Wave-particle association: uniqueness of the de Broglie assumption

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ABSTRACT. It is shown, in contrast with the two- or three-wave hypothesis proposed by some authors, that Lorentz invariance implies the de Broglie assumption in associating waves to particles.

RÉSUMÉ. On montre, en opposition avec l'hypothèse à deux ou trois ondes proposée par certains auteurs, que l'invaraince de Lorentz implique l'hypothèse de de Broglie d'association entre ondes et particules.

Louis de Broglie solved the particle - wave dualism by setting, for a free, spinless particle,

$$\omega = \frac{E}{\hbar}, \qquad \mathbf{k} = \frac{\mathbf{p}}{\hbar}, \tag{1}$$

where E and $\mathbf{p}$ are, respectively, the energy and momentum of the particle, while ω and $\mathbf{k}$ correspond to the angular frequency and wavenumber of the wave. From statement (1) it follows

$$\mathbf{v}_{particle} = \frac{c^2\mathbf{p}}{E} = \mathbf{v}_{group}. \tag{2}$$

In more recent times some authors[1-4] proposed to associate other kinds of waves to a free particle. In particular Das[2] suggested a complementary view to the de Broglie wave, that is, the "transformed Compton wave", by identifying

$$\omega = \frac{c|\mathbf{p}|}{\hbar}, \qquad \mathbf{k} = \frac{E\mathbf{p}}{\hbar c|\mathbf{p}|}, \tag{3}$$

which implies

$$\mathbf{v}_{particle} = \mathbf{v}_{phase}. \tag{4}$$

Das stresses that for a massive particle the wavelength so defined reduces to the usual Compton wavelength in the particle rest frame, where the de Broglie wavelength becomes infinite. Moreover he asserts that identification (3) can be done simultaneously in any Lorentz frame, i. e., that such an identification is covariant. He also gives arguments in favour of this assertion, moreover he rules out the "dual wave" proposed by Horodecki[1], showing that such a dual wave cannot be defined simultaneously in any reference frame.

In this letter we show that the "transformed" Compton wave cannot be associated covariantly to a massive particle and that the de Broglie choice is the only possible one.

Relativistically, a plane scalar wave is represented by the function

$$\psi \quad \propto \quad exp[i(\omega t - k_i x^i)], \tag{5}$$

in such a way that its phase is Lorentz invariant, that is, in such a way that the four quantities

$$k_0 = \frac{\omega}{c}, \qquad k_1, \qquad k_2, \qquad k_3$$

transform as the components of a four-vector under Lorentz transformations. Then

$$k^\nu k_\nu = \frac{\omega^2}{c^2} - \mathbf{k}^2 = \mu \tag{6}$$

is a Lorentz invariant quantity. By differentiating the dispersion relation (6), we get the group velocity, i. e.,

$$\mathbf{v}_{group} = \nabla_{\mathbf{k}}\omega = \frac{c^2}{\omega}\ \mathbf{k}, \tag{7}$$

whereas the phase velocity is given by

$$\mathbf{v}_{phase} = \frac{\omega}{\mathbf{k}^2}\mathbf{k}. \tag{8}$$

For $\mu > 0$ we have $|\mathbf{v}_{group}| < c$ and $|\mathbf{v}_{phase}| > c$, whereas for $\mu < 0$ the two inequalities are inverted.

Now we try to associate covariantly the scalar plane wave (5) to a free, spinless particle of mass $m > 0$. At first sight we have two possibilities:

i) According to de Broglie, we may identify the velocity of the particle, which is given by the first eq. (2), with the group velocity of the wave, see eq. (7). Such a comparison yields

$$\mathbf{p} = \sigma \mathbf{k}, \qquad E = \sigma \omega, \tag{9}$$

where σ is a Lorentz invariant quantity, which is found experimentally to be the same for all particles, i. e., $\sigma = \hbar$. In this case $\mu > 0$, therefore the group velocity of the wave is less than c, as expected. Eq. (9) is covariant, of course, since the (time-like) four-vector k_ν is proportional to the four-momentum of the particle. This equation implies the Klein - Gordon equation, i. e.,

$$\Box \psi = \frac{m^2 c^2}{\hbar^2} \psi. \tag{10}$$

ii) Alternatively we may identify the velocity of the particle with the phase velocity of the wave, eq. (8). In this case we set

$$E = \sigma' c |\mathbf{k}|, \qquad \mathbf{p} = \sigma' \, \frac{\omega \mathbf{k}}{c|\mathbf{k}|}, \tag{11}$$

where σ' is again a universal constant, to be determined experimentally; this constant has been defined in such a way to have the same dimensions as in eq. (9). From (11) we get

$$k_0 = \frac{\omega}{c} = \frac{|\mathbf{p}|}{\sigma'}, \qquad k_i = \frac{E p_i}{\sigma' c |\mathbf{p}|}. \tag{11'}$$

In this case $\mu < 0$, that is, the four-vector k_μ is space-like; therefore, as expected, the phase velocity is less than c. If we assume again $\sigma' = \hbar$, eqs. (11') turn out to coincide with eqs. (3); moreover eqs. (11) imply the dual Klein - Gordon equation, i.e.,

$$\Box \psi = -\frac{m^2 c^2}{\hbar^2} \psi. \tag{12}$$

This equation was studied firstly in 1910 by Ehrenfest[5], who showed in detail that the signal velocity - that is, the physically measurable quantity - is really the phase velocity, which we have proved to be less than c. Furthermore an equation of the type (12) may represent the equation for a wavepacket that describes an off-shell exchanged particle (both massless and massive) in the crossed t-channel; the massless case has been illustrated in detail by Barut and Chandola[6] (see also [7]). However in this case m is a parameter characterizing the wavepacket, not the real mass of the particle.

However the solution of such an equation cannot be associated to a free particle of mass m in any Lorentz frame, that is, eqs. (11') are not covariant. Let us consider a Lorentz transformation from a given inertial reference frame a to another inertial frame b and let $\Lambda_{\nu\rho}$ be the matrix that describes such a transformation in the Minkowski space-time. According to the transformation of the four-momentum $p_\nu \equiv (E, p_i)$ of the particle, the quantities on the r.h.s. of eqs. (11') transform as

$$k_0 = \frac{|\mathbf{p}|}{\sigma'} \qquad \longrightarrow \qquad \frac{(\Lambda_{i\nu}\Lambda_{i\rho}p^\nu p^\rho)^{\frac{1}{2}}}{\sigma'} \tag{13}$$

and

$$k_i = \frac{E}{c\sigma'|\mathbf{p}|}p_i \qquad \longrightarrow \qquad \frac{\Lambda_{0\alpha}p^\alpha \Lambda_{i\beta}p^\beta}{c\sigma' \ (\Lambda_{i\nu}\Lambda_{i\rho}p^\nu p^\rho)^{\frac{1}{2}}}, \tag{13'}$$

which generally do not coincide with the Lorentz-transformed quantities on the l.h.s. of (11'), i. e. with

$$k'_\nu = \frac{|\mathbf{p}|\Lambda_{\nu 0}}{\sigma'} - \frac{E\Lambda_{\nu i}p^i}{c\sigma'|\mathbf{p}|} \tag{14}$$

Expressions (13)-(13') turn out to coincide with (14) only in the case when a Lorentz boost in the direction of the momentum $\mathbf{p}$ of the particle is considered.

As a conclusion, we have shown that, contrarily to what asserted by some authors, the only possible covariant association of a wave to a particle is the one assumed by de Broglie.

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