with

$$\mathbf{P} \to \frac{\hbar}{i}\frac{\partial}{\partial x}$$

($m =$ uncorrected free particle mass),

$$\left(\sqrt{-\hbar^2c^2\frac{\partial^2}{\partial x^2} + m^2c^4}\right)\Psi = i\hbar\left(\frac{\Psi(t+\frac{\tau}{2}) - \Psi(t-\frac{\tau}{2})}{\tau}\right)\Psi$$
$$= i\hbar\left(\frac{e^{\frac{\tau}{2}\frac{\partial}{\partial t}} - e^{\frac{-\tau}{2}\frac{\partial}{\partial t}}}{\tau}\right)\Psi = \frac{2i\hbar}{\tau}\sinh\frac{\tau}{2}\frac{\partial}{\partial t}\Psi \tag{2.2}$$

Inverting the operator in Eq. (2.2) gives

$$\frac{\partial}{\partial t} = \frac{2}{\tau}\sinh^{-1}\left(-\frac{\tau i}{2\hbar}\sqrt{-\hbar^2c^2\frac{\partial^2}{\partial x^2} + m^2c^4}\right)$$

or

$$i\hbar\frac{\partial\Psi}{\partial t} = \frac{2i\hbar}{\tau}\sinh^{-1}\left(-\frac{\tau i}{2\hbar}\sqrt{-\hbar^2c^2\frac{\partial^2}{\partial x^2} + m^2c^4}\right)\Psi \tag{2.3}$$

Eq. (2.3) represents the discrete time difference amended wave equation, we note that Eq. (2.3) is highly non-local due to the infinite tower of derivatives on the right hand side. Actually, it is non-local in space and time. We now consider a free quantum particle represented by the wave function

$$\Psi = Ae^{\frac{i}{\hbar}(Px-Et)} \tag{2.4}$$

When Eq. (2.4) is substituted into Eq. (2.3) we have

$$E = \frac{2i\hbar}{\tau}\sinh^{-1}\left(-\frac{\tau i}{2\hbar}\sqrt{c^2P^2 + m^2c^4}\right) \tag{2.5}$$

using

$$\sinh^{-1}(x) \simeq x - \frac{x^3}{3!}$$

Eq. (2.5) becomes [13]

$$E =\simeq (c^2P^2 + m^2c^4)^{\frac{1}{2}} + \frac{\tau^2}{24\hbar^2}(c^2P^2 + m^2c^4)^{\frac{3}{2}} \tag{2.6}$$

Here we assume

$$\frac{E\tau}{2\hbar} << 1(m^2c^4 > P^2c^2)$$

Rewriting Eq. (2.6) gives

$$E = mc^2\left(1 + \frac{P^2}{m^2c^2}\right)^{\frac{1}{2}} + \frac{\tau^2m^3c^6}{24\hbar^2}\left(1 + \frac{P^2}{m^2c^2}\right)^{\frac{3}{2}} \cdots$$

or in the non-relativistic approximation

$$E \simeq mc^2\left(1 + \frac{P^2}{2m^2c^2}\right) + \frac{\tau^2m^3c^6}{24\hbar^2}\left(1 + \frac{3P^2}{2m^2c^2}\right)$$

or

$$E \simeq mc^2 + \frac{\tau^2m^3c^6}{24\hbar} + P^2\left(\frac{1}{2m} + \frac{mc^4\tau^2}{16\hbar^2}\right) \tag{2.7}$$

$$E = c^2\left(m + \frac{\tau^2m^3c^4}{24\hbar^2}\right) + P^2\left(\frac{1}{2m} + \frac{mc^4\tau^2}{16\hbar^2}\right) \tag{2.8}$$

In Eq. (2.8) we identify the first group of terms as the discrete time corrected passive gravitational mass which will couple to gravity independent of the momentum.

$$m_G = m + \frac{\tau^2m^3c^4}{24\hbar^2} \tag{2.9}$$

The second group of terms in Eq. (2.9) we identify with the discrete time corrected inertial mass

$$\frac{1}{2m_I} = \frac{1}{2m} + \frac{mc^4\tau^2}{16\hbar^2}$$

or

$$m_I = m - \frac{m^3 c^4 \tau^2}{8\hbar^2} \tag{2.10}$$

to order τ^2.

We see from Eq. (2.9) that the passive gravitational mass increased due to discrete time effects and the inertial mass is decreased accordingly to Eq. (2.10). By the limit established in the Eötvos experiment we have

$$\frac{\Delta m}{m} \simeq \frac{m_G - m_I}{m} \simeq \frac{1}{6}\frac{m^3 \tau^2 c^4}{\hbar^2 m} \simeq 10^{-12}$$

For an electron we have

$$\tau^2 \simeq \frac{6(10^{-54})}{10^{-54}}\left(\frac{10^{-12}}{10^{-42}}\right) \simeq 6 \times 10^{-54} \quad , \quad \tau \simeq 2.4 \times 10^{-27\prime\prime}$$

Thus any greater value of τ would lead to explicit violations of the equivalence principle set by the Eötvos experiment (Ref. 2).

Conclusion

From the above identification of the inertial mass and effective passive gravitational mass that receive different corrections from the discrete time difference quantum formalism, we see that the non- locality embodied in Eq. (2.3) has generated a violation of the equivalence principle that is in an opposite sense from that calculated in finite temperature field theory (Ref. 8). Since the non-locality has distributed a self-interaction over a finite spatial interval, it is not unexpected that there should be an equivalence principle violation.

In its original form the equivalence principle pictured a particle as separate from the surrounding space-time with no wave particle self- interaction. The present note suggests that these non-local self-interactions can violate the equivalence principle without any radiative considerations. With regard to black hole evaporation [14] it is known that the Hawking process requires a local Lorentz frame to view the decrease of the energy in the quantum field as representing an outgoing stream of Hawking radiative particle-anti-particle pairs. If this frame does not exist due to a violation of the equivalence principle at a fundamental level, then the entire Hawking process might be suspect.

Another alternative is that a fundamental geometric theory of space-time might be independent of an equivalence principle formulation for assumed point particles but rather based on a general theory of covariance with scalar, vector, spinor and tensor matter fields being the generic representation of matter without any geodesic condition for point-like particles.

Whatever the fundamental structure of space-time is at a primitive level, certainly finite temperature quantum field theory, torsion theory, non-symmetric gravitational theory and non-local quantum effects have given us enough reason to question the equivalence principle as a foundation of GR with possibly more geometric notions replacing the "principle" as the cornerstone of general relativity.

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