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# An Oscillator Naturally Satisfying Lorentzean Properties

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ABSTRACT. In his 1923 paper de Broglie suggested the presence of a small clock inside the particle, which is run by the self energy but does not contribute to momentum of the particle. He further indicated that its energy is similar to that of a heat-containing body in an internal state of equilibrium. The author examines aspects of an oscillator which satisfies many of such requirements, and show how it can generate a wavy trajectory with a superpositioning of a precession orbit.

*RÉSUMÉ. Dans son article de 1923 de Broglie suggérait la présence d'une petite horloge à l'intérieur de la particule, fonctionnant grâce à son énergie propre mais ne contribuant pas à son impulsion. Il indiquait en outre que son énergie est similaire à celle d'un corps contenant de la chaleur et en équilibre interne. L'auteur étudie un oscillateur qui satisfait à un grand nombre de ces exigences, et montre comment il engendre une trajectoire sinueuse se superposant à une orbite avec précession.*

## 1 Introduction

To make the argument of double solution and guidance wave workable as proposed by de Broglie, it is necessary to introduce a generator of the $u$-wave with a prescribed wavelength and phase, over which the point of singularity must ride for the particle position [1]. In his 1923 paper de Broglie discussed a possibility of the presence of a small internal clock, which was already at the very origin of wave mechanics, and suggested that the clock may be driven by the self energy of the particle somewhat similar to its internal heat [2]. However, any conventional clock or oscillator of known designs consumes energy, and therefore the suggested internal heat will be eventually dissipated. Once the internal heat is exhausted, the particle is expected to lose its means for generating the $u$-wave and the double solution should collapse.

The author proposes an internal oscillator of a given electrical charge of the particle under consideration propagating in case of the static particle along a circular orbit of an extremely small radius at the speed of light. An evaluation of the self energy of such an oscillator is found to be zero. In other words, such an oscillator can exist permanently as an internal structure of each particle. When the particle is in motion, the elongated corkscrew path of the charge propagation naturally satisfies Lorentz transformation. This is an extremely favorable finding for the consideration of modeling certain spin$-1/2$ particles, viz., charged leptons and quarks, in sub-particle level giving logical support for naturally defined particle spin and other inherent properties known in quantum mechanics.

## 2 An oscillator inside the particle

The photon in vacuum 3-space is the mediator of the electromagnetic force propagating at the speed of light $c$. It is plausible to stipulate that the electrical charge of a given static particle propagates at the same speed $c$ in the confined space internal to the particle unless some strong force constraints are imposed by the particle structure. Since observations of experimental physics confirm that no internal structure is known to exist in charged leptons and quarks, it is safe to postulate that the charge must satisfy the stipulated condition. The only possibility the author can then visualize is that in case of static particle the charge propagates along a circular orbit internal to the particle shell of some kind.

Experimental physicists know that the particle charges are somehow protected and not bare. If the charge propagates along an orbit, as proposed here, then the closed loop generates a spherical magnetic shell, very much like that of geomagnetosphere, enclosing the loop inside. The charge of a static particle will then be unexposed and protected by the magnetic shell. Physicists also know that at least charged leptons show point-like structures with no extension, very much like the mathematically postulated points in pure abstraction, when they collide. Since particles colliders provide relativistic particle velocities, the propagation paths of such internal oscillators form far extended corkscrews of nearly linear shape in the laboratory frame. The magnetic shells then cannot be observed in the laboratory frame. This will make the particles appear to consist nothing but of the point charges.

When the particle defined in this way is in a linear motion, the charge does not stay on the corkscrew path stretched out from the orbits of the static particle. Instead, the motion of the charge produces a magnetic field and yields magnetic moments which causes a precession. Superpositioning of the precession orbit produces a wavy trajectory of wavelength proportional to de Broglie wave. This explains the validy of $u$-wave.

The last question remains as to whether Maxwell equations will hold at the microscopic dimensions of quarks and charged leptons. The answer is affirmative as explained next. Ashtekar introduced loop metric and demonstrated that singularities experienced in the standard formulation inside a black hole or in a domain of high energy condensation could be avoided, and that Einstein's equations hold down to Planck scale of $10^{-33}$ cm [6,7]. Ashtekar and Rovelli then showed quantum Maxwell field formulated using loop physics [9]. Their findings support the validity and applicability of Maxwell equations in the standard formulations as practiced in classical physics down to Planck scale. In the following development the standard formulation in 3-space is used. The propagation of an electric charge along a circular orbit forms a current loop as previously envisioned [10].

## 3 The current loop model.

Consider a static current loop of radius $a$ and of an electric charge $q$ propagating at the velocity $c$. The loop generates a magnetic moment $\mu = qc\pi a$ with the current $i = qc/a$ at the angular velocity $c/a$ with the loop area of $\pi a^2$. Suppose the loop propagates the charge counterclockwise on the $(x, y)$-plane of an Euclidean 3-space with its loop center located at the origin.

The electric field surrounding the current loop is then given by

$$\mathbf{E}_0 = \frac{qR^{-3}}{4\pi\varepsilon_0}(x, y, z) \tag{3.1}$$

where $R^2 = x^2 + y^2 + z^2$ with the electrical vector pointing radially from the origin to each surrounding coordinate position $(x, y, z)$. The current loop also generates a magnetic shell defined by

$$\begin{aligned}\mathbf{H}_0 &= \frac{qaR^{-5}}{8\varepsilon_0 c}(3xz, 3yz, -R^2 + 3z^2) \\ &= \frac{qaR^{-3}}{4\pi\varepsilon_0}(3\cos\phi\cos\theta, 3\sin\phi\cos\theta, -1 + 3\sin\theta)\end{aligned} \tag{3.2}$$

at $(x, y, z)$, cf. p. 14-8 of Feynman's notes [11], or at $(R, \phi, \theta)$ using a polar coordinate expression. Note that the $z$-component has the $(-1)$-term in the polar coordinates identifying the magnetic flux of strength $R^{-3}$ at distance $R$ from the origin, pointed perpendicular to the loop. The magnetic shell consists of multiple layers, each layer with strength proportional to $R^{-3}$. As the generic form of the model particle, the flux component should represent an electromagnetically defined spin. Weisskopf describes Schrödinger's notion of an irregular circular fluctuation movement (Zitterbewegung) as a possible explanation for the electron spin [12]. The present model has instead a built-in circular current loop located inside a magnetic shell. Two spin orientations are defined as "parallel" and "antiparallel" orientations with respect to an external magnetic field. The definition of the helicity (right, or left handed screw along the direction of the particle movement) results empirically from variations of the precession orbit of the model particle discussed in Section 4.

By examining (3.2), it is seen that, if $q > 0$, the flux is along the positive $z$-axis, and individual magnetic field vectors deviate away from the $z$-axis outward for positive $z$, point downwards near and on the equatorial $(x, y)$-plane, and inward towards the $z$-axis for negative $z$ eventually converging back towards the spin flux direction along the $z$-axis, thus forming circulation patterns of the field vectors on each layer very much like that of geomagnetosphere. If $q < 0$, the magnetic shell is inverted.

The total magnetic moment acting on the static current loup must be computed by

$$p = \int [\mathbf{E}_0, \mathbf{H}_0] d\mathbf{r} \tag{3.3}$$

where $[a, b] = a \times b$ for the $\mathbf{E}_0$ and $\mathbf{H}_0$ of (3.1) and (3.2). The individual magnetic moment vectors $\mathbf{g}(\mathbf{r})$ at $\mathbf{r} = (x, y, z)$ are given by

$$\mathbf{g}(\mathbf{r}) = [\mathbf{E}_0, \mathbf{H}_0] = \frac{q^2 aR^{-6}}{16\pi\varepsilon_0^2 c}(-y, x, z) \tag{3.4}$$

showing that each $\mathbf{g}(\mathbf{r})$ is perpendicular to the radial vector $\mathbf{r}$ and parallel to the $(x, y)$-plane originating from the endpoint of the $\mathbf{r}$, and all of them pointing counterclockwise on every plane parallel to the $(x, y)$-plane. On each of these planes parallel to the $(x, y)$-plane, the integration of the $\mathbf{g}(\mathbf{r})$ becomes null due to the circular distribution pattern of the individual vectors all pointed counterclockwise. Inside the loop for $R < a$, only the spin magnetic flux exists, and the total magnetic moment becomes null, because both $\mathbf{E}_0$ and $\mathbf{H}_0$-vectors point along the same or the opposite directions, and do not contribute in total to (3.3). Therefore the total integration of (3.3) becomes null.

The above finding of the totel magnetic moments of (3.3) being null is extremely significant. This a conclusion that one hopes to find in the proposed current loop model in which the charge is propagating at the speed $c$, if possible at all, without any dissipation of energy while keeping the oscillator going. A further consideration is given in Section 7 on this subject. The current loop protected by the magnetic shell should then be able to maintain its physical entities as a static particle and keep the structure permanently without energy loss.

The total energy density $s$ contained in the compact space surrounding the current loop is computed using

$$s = \int (\mathbf{E}_0^2 + \mathbf{H}_0^2) d\mathbf{r} \tag{3.5}$$

for the electric and magnetic vectors of (3.1) and (3.2). The lower limit of the integration with respect to the variable $R$ must be set equal to the current loop radius $a$ at which (3.2) ceases to apply. Calculations yield an expression in the order of $a^{-3}$. This is the second most significant finding, implying that a high energy density, in the order of $a^{-3}$, is concentrated and always contained inside the magnetic shell.

## 4 The current loop in motion.

Consider the above defined static model particle moving freely along the $x$-axis at a uniform nonrelativistic velocity $\mathbf{v} = (v, 0, 0)$. The moving charge $\mathbf{E}_0$ induces the magnetic field

$$\mathbf{H}_v = c^{-2}[\mathbf{v}, \mathbf{E}_0] \tag{4.1}$$

for which the generated magnetic moment along the $x$-axis is given by

$$p_x = \frac{2}{3}(\frac{q}{4\pi\varepsilon_0})^2 \frac{v\varepsilon_0}{ac^2}. \tag{4.2}$$

When the expression of this (4.2) is interpreted in the classical formula of $p = mv$, the corresponding static mass $m$ of the particle in motion, as the coefficient of the $v$ in (4.2), becomes inversely proportional to the loop radius. This is consistent with the earlier findings mentioned on p. 28-3 of Feynmann, loc. cit., and can be interpreted as the one-dimensional component of the energy density $s$ sensed along the direction of the particle movement (or along the direction of the gravitational pull).

The total magnetic moment along the $y$-axis for the half space of $x > 0$ is shown to be

$$p_y(x > 0) = \frac{q^2\pi}{32\varepsilon_0 ca} \tag{4.3}$$

For $x < 0, p_y(x < 0) = -p_y(x > 0)$ holds. The current loop with $\mathbf{v}$ then experiences a precession due to the rotation of the magnetic sphere with a differential angular increment

$$d\rho = \tan^{-1}[p_y(x > 0)/p_x]dt = \frac{3\pi^2 c}{4v}dt. \tag{4.4}$$

From the definitions of Planck length $L_p = (\frac{\hbar G}{c^3})^{1/2} = 1.6 \times 10^{-33}$ cm and Planck mass $M_p = (\frac{\hbar c}{G})^{1/2} = 2.2 \times 10^{-5}$ g, one obtains $L_p M_p = \hbar/c$. Using this result, (6.4) yields

$$d\rho = m(.669 \times 10^{38})\lambda dt\,\mathrm{cm}^{-1} \tag{4.5}$$

where $\lambda = h/p$ for the particle static mass $m$. The precession orbit superimposed on top of a linear or a quasi-linear trajectory of a freely moving model particle therefore shows a wavy motion, whose wavelength must be proportional to the precession angular rate $\rho$, which is proportional to the de Broglie wavelength $\lambda$.

## 5 Wavy motion of the current loop.

To associate the above $d\rho$ with the observed wavelength, the angular velocity $\theta$ of the precession orbit must satisfy

$$\theta = 2\pi v/\lambda = 2E/\hbar$$

where

$$E = mv^2/2. \tag{5.1}$$

(Note the presence of "2" in the last expression in comparison to that of the group velocity of a wave packet). Let $\eta$ denote the radius of the precession orbit. Suppose the particle initially moves from the origin along the $x$-axis and the precession orbit lies in the $(x, y)$-plane. Then the physical de Broglie wave is defined by

$$\boldsymbol{\chi}_t = (x_t, y_t, 0) = (vt + \eta \sin \theta t, \eta \cos \theta t, 0) \tag{5.2}$$

near the origin for a linear segment of the trajectory. The temporal travel density of the $\boldsymbol{\chi}_t$ is given by the delta length of the sinusoid

$$dL(x_t, y_t) = [(dx_t/dt)^2 + (dy_t/dt)^2]^{1/2} dt \sim v(1 + \frac{\eta\theta}{v})dt \tag{5.3}$$

ignoring the second term. Note that $\eta\theta dt = \eta \sin \theta dt$, and therefore

$$\int_0^t dL(x_t, y_t) = vt + \sin \theta t \tag{5.4}$$

The $vt$ term represents the distance of the particle travel along the $x$-axis, and should be subtracted from this to obtain the temporal density. Finally, this yields the sinusoidal temporal density fluctuation of the model particle of unit charge. The above precession orbit was defined in the $(x, y)$-plane for the ease of discussion, but need not to. Variations of the precession orbit, for instance defined in $(y, z)$-plane, can introduce the helicity.

In presence of an external magnetic field, an additional field term must be added to the $\mathbf{H}_0$ in (3.2), (3.3), (3.4), and to the $\mathbf{H}_v$ of (4.1). As the result the precession orbit is significantly perturbed, and the radius $\eta$ of the precession in (5.2) becomes enlarged and discernible under microscopic inspection, e.g., seen as electron tracks on cloud chamber photographs. To produce diffraction or interference patterns, a coherent particle beam of uniform particle velocity is required. In case of the famous twin slit experiment, the insertion and on/off switching activation of an electrode or an electromagnet behind the central wall separating the two slits should affect the interference pattern due to the electromagnetic nature of (4.4) as predicted by Aharonov-Bohm [13].

## 6 Naturally defined Lorentz transformation.

The angular velocity of a static current loop with radius $a$ is given by $\omega_0 = c/a$. When the model particle moves with velocity $v$ with respect to an inertial coordinate system, the angular velocity of the moving current loop measured in the inertial frame is given by

$$\omega_v = (c^2 - v^2)^{1/2}/a \tag{6.1}$$

due to the constancy of $c$ along the stretched corkscrew propagation path. Obviously

$$\omega_v = \omega_0\sqrt{1-\beta^2} \tag{6.2}$$

where $\beta = v/c$ holds. Suppose the loop period defines the internal clock time of the current loop. The time differential of the current loop with velocity $v$ with respect to an inertial frame is then given by

$$dt_v = dt_0\sqrt{1-\beta^2} \tag{6.3}$$

In correspondence, one gets easily the coordinate relationship

$$dx_t = dx_0\sqrt{1-\beta^2}$$

and

$$x_v = \frac{x_0 - vt}{\sqrt{1-\beta^2}} \tag{6.4}$$

Using $t_v = t_0\sqrt{1-\beta^2} + x_v/c$,

$$t_v = \frac{t_0 - vx_0/c^2}{\sqrt{1-\beta^2}} \tag{6.5}$$

is obtained. As deduced from (4.2), the static mass $m_0$ of the current loop is proportional to the $\omega_0$, and therefore the one moving at speed $v$ should satisfy

$$m_v = m_0/\sqrt{1-\beta^2} \tag{6.6}$$

Here the Lorentz transformation is naturally defined and derived from the model stipulation.

## 7 Null self force and no radiation of the current loop.

The revolving point charge moving along the $x$-axis at a constant velocity $v$ shows obviously an electric oscillator defined in the $(x, y)$-plane, where $x = vt + a\sin\omega t, y = a\cos\omega t, \omega = \omega_0$ of (6.2), and therefore the point charge has acceleration in both $x$ and $y$-axes. The self force function of the electron acting on itself is given by Eq. (28.6) on p. 28-6 of Feynman [11] for a single axis component as

$$F = \alpha \frac{e^2}{ac^2}\ddot{x} - \frac{2}{3}\frac{e^2}{c^3}\dddot{x} + \gamma\frac{e^2 a}{c^4}\ddddot{x} + \ldots \tag{7.1}$$

where $e = q/4\pi\varepsilon_0$, and $\alpha$ and $\gamma$ are numerical constants of the order of unity. By inserting the derivatives in (7.1), one obtains

$$\frac{F}{a\omega^2} = -\alpha\frac{e^2}{ac^2}\sin\omega t + \frac{2}{3}\frac{e^2\omega}{c^3}\cos\omega t + \frac{\gamma e^2 a\omega^2}{c^4}\sin\omega t + \ldots \tag{7.2}$$

It is surprising to observe that, if

$$\cot\omega t = \frac{3}{2a\omega c}[\alpha c^2 - \gamma a^2\omega^2] \tag{7.3}$$

is satisfied for all $t$, the null force $F = 0$ can be achieved. The right hand side of (7.3) is not a function of $t$, and therefore $F = 0$ is satisfied only when $\cot\omega t = 0$. This yields

$$\omega = \frac{c}{a}\sqrt{\frac{\alpha}{\gamma}} \tag{7.4}$$

indicating that the state of null self energy $F = 0$ is achieved when the $\omega$ takes on the largest possible value $\omega_0 = c/a$ as postulated in the current loop model and when the effect of $\alpha$ and $\gamma$ becomes negligible. Exactly the same result is obtained for the orthogonal $y$-axis component.

That no energy is lost in the current loop is an obvious logical corollary to the charge being carried by the photon, because the photon should propagate the charge through space without dissipating its energy by definition.

An equivalent result can be obtained by demonstrating that the moving point charge at the velocity $c$ along the loop does not radiate. Jackson [14] pp. 654-657 shows a formula for evaluating the amount of radiation of a moving charge. Let $\mathbf{r}(t) = (-a(1 - \cos\mid\omega t\mid), -a\sin\mid\omega t\mid, 0)$ denote the position of the point charge at $t < 0$. Let $\mathbf{n}$ denote a unit vector from the $\mathbf{r}(t)$ pointed to the origin with $R\mathbf{n} = -\mathbf{r}(t)$, and let $\boldsymbol{\beta}$ denote the normalized velocity $\boldsymbol{\beta} = \mathbf{v}(t)/c$, then

$$R = 2a\sin(\mid \omega t \mid /2)$$
$$\mathbf{n} = a((1 - \cos \mid \omega t \mid)/R, \sin \mid \omega t \mid /R, 0)$$
$$\boldsymbol{\beta} = (\sin \mid \omega t \mid, \cos \mid \omega t \mid, 0) \quad \text{and} \quad \dot{\boldsymbol{\beta}} = (\omega \cos \mid \omega t \mid, -\omega \sin \mid \omega t \mid, 0) \tag{7.5}$$

According to Eq. (14.13) and (14.14) on p. 657, the generated fields are given by

$$\mathbf{B} = [\mathbf{n} \times \mathbf{E}]_{ret} \tag{7.6}$$

$$\mathbf{E}[\mathbf{r}, t]_{ret} = e\Big[\frac{\boldsymbol{\zeta}}{\gamma^2\eta^3R^2}\Big]_{ret} + \frac{e}{c}\Big[\frac{\mathbf{n} \times (\boldsymbol{\zeta} \times \dot{\boldsymbol{\beta}})}{\eta^3 R}\Big]_{ret} \tag{7.7}$$

where $\boldsymbol{\zeta} = (\mathbf{n} - \boldsymbol{\beta}), \eta = (1 - \boldsymbol{\beta}.\mathbf{n})$, and $\gamma = (1 - (v/c)^2)^{-1/2}$, and the subscript "ret" identifies that the quantities are evaluated at retarded time. Consider the field $\mathbf{E}$ at $t = \Delta t$ for the ease of computation. The first "velocity field" term of the $\mathbf{E}$ vanishes due to the infinity of the $\gamma$ in the denominator for $v = c$, and the second "acceleration field" term becomes infinitesimal for $\omega\Delta t$ as verified from

$$\begin{aligned} \boldsymbol{\beta}.\mathbf{n} &= a/R \\ \mathbf{n} - \boldsymbol{\beta} &\sim (- \mid \omega\Delta t \mid /2, 0, 0) \\ (\mathbf{n} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}} &\sim (0, 0, 0) \end{aligned} \tag{7.8}$$

which hold for a sufficiently small $\Delta t$. Consequently the $\mathbf{B}$ field becomes also null.

This finding is explained by the fact that the charge being carried by the photon is by itself an element of radiation, and restates that a radiating element cannot radiate by itself.

## 8 The loop radius.

The above discussion indicates the permanence of both the charge $q$ and the loop radius $a$, the latter inversely proportional to the mass of the particle (or as a single axis component of the total energy density of the current loop). Since the energy density is proportional to $a^{-3}$, a larger energy density is achieved by making the radius $a'$ smaller ($a' < a$). The currently accepted view of three generations and three flavors of quarks and three geerations of leptons may be explained in terms of contemporaneous energy density of the universe at the time of the respective individual particle creations and at the time of the shifting from one generation to the next. The ability of making a direct translation of the energy and mass relationship via the current loop radius renders the present model quite attractive to the future research.

## References

[1] L. de Broglie, Com. Rend. Acad. Sci. Paris, 177, 506, 548, 630, 1923; 183, 447-448, 1926; and 184, 273-274, 1927.
[2] L. de Broglie, *Non-Linear Wave Mechanics*, translated by A.J. Knodel & J.C. Miller, Elsevier Publ. Co, New-York, 1960.
[3] L. de Broglie, *The Current Interpretation of Wave Mechanics*, Elsevier Publ. Co, New-York, 1964.
[4] L. de Broglie, d'Espagnat Edit. "Il Corso", 346-367, Academic Press, NY 1971.
[5] L. de Broglie, (English translation) Ann. de la Fond. L. de Broglie, 12, 399-421, 1987.
[6] A. Ashtekar, Phys. Rev. Lett., 57, 18, 2244, 1986.
[7] A. Ashtekar, Phys. Rev. D, 36, 6, 1587, 1987.
[8] C. Rovelli, Class. Quantum Grav., 8, 1613, 1991.
[9] A. Ashtekar and C. Rovelli, Class. Quantum Grav., 9, 1121, 1992.
[10] M. Mizuki, Bulltn Inst. Math. Statistics, 87t-46, 87t-47, 1987.
[11] R.P. Feynman et al, The Feynman Lecture on Physics, Vol.II, Addison-Wesley, Reading, Mass, 1964.
[12] V.F. Weisskopf, Phys. Rev., 56, 72, 1939.
[13] H.H. Aharonov & D. Bohm, Phys. Rev., 115, 485-491, 1959.
[14] W.D. Jackson, Classical Electrodynamics, 2nd ed., John Wiley, New-York, 1975

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