Dynamics and Interpretation in Quantum Theory

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ABSTRACT. A dynamic system $\mathcal{S}_{KG}$, described by the Klein-Gordon equation is shown to be a special case of a more general dynamic system $\mathcal{S}$ which can be considered as a set of identical classical particles, interacting via a self-consistent field κ. The κ-field is responsible for quantum effects, it is able to generate pairs and to escape from matter. It can be interpreted as a force field, describing dynamically a statistical effect of the world lines reconnection. Quantum effects can be considered as dynamical effects without a reference to QM principles.

RÉSUMÉ. Il est demontrè qu'un systeme dynamique $\mathcal{S}_{KG}$ décrit par l' equation de Klein-Gordon est un cas particulier du systeme dynamique général $\mathcal{S}$ qui peut-étre considerè comme un ensemble de particules classiques identiques, interagissant entre elles à l'aide d'un champ self-consistant κ. Le champ κ est responsable des effets quantiques et est capable de donner naissance aux paires de particules et peut se détacher de la matiere. Ce champ peut être interpreté comme un champ de forces décrivant dynamiquement un effet statistique de reconnection des lignes d'univers. Les effets quantiques peuvent etre considerè comme effets dynamiques sans recourir aux principes de la Mécanique Quantique.

1 Introduction

In the paper dynamic properties of a quantum system are investigated with the QM principles being reserved. Adding the reserved QM principles, one obtains the conventional presentation of the quantum mechanics.

Conventional scheme (C-scheme) of the quantum mechanics presentation can be written as follows:

$$\text{Single system} \quad \Rightarrow \quad \text{Stat. propositions} \quad \Rightarrow \quad \text{Dynamics}$$

where "Single system" means a real single physical system, and "statistical propositions" means a set of rules for calculations of average values of physical quantities. These rules describe the way of calculation of the mean value of a physical quantity R in the pure state ψ and can be written in the form [**?**]

$$\langle R \rangle_\psi = \langle \psi^* \mid \hat{R} \mid \psi \rangle \tag{1.1}$$

Here $\hat{R}$ means linear operator corresponding to the physical quantity R. Dynamics describes time evolution of mean values of physical quantities (in the Heisenberg picture), or the time evolution of the state vector ψ of the physical system (in the Schrödinger picture). In both cases the dynamics is described by fixing the evolution operator H called Hamiltonian. The arrows show the logical connection between the terms of the scheme. In particular, the second arrow shows that the statistical propositions are formulated before the dynamics, whereas the dynamics and dynamic equations are formulated after and in terms of the statistical propositions. In such a scheme practically all quantum properties [**?**] are contained in the statistical propositions. In particular, the QM principles can be derived from the statistical propositions [**?**].

Dynamical equations and statistical propositions generate some associations with the classical mechanics or with some elements of the classical mechanics. These associations are commonly considered as quantum mechanics interpretations. These associations are rather indefinite in the sense that as a rule they are not formulated in a mathematical form. It is a reason why some scientists believe that the quantum mechanics can exist without any interpretations, and only mathematical statements of the statistical propositions and the dynamic equations are of importance. Nevertheless, a lot of papers [**?**] - [**?**] is devoted to development of different interpretations of the quantum mechanics. It is commonly supposed that a valid interpretation is useful for a correct development of the quantum theory.

Here another scheme of quantum mechanics presentation is used. It will be refered to as a SET-scheme (SET is an abbreviation of "statistical ensemble technique"). The SET-scheme can be presented in the form:

$$\text{Single system} \quad \Rightarrow \quad \text{Dynamics} \quad \Rightarrow \quad \text{Interpretation}$$

where "Single system" means a single stochastic physical system $\mathcal{S}_{st}$, "Dynamics" denote a dynamical system $\mathcal{E}$, its attributes and its dynamic equations, describing the time evolution. This dynamic system $\mathcal{E}$ associates with the single stochastic system $\mathcal{S}_{st}$. "Interpretation" means a correlation between the dynamic variables of the dynamic system $\mathcal{E}$ and mean values related to the single stochastic system $\mathcal{S}_{st}$. The interpretation means the same statistical propositions and, maybe, some other statements which admit to formulate properties of the stochastic system $\mathcal{S}_{st}$ in terms of dynamic variables of the dynamic system $\mathcal{E}$. The arrows show also the logical connection between the terms of the scheme. In particular, the second arrow shows that the dynamics is formulated before and independently of the interpretation (statistical propositions). It means also that the interpretation (statistical propositions) is formulated after and in terms of the dynamics.

A usage of the SET-scheme can be explained in the following example. Precise experiments with a single electron are irreproducible, in general. It means by definition that a single electron is a stochastic system $\mathcal{S}_{st}$, and there are no dynamic equations for the time evolution of the single electron. At the same time a series of many precise independent experiments with the stochastic system $\mathcal{S}_{st}$ gives distributions of physical quantities. These distributions are reproducible in other series of like experiments with the stochastic system $\mathcal{S}_{st}$. It means that the set of many similar independent stochastic systems $\mathcal{S}_{st}$ can be considered as a deterministic dynamic system $\mathcal{E}$, associated with the stochastic system $\mathcal{S}_{st}$. It is a common practice to call such a dynamic system $\mathcal{E}$ as a statistical ensemble of stochastic systems $\mathcal{S}_{st}$. The time evolution of the statistical ensemble is described by dynamic equations which are determined by properties of stochastic systems $\mathcal{S}_{st}$ constituting the statistical ensemble $\mathcal{E}$.

Interpretation (statistical propositions) determines mean values of physical quantities of $\mathcal{S}_{st}$ in terms of dynamic variables of $\mathcal{E}$. It is an interpretation of the stochastic system in terms of a deterministic dynamic system, i.e. it is an interpretation in a conventional sense of this word. In particular, the interpretation (statistical propositions) determines the time evolution of the mean values of $\mathcal{S}_{st}$.

It should note a conceptual difference between the C-scheme and the SET-scheme. In the C-scheme all quantum properties are contained in the statistical propositions, whereas in the SET-scheme they are contained in dynamics. Indeed, removing the statistical propositions from

the C-scheme, one removes the quantum mechanics as a whole. At the same time in the SET-scheme one can derive practically all quantum effects (including the uncertainty principle, interference and diffraction) only from dynamics, i.e. without a usage of the interpretation (statistical propositions). From the viewpoint of the C-scheme it seems rather strange and unexpected. Nevertheless, it will be demonstrated below.

In the second section one considers dynamics of a set of identical classical particles interacting via a self-consistent field κ^l. Properties of this field are investigated in the third section. An interpretation of the κ-field is given in the fourth section. The fifth section is devoted to an interpretation of the quantum effects in dynamical terms.

2 Dynamics of a set of relativistic particles

Let us consider a dynamic system $\mathcal{S}$, which is a set of identical classical particles of the mass m, interacting via some self-consistent field κ. The action has the form

$$\mathcal{A}_L[x,\kappa] = -\int mcK\sqrt{g_{kl}\dot{x}^k\dot{x}^l}d^4\xi, \quad \dot{x}^k \equiv dx^k/d\tau, \quad \tau = \xi_0 \qquad (2.1)$$

$$K \equiv \sqrt{1+\lambda^2(\partial_l\kappa^l + \kappa^l\kappa_l)}, \qquad (2.2)$$

where c is the speed of the light, g_{kl} =diag$\{c^2,-1,-1,-1\}$ is the metric tensor and $\lambda = \hbar/mc$ is the Compton wave length. $\hbar$ is the Planck constant. $x = \{x^l\}$ and $\kappa = \{\kappa^l\}$, $l = 0,1,2,3$ are dependent dynamic variables. $\xi = \{\xi_0, \boldsymbol{\xi}\} = \{\xi_k\}$, $k = 0,1,2,3$ are independent dynamic variables. κ depends on ξ only via x:

$$\kappa^k = \kappa^k(x), \quad k = 0,1,2,3; \quad x^k = x^k(\xi), \quad \xi = \{\xi_k\}, \quad k = 0,1,2,3. \qquad (2.3)$$

Here and further a summation is produced over repeated Latin indices $(0-3)$ and over the Greek ones $(1-3)$. The dynamic system $\mathcal{S}$ is a distributed system which can be considered as a fluid. $\boldsymbol{\xi} = \{\xi_\alpha\}, \alpha = 1,2,3$ are Lagrangian coordinates labelling particles of the fluid, $\xi_0 = \tau$ is the time Lagrangian coordinate along the world line of the fluid particle. Dynamic equatons for the field κ^k are determined as a result of a variation of the action (2.1) with respect κ^k. $\delta\mathcal{A}_L/\delta\kappa^k = 0$, $k = 0,1,2,3$.

Let us derive dynamic equations for the system $\mathcal{S}$.

$$\frac{\delta \mathcal{A}_L}{\delta x^l} = -\frac{dp_l}{d\tau} - mc\sqrt{\dot{x}^s \dot{x}_s}\partial_l K = 0, \quad \partial_l \equiv \frac{\partial}{\partial x^l}, \quad l = 0,1,2,3 \tag{2.4}$$

$$p_l = -\frac{mcK}{\sqrt{\dot{x}^s \dot{x}_s}}\dot{x}_l, \quad l = 0,1,2,3 \tag{2.5}$$

At variation with respect to κ^l one should take into account that κ^l is a function of x, and the action (2.1) has to be written as an integral over x

$$\int (.) d^4\xi = \int (.) J d^4 x, \quad J = \det \parallel \xi_{i,k} \parallel, \quad \xi_{i,k} \equiv \partial_k \xi_i, \quad i,k = 0,1,2,3 \tag{2.6}$$

where J is a Jacobian of the transformation from the Eulerian coordinates x to the Lagrangian coordinates ξ. Then variation with respect to κ^l gives

$$\frac{\delta \mathcal{A}_L}{\delta \kappa^l} = -\frac{\lambda^2 mc\sqrt{\dot{x}^s \dot{x}_s} J}{K}\kappa_l + \partial_l \frac{\lambda^2 mc\sqrt{\dot{x}^s \dot{x}_s} J}{2K} = 0, \quad l = 0,1,2,3 \tag{2.7}$$

Let us introduce designations

$$j^k = J\dot{x}^k \equiv \frac{\partial J}{\partial \xi_{0,k}} \equiv \frac{\partial(x^k, \xi_1, \xi_2, \xi_3)}{\partial(x^0, x^1, x^2, x^3)}, \quad k = 0,1,2,3 \tag{2.8}$$

$$\rho = \frac{\sqrt{\dot{x}^s \dot{x}_s} J}{mcK} = \frac{\sqrt{j^s j_s}}{mcK} \equiv \frac{\sqrt{j^s j_s}}{mc\sqrt{1 + \lambda^2(\partial_k \kappa^k + \kappa_k \kappa^k)}}. \tag{2.9}$$

Then the equation (2.7) takes the form

$$\kappa_k = \partial_k \rho / 2\rho. \tag{2.10}$$

It means that the field κ^l has always a potential $\kappa = \frac{1}{2}\log(\rho/a^2)$, where a =const. Thus,

$$\rho = \frac{\sqrt{j^s j_s}}{mc\sqrt{1 + \lambda^2(\partial_k \kappa^k + \kappa_k \kappa^k)}} = a^2 e^{2\kappa}, \quad \kappa_l \equiv \partial_l \kappa \tag{2.11}$$

Eq.(2.11) is a dynamic equation for the potential κ. It can be written also in the form

$$\lambda^2 \partial_l \partial^l e^\kappa = [\frac{e^{-4\kappa} j^s j_s}{a^4 m^2 c^2} - 1] e^\kappa. \tag{2.12}$$

In virtue of Eq.(2.8) after substitution of $\partial J/\partial\xi_{0,k}$ by j^k the identity

$$\partial_k \frac{\partial J}{\partial \xi_{0,k}} \equiv 0 \tag{2.13}$$

turns to the continuity equation

$$\partial_k j^k = 0 \tag{2.14}$$

for the current $j^k \equiv J\dot{x}^k$. Let us multiply Eq.(2.4) by J, use the designation (2.8) and the relation

$$\frac{dp_l}{d\tau} = \dot{x}^k \partial_k p_l, \tag{2.15}$$

which is valid provided p_l is considered as a function of only x. After simple transformations one obtains

$$j^k(\partial_k p_l - \partial_l p_k) = 0, \quad l = 0,1,2,3 \tag{2.16}$$

$$p_l = -a^{-2}e^{-2\kappa} j_l \tag{2.17}$$

Only three of four equations (2.16) are independent, because a convolution of Eq.(2.16) with j^l leads to an identity. Thus, equations (2.12), (2.14), (2.16), (2.17) form a system of the hydrodynamic type equations for the current j^l, $l = 0,1,2,3$ of the fluid and for the potential κ of the self-consistent κ-field.

Equations (2.14), (2.16), (2.17) can be integrated in a general form

$$p_l = -a^{-2}e^{-2\kappa} g_{lk} \frac{\partial J}{\partial \xi_{0,k}} = b\partial_l\varphi + bg^\alpha(\boldsymbol{\xi})\xi_{\alpha,l}, \quad l = 0,1,2,3 \tag{2.18}$$

$$j^l = \frac{\partial J}{\partial \xi_{0,l}}, \quad l = 0,1,2,3 \tag{2.19}$$

where φ is a new dynamic variable, $\partial J/\partial\xi_{0,k}$ is the function of first derivatives $\xi_{\alpha,k} \equiv \partial_k\xi_\alpha$, $k = 0,1,2,3$, $\alpha = 1,2,3$, which is defined by the relation (2.8), $g^\alpha(\boldsymbol{\xi})$, $\alpha = 1,2,3$ are arbitrary functions of $\boldsymbol{\xi}$, b is a constant used to make the functions $g^\alpha(\boldsymbol{\xi})$ dimensionless ($\boldsymbol{\xi}$ are supposed to be dimensionless and $[b]$=[action]). The relations (2.18), (2.19) satisfy equations (2.14), (2.16) for arbitrary functions $g^\alpha(\boldsymbol{\xi})$. One can verify

this, substituting Eqs. (2.18), (2.19) into Eqs. (2.14), (2.16) and using identities (2.13) and

$$\frac{\partial J}{\partial \xi_{0,l}} \xi_{\alpha,l} \equiv 0, \quad \alpha = 1,2,3 \tag{2.20}$$

Equations (2.18), (2.12) are four first order dynamic equations for four hydrodynamic potentials $\varphi = \xi_0$, ξ_α, $\alpha = 1,2,3$, and a second order equation for the potential κ of the κ-field.

Description in terms of hydrodynamic equations (2.12), (2.14), (2.16), (2.17) contains only information about the particle velocities in the sense that, if $\kappa(x)$, $j^k(x)$ is a solution of equations (2.12), (2.14), (2.16), then the velocity is determined by the relation

$$\frac{dx^\alpha}{dx^0} = \frac{j^\alpha}{j^0}, \quad \alpha = 1,2,3. \tag{2.21}$$

For determination of the world line of a particle one has to integrate Eq.(2.21)

The description (2.12), (2.18) in terms of potentials is more informative in the sense that it contains information about both position and velocity of a particle. If $\boldsymbol{\xi} = \boldsymbol{\xi}(x)$ is a solution of Eq.(2.18), the world lines are determined by the finite relations

$$\boldsymbol{\xi}(t, \mathbf{x}) = \boldsymbol{\xi}_0 = \text{const}, \tag{2.22}$$

which should not be integrated. On the other side the description in terms of potentials is physically indefinite in the sense that there is not one-to-one correspondence between the fluid flow and potentials φ, ξ. The same flow can be described by different sets of potentials $\varphi = \xi_0$, $\boldsymbol{\xi}$. This fact is displayed mathematically as an invariancy of the action (2.1) and that of dynamic equations (2.18), (2.19) with respect to the group of transformations of Lagrangian coordinates

$$\xi_\alpha \to \tilde{\xi}_\alpha = f_\alpha(\boldsymbol{\xi}), \quad \det \parallel \partial f_\alpha / \partial \xi_\beta \parallel = 1, \quad \alpha, \beta = 1,2,3 \tag{2.23}$$

$$\xi_0 \to \tilde{\xi}_0 = f_0(\xi_0, \boldsymbol{\xi}), \quad \partial f_0 / \partial \xi_0 > 0 \tag{2.24}$$

where $f_\alpha(\boldsymbol{\xi})$, $f_0(\xi)$ are arbitrary functions of their arguments restricted only the second relations (2.23), (2.24). In virtue of Eq.(2.8) the equation (2.18) is invariant with respect to transformations (2.23), (2.24). Under

these transformations the functions $g^\alpha(\boldsymbol{\xi})$ transform as vectors in the space of Lagrangian coordinates $\boldsymbol{\xi}$.

$$g^\alpha(\boldsymbol{\xi}) \to \tilde{g}^\alpha(\tilde{\boldsymbol{\xi}}) = \frac{\partial \xi_\beta}{\partial \tilde{\xi}_\alpha} g^\beta(\boldsymbol{\xi}), \quad \alpha = 1,2,3 \tag{2.25}$$

The world lines of the fluid are labelled by the Lagrangian coordinates $\boldsymbol{\xi}$. The labelling is arbitrary and does not influence on the dynamics of the system. The transformation (2.23) describes relabelling of the world lines.

The description in terms of the wave function is a kind of description in terms of hydrodynamic potentials [**?**]. To obtain this description let us convolute equations (2.18) with $\partial^l g^\alpha(\boldsymbol{\xi})$. In virtue of identities (2.20) one obtains three first order equations for potentials $\boldsymbol{\xi}$

$$[b\partial^l \varphi + b g^\beta(\boldsymbol{\xi}) \partial^l \xi_\beta] \partial_l g^\alpha(\boldsymbol{\xi}) = 0, \quad \alpha = 1,2,3 \tag{2.26}$$

Let us substitute $\partial J/\partial \xi_{0,k}$ from Eq.(2.18) into Eq.(2.13). One obtains the second order equation for φ in the form

$$\partial_k [e^{2\kappa}(b\varphi^k + b g^\alpha(\boldsymbol{\xi}) \partial^k \xi_\alpha)] = 0, \quad \varphi^k \equiv \partial^k \varphi \tag{2.27}$$

Finally the substitution $\partial J/\partial \xi_{0,k}$ into Eq.(2.12) leads to generalized Hamilton-Jacobi equatioin

$$m^2 c^2 [1 + \lambda^2(\kappa^k \kappa_k + \partial_k \kappa^k)] = [b\partial_l \varphi + b g^\beta(\boldsymbol{\xi}) \partial_l \xi_\beta][b\partial^l \varphi + b g^\beta(\boldsymbol{\xi}) \partial^l \xi_\beta] \tag{2.28}$$

which should be considered as a dynamic equation for κ. The system of equations (2.26) -(2.28) is of seventh order with respect to temporal derivatives.

The equations (2.26), (2.27), (2.28) can be obtained as a result of varation of the action

$$\mathcal{A}_E[\varphi, \boldsymbol{\xi}, \kappa] = \frac{a^2}{2} \int e^{2\kappa} \{[b\partial_k \varphi + b g^\alpha(\boldsymbol{\xi}) \partial_k \xi_\alpha][b\partial^k \varphi + b g^\beta(\boldsymbol{\xi}) \partial^k \xi_\beta] -$$

$$- m^2 c^2 [1 - \lambda^2 (\partial_k \kappa)(\partial^k \kappa)]\} d^4 x \tag{2.29}$$

respectively with respect to $\boldsymbol{\xi}$, φ and κ.

Let $\psi = \{\sqrt{\rho}e^{i\varphi}v_\alpha(\boldsymbol{\xi})\}, \quad \alpha = 1, 2, \ldots n$, $\rho = a^2e^{2\kappa}$ be n-component complex function having the property $\sum_{\alpha=1}^{n} v_\alpha^* v_\alpha = 1$, and v_α be functions of only $\boldsymbol{\xi}$. $(^*)$ means a complex conjugation. Then

$$\rho = \psi^*\psi, \quad j_k \equiv \frac{ib}{2}(\psi^*\partial_k\psi - \partial_k\psi^*\psi) = -b\rho[\partial_k\varphi + f^\beta(\boldsymbol{\xi})\partial_k\xi_\beta], \quad (2.30)$$

$$k = 0, 1, 2, 3$$

where

$$f^\beta(\boldsymbol{\xi}) = -\frac{i}{2}\sum_{\alpha=1}^{n}(v_\alpha^*\frac{\partial v_\alpha}{\partial\xi_\beta} - \frac{\partial v_\alpha^*}{\partial\xi_\beta}v_\alpha), \quad \beta = 1, 2, 3 \qquad (2.31)$$

(2.31) and (2.18) coincide, provided $f^\alpha = g^\alpha$, $\alpha = 1, 2, 3$. It means that five real variables $\rho, \varphi, \boldsymbol{\xi}$ can be substituted by one three-component complex function ψ, described by six real variables. To make such a change of variables it is sufficient to find some solution of the system of equations

$$-\frac{i}{2}\sum_{\alpha=1}^{3}(v_\alpha^*\frac{\partial v_\alpha}{\partial\xi_\beta} - \frac{\partial v_\alpha^*}{\partial\xi_\beta}v_\alpha) = g^\beta(\boldsymbol{\xi}), \quad \beta = 1, 2, 3, \quad \sum_{\alpha=1}^{3} v_\alpha^* v_\alpha = 1, \quad (2.32)$$

The form of the solution depends essentially on the form of functions $g^\alpha(\boldsymbol{\xi})$. Then

$$\psi = \{ae^{\kappa+i\varphi}v_\alpha(\boldsymbol{\xi})\}, \quad \alpha = 1, 2, 3 \qquad (2.33)$$

ánd the action (2.29) takes the form

$$\mathcal{A}_E[\psi, \psi^*] = \int\Big\{\frac{1}{\psi^*\psi}\{-\frac{b^2}{4}(\psi^*\psi_k - \psi_k^*\psi)(\psi^*\psi^k - \psi^{*k}\psi) -$$

$$-m^2c^2[(\psi^*\psi)^2 - \frac{\lambda^2}{4}\partial_k(\psi^*\psi)\partial^k(\psi^*\psi)]\}\Big\}d^4x, \quad \psi_k \equiv \partial_k\psi \qquad (2.34)$$

In reality there is no necessity to solve the system (2.32). It is sufficient to be sure that the system has a solution for any functions $g^\alpha(\boldsymbol{\xi})$. In the general case the dynamic equation for the ψ-function is nonlinear.

Note that the constant b appeared in Eq. (2.18) as an integration constant which has nothing to do with quantum effects, whereas λ appeared in the action (2.1) as a constant introducing the κ-field responsible for quantum effects. Identifying b with $\hbar = mc\lambda$, one can obtain the

following form of the action (2.34)

$$\mathcal{A}_E[\psi,\psi^*]=\frac{1}{2}\int[\hbar^2\partial_k\psi^*\partial^k\psi-(m^2c^2+\frac{\hbar^2}{2}\sum_{\alpha,\beta=1}^{n}Q_{\alpha\beta,k}Q^*_{\alpha\beta}{}^k)\psi^*\psi]d^4x, \tag{2.35}$$

where

$$Q_{\alpha\beta,k}\equiv\frac{1}{\psi^*\psi}\begin{vmatrix}\psi_\alpha & \psi_\beta\\ \partial_k\psi_\alpha & \partial_k\psi_\beta\end{vmatrix},\quad Q_{\alpha\beta}{}^k=g^{kl}Q_{\alpha\beta,l} \tag{2.36}$$

For some functions $g^\alpha(\boldsymbol{\xi})$ the ψ-function can have less number of essential components. For instance, if $g^\alpha(\boldsymbol{\xi})\equiv 0$, the ψ-function has one essential component $\psi=ae^{\kappa+i\varphi}$, and the action (2.35) turns to the action

$$\mathcal{A}_{KG}[\psi,\psi^*]=\frac{1}{2}\int[\hbar^2\partial_k\psi^*\partial^k\psi-m^2c^2\psi^*\psi]d^4x \tag{2.37},$$

for the Klein-Gordon equation

$$\hbar^2\partial_k\partial^k\psi+m^2c^2\psi=0 \tag{2.38}$$

The following expressions for the current j^k and the energy-momentum tensor T^l_k correspond to the action (2.37)

$$j^k=\frac{i\hbar}{2}(\psi^*\psi^k-\psi^{*k}\psi),\quad k=0,1,2,3,\quad \psi^k\equiv\partial^k\psi \tag{2.39}$$

$$T^l_k=\frac{\hbar^2}{2}(\psi^{*l}\psi_k+\psi^*_k\psi^l-\delta^l_k\psi^{*s}\psi_s)+\frac{1}{2}\delta^l_k m^2c^2\psi^*\psi, \tag{2.40}$$

$$k,l=0,1,2,3$$

The ψ-function can be considered as a wave function, because it satisfies the linear Klein-Gordon equation that is compatible with the quantum axiomatics. The one component ψ-function describes a potential flow of the fluid, as it folows from Eq.(2.18).

The current $j^k=\dot{x}^kJ$ and the energy-momentum tensor T^l_k are attributes of the dynamic system $\mathcal{S}$ (not attributes of statistical propositions (1.1)). The canonical energy-momentum tensor has the form

$$T^l{}_k=-p_kj^l-\frac{1}{2}\hbar^2a^2e^{2\kappa}\partial_k\kappa^l=\frac{mcKj^lj_k}{\sqrt{j^sj_s}}-\frac{1}{2}\hbar^2a^2e^{2\kappa}\partial_k\kappa^l. \tag{2.41}$$

In the potential case j^k and T^l_k can be expressed via the wave function ψ and coincide respectively with expressions (2.39) and (2.40). Thus, a potential flow in the system $\mathcal{S}$ is described completely in terms of the dynamic system $\mathcal{S}_{KG}$.

3 Properties of the κ-field

Let us list the properties of the κ-field introduced by the relations (2.1), (2.2).

(1) The κ-field is responsible for quantum effects.

(2) The κ-field can escape from the matter and exist separately in the "empty" space-time.

(3) The κ-field enables to generate pairs.

The first property is rather evident from Eqs.(2.1), (2.2). Setting $\kappa^l \equiv 0$, $l = 0, 1, 2, 3$, the action (2.1) turns to the action for the classical statistical ensemble of independent classical particles.

The second property follows from the dynamic equation (2.12). Setting $j^l \equiv 0$, $l = 0, 1, 2, 3$, Eqs. (2.14), (2.16) are satisfied identically. Eq.(2.12) turns to the Klein-Gordon equation for e^κ

$$\lambda^2 \partial_l \partial^l e^\kappa + e^\kappa = 0. \tag{3.1}$$

In the case $j^l = 0$, $l = 0, 1, 2, 3$ the wave function (2.33) turns to $\psi = ae^\kappa$. In other words, real wave function ψ, satisfying the Klein-Gordon equation, describes free κ-field without a matter. According to Eq. (2.40) the energy-momentum tensor of the free κ-field is determined by the relation

$$T^l_k = \frac{1}{2} m^2 c^2 a^2 e^{2\kappa} [\delta^l_k + \lambda^2 (2\kappa^l \kappa_k - \delta^l_k \kappa^s \kappa_s)], \quad k, l = 0, 1, 2, 3 \tag{3.2}$$

$$T^0_0 = \frac{1}{2} m^2 c^2 a^2 e^{2\kappa} [1 + \lambda^2 \sum_{l=0}^{3} \mid \kappa_l \mid^2] \geq 0 \tag{3.3}$$

If j^l changes rather slightly over the distance of the Compton length λ, the dynamic equation (2.12) for the κ-field is a differential equation with a small parameter at the highest derivative. In this case rhs of Eq. (2.12) is to vanish, and its solution has the form

$$\kappa \simeq \frac{1}{2} \log \frac{\sqrt{j^s j_s}}{a^2 mc}, \quad K \simeq 1, \quad \lambda \mid \partial_l j^s \mid \ll j^s \tag{3.4}$$

In this case the κ-field is coupled with the matter (like the Coulomb electric field) and has no proper degrees of freedom. In the non-relativistic case the energy difference $mKc^2 - mc^2 = -\frac{1}{2}\lambda^2 mc^2 (j_0)^{-1/2}\nabla^2\sqrt{j_0}$ associates with the Bohm potential $V_{\rm B} = [\hbar\nabla \log(\psi^*\psi)]^2/8m$ [**?**].

Before a consideration of the pair production problem let us discuss some conceptual points, connected with a description of particles and antiparticles in classical terms. First, about terminology. The terms "world line of a particle" and "world line of an antiparticle" are essentially non-relativistic. They suppose that a world line is an *attribute* (history) of a pointlike physical object (particle or antiparticle). This physical object is a point in the 3-dimensional space, and its trajectory in the space-time is its world line. From the non-relativistic viewpoint a particle and an antiparticle are different physical objects. From relativistical viewpoint a world line (*WL*) is a real physical object. This object is a one-dimensional line in the 4-dimensional space-time. *WL* has two distinct orienatations which can be considered as one of two possible directions of motion along *WL* [**?**, **?**]. Further the term "*WL*" will be used instead of the term "world line" in those cases, when the world line is considered as a physical object (not as a history of a particle). Particles and antiparticles are derivative pointlike physical objects which arise as intersections of *WL* with the hyperplane t=const at some coordinate system. One *WL* can describe a few particles and antiparticles, placed in different regions.

A particle and an antiparticle are distinguished by the orientation of *WL* at its intersection with the hyperplane t=const. From relativistical viewpoint a particle and an antiparticle are two different states (or attributes) of a *WL*, whereas from the non-relativistic viewpoint they are two different physical objects. The term "*SWL*" (abbreviation of "section of world line") will be used as a common concept with respect to concepts "particle' and "antiparticle". *SWL* means any point of intersection between the *WL* and a hyperplane t=const independently of an orientation of the *WL* at this point.

Describing a particle and an antiparticle, one has to distinguish between the energy-momentum vector $P_l = \{P_0, \mathbf{P}\}$ and the canonical momentum vector $p_l = \{p_0, \mathbf{p}\}$, especially between the energy $E = P_0$ and the time component p_0 of the canonical momentum. These quantities are defined in different way. The energy-momentum vector is defined by

the relation

$$P_l = \int\limits_V T_l^0 d\mathbf{x}, \quad l = 0,1,2,3, \quad E = P_0 \tag{3.5}$$

where T_k^l is the energy-momentum tensor and V is a 3-volume around SWL. The canonical momentum p_l is defined as a quantity, canonically conjugate to the position x^l, $\quad l = 0,1,2,3$ of SWL. In classical physics it means that $p_l = \partial L/d\dot{x}^l$, $\dot{x}^l \equiv dx^l/d\tau$, $l = 0,1,2,3$, where L is a Lagrangian, and τ is a parameter along WL. In the quantum physics the operator $\hat{p}_l$ of the canonical momentum is defined by the relation

$$[\hat{u}, \hat{p}_l]_- = i\hbar \frac{\partial \hat{u}}{\partial x^l} \tag{3.6}$$

where $[\ldots]_-$ denotes a commutator and $\hat{u}$ means an operator of any dynamic variable in the Heisenberg representation.

The fact that the energy E of a free particle coincide with the time component $-p_0$ of the canonical momentum taken with the opposite sign $E = -p_0$ does not mean that the same relation is valid for a free antiparticle. Indeed, setting for simplicity $\kappa^l \equiv 0$, $l = 0,1,2,3$ in the relations (2.41), (2.5), one obtains

$$T_0^0 = \frac{mcKj^0 j_0}{\sqrt{j^s j_s}} \geq 0, \quad p_0 = -\frac{mcKj_0}{\sqrt{j^s j_s}} = -\frac{mcK\dot{x}_0}{\sqrt{\dot{x}^s \dot{x}_s}} \tag{3.7}$$

$\mathrm{sgn}(K\sqrt{j^s j_s})$ $=\mathrm{sgn}(K/\sqrt{j^s j_s}) = 1$ always. Then it follows from Eqs. (3.5), (3.7) that the energy is positive always, whereas the sign of p_0 depends on the sign of j^0 (or $\dot{x}^0$), and is different for a particle and for an antiparticle. If p_0 can be considered as a constant, then it follows from Eqs. (3.5), (3.7)

$$E = P_0 = -p_0 N, \quad N = \int j^0 d\mathbf{x}, \tag{3.8}$$

where N is the number of SWLs. N is positive for particles and negative for antiparticles For one WL the relation (3.8) reduces to $E = -p_0 \mathrm{sgn}(j^0) =\mid p_0 \mid$.

A like relation

$$Q = e\ \mathrm{sgn}(j^0) = e\ \mathrm{sgn}(\dot{x}^0), \quad \mathrm{sgn}(J) = 1 \tag{3.9}$$

is valid for the connection between the electric charge Q of SWL, defined as a source of the electromagnetic field and the constant e, describing

interaction of a charged *WL* with the electromagnetic field [**?**]. The electric charge Q is defined by the relation (3.9) which describes the well known fact that a particle and an antiparticle have opposite electric charges.

Identification of P_l with p_l is possible [**?**, **?**], provided particles and antiparticles are considered as different objects, (but not attributes of a whole *WL*). In the up-to-date quantum field theory such an identification is produced almost always (see, however, [**?**]) As a result one deals with indefinite number of *SWL* (instead of a few *WL*s), and many additional problems arise. In particular, conventionally one claims [**?**, **?**] that some solutions of the Klein-Gordon equation has a negative energy and are not physical. Here one has a confusion of the energy E with the canonical momentum p_0. The energy of any solution is non-negative, as it follows from Eqs.(3.5), (3.7), or (2.40), whereas the p_0 can have both signs, but it does not mean that the solution with $-p_0 < 0$ is not physical. It means only an appearance of antiparticles.

The problem of pair production is connected with a possibilty of changing sign of p_0, or $\dot{x}^0$. To change $\mathrm{sgn}(\dot{x}^0)$, the vector $\dot{x}^l$ is to be spacelike $\dot{x}^k\dot{x}_k < 0$. It is a necessary condition of changing sign of $\dot{x}^0$. The first term of the action (2.1) remains real, provided $\dot{x}^k\dot{x}_k < 0$ and

$$K^2 = 1 + \lambda^2(\kappa_k\kappa^k + \partial_k\kappa^k) < 0 \tag{3.10}$$

To satisfy the last condition, the κ-field is to be large enough and to change rather rapidly.

To distinguish between particles and antiparticles, let us introduce the distribution function over position $\mathbf{x}$, canonical momentum $\mathbf{p}$ and ε=$\mathrm{sgn}(j^0) = -\mathrm{sgn}(p_0)$:

$$F(\mathbf{x},\mathbf{p},\varepsilon) = \frac{\sqrt{j^s j_s}}{mcK}\prod_{\alpha=1}^{3}\delta(p_\alpha + \frac{mcKj^\alpha}{\sqrt{j^s j_s}})\delta_{\varepsilon,\mathrm{sgn}(j^0)} \tag{3.11}$$

where K is defined by the relation (2.2). One distinguishes between the canonical momentum (c-momentum) $\mathbf{p} = \{p_1, p_2, p_3\}$ and the momentum $\mathbf{P} = \{P_\alpha\}, \alpha = 1, 2, 3$ which are coupled between themselves by the relation

$$\mathbf{P} = \varepsilon\mathbf{p}, \quad \varepsilon = -\mathrm{sgn}(p_0) = \pm 1 \tag{3.12}$$

$\mathbf{P}$ is the spatial component of the energy-momentum vector (3.5), whereas $\mathbf{p}$ is defined by Eq.(2.5).

The distribution function permits to calculate the mean value $\langle R\rangle$ of any function $R(\mathbf{x},\mathbf{P},\varepsilon)$ by means of the relation

$$\langle R(\mathbf{x},\mathbf{P},\varepsilon)\rangle = A^{-1}\sum_{\varepsilon=\pm 1}\int\int R(\mathbf{x},\varepsilon\mathbf{p},\varepsilon)F(\mathbf{x},\mathbf{p},\varepsilon)d\mathbf{x}d\mathbf{p} \qquad (3.13)$$

$$A = \sum_{\varepsilon=\pm 1}\int\int F(\mathbf{x},\mathbf{p},\varepsilon)d\mathbf{x}d\mathbf{p} = \int (mcK)^{-1}\sqrt{j^s j_s}d\mathbf{x} = \int \rho d\mathbf{x} \quad (3.14)$$

Relations (3.13), (3.14) are some new statistical propositions alternative to Eq.(1.1), which will be referred to as a dynamical interpretation, or as *WL*-interpretation (interpretation in terms of *WL*). They realize in a mathematical form the "classical associations" arising at the description of the dynamic system $\mathcal{S}$ by means of the action (2.1). Until formulation of the *WL*-interpretation in the form (3.13) all "classical considerations" were only a description of the dynamic system $\mathcal{S}_{KG}$ in terms of other dynamic variables and nothing more.

A possiblity of an alternative interpretation arises only after mathematical formulation of alternative statistical propositions. The *WL*-interpretation is more general, than the conventional interpretation (1.1), because it is applicable in the case, when ψ satisfies nonlinear dynamic equation and the linear superposition principle does not take place. The distribution function (3.11) describe a classical particle of alternating mass

$$\mu = mK = m\sqrt{1+\lambda^2(\kappa_k\kappa^k + \partial_k\kappa^k)} \qquad (3.15)$$

with the mass depending on the κ-field magnitude. μ is imaginary for spacelike momenta.

ε=sgn(j^0) is not a relativistic invariant for the spacelike vector j^k. It means that the spacelike *WL* describes a particle ($\varepsilon = 1$) in one coordinate system and an antiparticle ($\varepsilon = -1$) in other one. Spacelike *WL* are possible only inside a region with a large κ-field which associates with an existence of pairs. Apparently, spacelike *WL* should be regarded as the mean *WL*, describing simultaneously one *SWL* and a few pairs.

Let us stress that we do not insist on the *WL*-interpretation. It is important only that the most of quantum effects can be explained independently of which of the two interpretations [(1.1), or (3.11), (3.13)] is used. At the same time most of relations (1.1), or (3.11), (3.13) cannot be proved experimentally [**?**]. They can be tested in the case, when $R = R(x)$, but in this case the statistical propositions (1.1) and (3.11), (3.13) coincide.

4 Interpretation of the κ-field on the dynamic-statistical base

Let us discuss interpretation of the κ-field. For simplicity only the case of two-dimensional space-time is considered. Generalization for the case of four dimensions can be obtained easily. Let us consider a statistical ensemble $\mathcal{E}$ of free classical (deterministic) *WL*s in the two-dimensional space-time. All *WL*s have the same mass m and the constant e. The state of the statistical ensemble $\mathcal{E}$ is described by the distribution function F as a function of coordinates $x = \{x^0, x^1\}$ and momentum $p = \{p_0, p_1\}$. As far as variables p_0 and p_1 are not independent because of the relation $p_l p^l = m^2, \quad (c = 1)$, it is convenient to use variables p_1 and $\varepsilon = -\mathrm{sgn}(p_0), \quad \varepsilon = \pm 1$ as independent arguments of the distribution function $F = F(x, p_1, \varepsilon)$. The discrete variable ε is different for particles and antiparticles ($\varepsilon = 1$ for particles and $\varepsilon = -1$ for antiparticles).

The distribution function F satisfies the free Liouville equation

$$\partial_l [p^l F(x, p_1, \varepsilon)] = 0, \tag{4.1}$$

$$p^1 = -p_1, \quad p_0 = -\varepsilon E(p_1), \quad E(p) \equiv \mid \sqrt{m^2 + p_1^2} \mid$$

The mean current in the statistical ensemble $\mathcal{E}$ is defined by the relation

$$j^l(x) = \sum_{\varepsilon = \pm 1} \int p^l F(x, p_1, \varepsilon) dp_1, \quad l = 0, 1, \tag{4.2}$$

Let us consider a very important case of the state of $\mathcal{E}$

$$F(x, p_1, \varepsilon) = f(x, \varepsilon) \delta(p_1 - k) \tag{4.3}$$

where k is a constant, or a slowly varying function of x. If *WL*s of the state (4.3) of $\mathcal{E}$ do not intersect (have no common points) inside some region $\mathcal{R}$ of the space-time, then by definition the $\mathcal{E}$ is a simple ensemble (more exactly an ensemble simple inside $\mathcal{R}$).

A simple ensemble has the remarkable property that its state $F(x, p_1, \varepsilon)$ is determined by its current $j^l(x)$.

$$F(x, p_1, \varepsilon) = m^{-1} \sqrt{j_s j^s} \delta[p_1 + m j_1 (j^s j_s)^{-1/2}] \delta_{\varepsilon, \mathrm{sgn}(j^0)}, \tag{4.4}$$

Besides it is possible to write the action and dynamic equations for the current of the simple ensemble state

$$\mathcal{A}[j, \varphi] = \int \{-m \sqrt{j_s j^s} - \hbar j^l \partial_l \varphi\} d^2 x \tag{4.5}$$

After elimination of φ the dynamic equations for the action (4.5) reduce to the form

$$\partial_l j^l = 0, \quad \partial_l T^{lk} = 0, \quad k = 0, 1 \tag{4.6}$$

$$T^{kl} = \frac{m j^k j^l}{\sqrt{j_s j^s}}, \quad k, l = 0, 1 \tag{4.7}$$

In such a form the dynamic equations (4.6) describe the conservation law of *SWL*s and the energy-momentum. One can verify that in virtue of the dynamic equations (4.6) the distribution function (4.4) satisfies the Liouville equation (4.1).

Let us consider two different simple states F_1 and F_2 of the statistical ensemble $\mathcal{E}$, which will be referred to as simple statistical ensembles $\mathcal{E}_1$ and $\mathcal{E}_2$ of free *WL*s.

$$\mathcal{E}_1: \quad F_1(x, p_1, \varepsilon) = f_1(x^1 - x^0 p^1/p^0)\delta(p_1 - k)\delta_{\varepsilon\varepsilon_1}, \quad p_0 = -\varepsilon E(p_1) \tag{4.8}$$

$$\mathcal{E}_2: \quad F_2(x, p_1, \varepsilon) = f_2(x^1 - x^0 p^1/p^0)\delta(p_1 + k)\delta_{\varepsilon\varepsilon_2}, \quad p_0 = -\varepsilon E(p_1) \tag{4.9}$$

where k =const, $E(p) \equiv \mid (m^2 + p^2)^{1/2} \mid$, ε_1 and ε_2 are fixed numbers equal to ± 1 and

$$f_s(x) = \begin{cases} A, & \mid x \mid < L_s \\ 0, & \mid x \mid > L_s \end{cases} \qquad s = 1, 2, \tag{4.10}$$

$$A, L_1, L_2 = \text{const}, \quad A > 0.$$

If in Eqs. (4.8), (4.9) $p_0 < 0$, then $\varepsilon = 1$, and *WL*s of the ensemble describe particles. If $p_0 > 0$, then $\varepsilon = -1$, and *WL*s describe antiparticles. The ensembles $\mathcal{E}_1$ and $\mathcal{E}_2$ are described respectively by the currents $j^0_{(1)}$ and $j^0_{(2)}$

$$\mathcal{E}_1: \quad j^0_{(1)} = k_0 f_1(x^1 + x^0 k/k_0), \quad j^1_{(1)} = -k f_1(x^1 + x^0 k/k_0), \tag{4.11}$$

$$\mathcal{E}_2: \quad j^0_{(2)} = k_0 f_2(x^1 - x^0 k/k_0), \quad j^1_{(2)} = k f_2(x^1 - x^0 k/k_0), \tag{4.12}$$

$$k_0 = -\varepsilon E(k)$$

As far as the distribution functions F_1 and F_2 satisfy the linear equation (4.1) and describe two different states of the same dynamic system $\mathcal{E}$, the distribution function $F = F_1 + F_2$ describes also a state of $\mathcal{E}$. This state F can be considered as a result of composition of two statistical ensembles

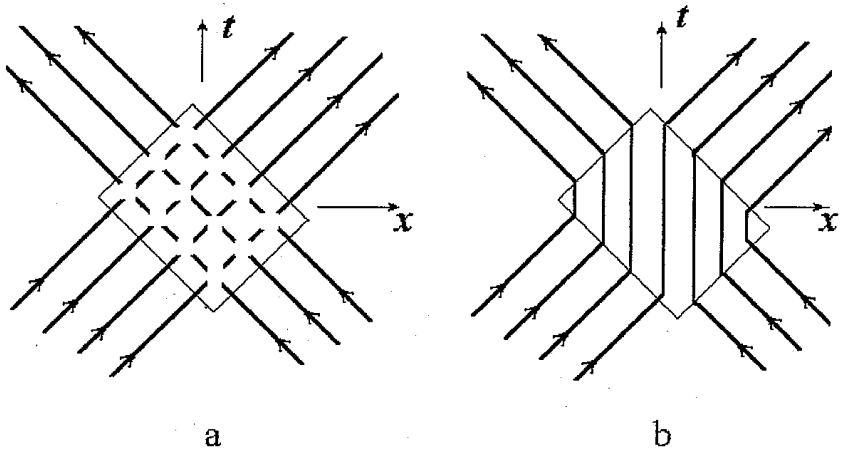

Figure 1: World lines of classical particles: a) for single particles, b) for mean world lines.

$\mathcal{E}_1$ and $\mathcal{E}_2$. Thus, the statistical ensemble $\mathcal{E} = \mathcal{E}_1 + \mathcal{E}_2$ is simple everywhere in the space-time except for the region $\mathcal{R}_c$. The region $\mathcal{R}_c$ is located inside the parallelogram formed by the straight lines $x^1 + kx^0/k_0 = \pm L_1$, $x^1 - kx^0/k_0 = \pm L_2$. Inside $\mathcal{R}_c$ any point belongs to two different *WL*s of the statistical ensemble $\mathcal{E}$. These intercepts of *WL*s are shown in Fig.1a, 2a. as dashed lines. In Fig.1 $\varepsilon > 0$ in both ensembles $\mathcal{E}_1$ and $\mathcal{E}_2$, and *SWL*s are particles. Thus, the statistical ensemble $\mathcal{E}$ is not simple, in general.

The total current j^l of $\mathcal{E}$ is determined by Eq.(4.2). One obtains the relation

$$j^l = j^l_{(1)} + j^l_{(2)} \tag{4.13}$$

as a corollary of Eqs. (4.8), (4.9) and constructs the mean *WL*s as lines tangent to the total current j^l. Then the ensemble $\mathcal{E}$ becomes a simple ensemble $\mathcal{E}_m$ of mean *WL*s, shown in Fig.1b. But now $\mathcal{E}_m$ is not a statistical ensemble of mean *WL*s, because by definition a statistical ensemble is a set of independent and, hence, non-interacting elements (*WL*s).

Outside the region $\mathcal{R}_c$ the mean *WL*s coincide with *WL*s of the ensembles $\mathcal{E}_1$ or $\mathcal{E}_2$, but inside $\mathcal{R}_c$ the mean *WL*s distinguish from *WL*s of

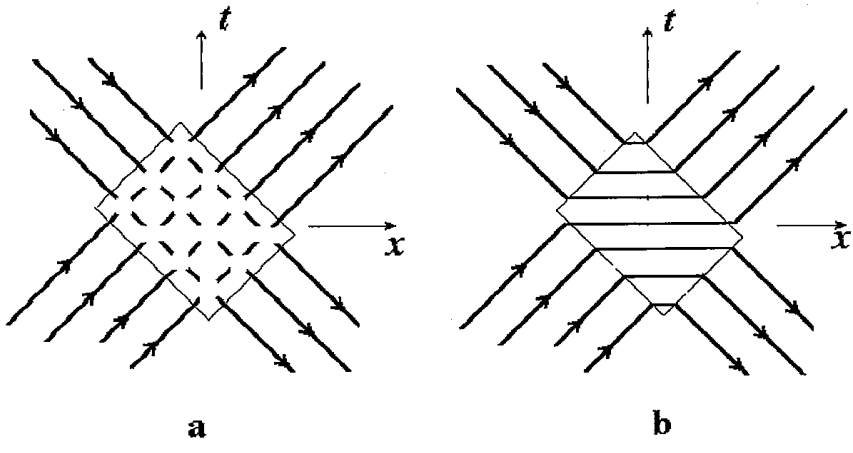

Figure 2: World lines of classical particles and antiparticles: a) for single particles and antiparticles, b) for mean world lines.

both ensembles $\mathcal{E}_1, \mathcal{E}_2$. Looking at Fig.1*b*, one can see that after the unification (4.13) a reconnection of *WL*s take place. From the non-relativistic viewpoint this reconnection can be interpreted as an exchange effect between the particles of the ensembles $\mathcal{E}_1$ and $\mathcal{E}_2$. This reconnection (or the exchange effect) is interpreted as some statistical effect. There is no dynamic interaction between *WL*s here, nevertheless the mean *WL*s of the ensemble $\mathcal{E}_m$ look, as if they interact very strongly inside the region $\mathcal{R}_c$. In other words, considering non-simple statistical ensemble $\mathcal{E}$ of deterministic *WL*s as a simple ensemble $\mathcal{E}_m$ of mean *WL*s, one introduces some *effective dynamic interaction between the mean WLs.* This dynamic interaction imitates the statistical effect of reconnection of *WL*s (from relativistic viewpoint), or the effect of the particle exchange (from the non-relativistic standpoint).

Now let us look at Fig.2b. In this case ε is different for $\mathcal{E}_1$ and $\mathcal{E}_2$. Here one sees the effect of the *WL*s reconnection, but this effect can be hardly interpreted from the non-relativistic viewpoint as an exchange effect, because from this point of view particles and antiparticles are not identical physical objects. Besides, the picture of mean world lines in Fig.2b should be interpreted as follows. Particles and antiparticles

annihilated in the region $\mathcal{R}_c$, are converted into some field, and thereafter this field produces particle-antiparticle pairs. Such a situation, when particles disappear and arise, can be hardly explained from the non-relativistic standpoint.

The dynamic interaction is self-consistent in the sense that it depends on the state of the whole ensemble $\mathcal{E}$. For instance, the fact that the mean *WL*s are straight lines inside $\mathcal{R}_c$ (Fig.1b,2b) is a corollary of the fact that $f_s(x) = A$=const inside $\mathcal{R}_c$. If one violates the form of the functions f_s, the shape of the mean *WL*s in $\mathcal{R}_c$ changes.

From the relativistical viewpoint both pictures in Fig.1,2 describe some statistical effect, when non-simple statistical ensemble $\mathcal{E}$ of free *WL*s is described in terms of a simple ensemble $\mathcal{E}_m$ of *mean* interacting *WL*s. This statistical effect is described as a reconnection of *WL*s and can be imitated by means of dynamic interaction between mean *WL*s.

Thus, there are two different ways of statistical description of deterministic *WL*s: (1) description F in terms of the distribution function, (2) description j in terms of currents. The description F is the most detailed one. It is produced in the 3-dimensional phase space and uses dynamic equations of the type (4.1). The description j is less detailed one. It is produced in the 2-dimensional space-time and uses dynamic equations of the type (4.6).

If there is a non-simple ensemble, then it is possible to describe each beam of *WL*s in the space-time by means of dynamic equations of the type (4.6), (4.7). The mean *WL*s are described by the mean current j^l of the type (4.13). But the dynamic equations cannot be written only in terms of j^l. They contain also variables like $j^l_{(1)} - j^l_{(2)}$ which imitate some self-consistent field describing the interaction between mean *WL*s. In the case of the ensemble of determinisitc *WL*s one knows definitely that real *WL*s do not coincide with the mean *WL*s. It is valid also in the case of the ensemble of stochastic *WL*s.

At the F-description in the phase space the *WL*s of the ensembles $\mathcal{E}_1$ and $\mathcal{E}_2$ do not interact. At the j-description in the space-time they do. Of course, in reality the *WL*s do not interact (at any rate they do not interact dynamically), but describing stochastic *WL*s in quantum systems, one is forced to use only j-description in the space-time (but not the F-description in the phase space). Mathematically an interplay between descriptions of stochastic *SWL*s and deterministic *WL*s can be established only for j-description of mean *WL*s. Although intuitively

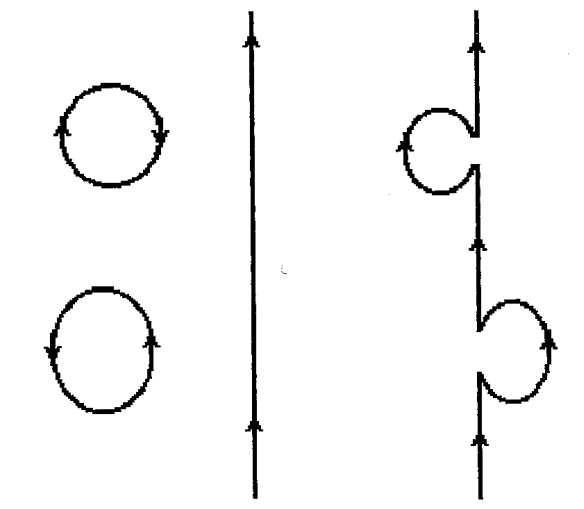

Figure 3: Reconnection of world lines, described by the κ-field.

*WL*s of ensembles $\mathcal{E}_1$ and $\mathcal{E}_2$ seem to be independent, nevertheless the imitation by means of mean interacting *WL*s seems to be more effective mathematically in the case of stochastic *WL*s.

A possibility of imitating statistical effects by means of dynamic interaction is very important for a practical use. For instance, *WL*s describing microparticles (electron, proton,...etc) are stochastic, and statistical ensembles of such stochastic *WL*s cannot be simple, because they cross between themselves many times. But it is possible to imitate such non-simple statistical ensembles of *WL*s by simple ensembles of mean *WL*s which do not cross by definition. But in this case the mean *WL*s of the ensemble interact between themselves via some self-consistent field. Thus, considering an ensemble $\mathcal{E}$ (not a statistical ensemble) of *WL*s, interacting via some self-consistent field, it is possible to take into account statistical effects connected with the stochasticity and substitute them by dynamic interaction with some self-consistent field.

One can imagine such a field as a field describing closed *WL*s, or closed current loops associated with particle-antiparticle pairs. A join of such closed *WL*s to a *WL* describing a particle (or an antiparticle) changes effectively a shape of the *WL* (see Fig.3).

Let us return to consideration of $\mathcal{S}_{KG}$ and consider the special case of a stationary state in the two-dimensional space-time, when the state of $\mathcal{S}_{KG}$ does not depend on the temporal coordinate. It is described by the wave function $\psi_s(t,x,W)$

$$\psi = \psi_s(t,x,W) = (C_1 e^{ik_1 x + i\varphi_1} + C_2 e^{-ik_1 x + i\varphi_2}) e^{-ik_0 t} \tag{4.14}$$

$$\psi^* = \psi_s^*(t,x,W) = (C_1 e^{-ik_1 x - i\varphi_1} + C_2 e^{ik_1 x - i\varphi_2}) e^{ik_0 t}, \quad k_0^2 - k_1^2 = m^2 \tag{4.15}$$

where $W = \{k_0, k_1, C_1, C_2, \varphi_1, \varphi_2\}$ are real parameters. Then one obtains for the current and energy-momentum tensor

$$j_0 = k_0[C_1^2 + C_2^2 + 2C_1 C_2 \cos\nu], \quad j_1 = -k_1(C_1^2 - C_2^2), \tag{4.16}$$

$$T^{00} = k_0^2(C_1^2 + C_2^2) - 2m^2 C_1 C_2 \cos\nu, \quad T^{01} = -k_0 k_1 (C_1^2 - C_2^2),$$
$$T^{11} = k_1^2(C_1^2 + C_2^2), \quad \nu = 2k_1 x + \varphi_1 - \varphi_2 \tag{4.17}$$

Let us consider a state of $\mathcal{S}_{KG}$ in the form of two wave packets of size $L \gg m^{-1}$ which pass one through another in vicinity of the coordinate origin. The state can be described by the wave function

$$\psi(t,x,q_1) = \int \psi_s(t,x,W)\Psi_0(q_1 - k_1)dk_1 \tag{4.18}$$

where ψ_s is determined by Eq.(4.14) with

$$W = \{\sqrt{m^2 + k_1^2}, k_1, 1, 1+\alpha, 0, 0\} \tag{4.19}$$

$$\Psi_0(k_1) = A(L^2/2\pi)^{1/2} \exp(-L^2 k_1^2/2), \quad \epsilon = (Lm)^{-1} \ll 1, \quad \alpha, A = \text{const} \tag{4.20}$$

Let us consider the region

$$\mathcal{R}_s: \quad |\, t \,| < L\epsilon^{-1/2} = m^{-1}\epsilon^{-3/2}, \quad |\, x \,| < L\epsilon^{-1/2} = m^{-1}\epsilon^{-3/2} \tag{4.21}$$

Calculation of the integral (4.18) inside this region leads to the following result

$$\psi(t,x,q_1) = \psi_s(t,x,W(q_1,T,X)) + o(\epsilon^2) \tag{4.22}$$

Now parameters $W = \{k_0, k_1, C_1, C_2, 0, 0\}$ of the function (4.14) are some functions of $q_1, T = \epsilon t, X = \epsilon \mathrm{x}$. The parameter $\epsilon = (Lm)^{-1} \ll 1$

is small, and parameters W are slowly varying functions of x and t. q_1 is a parameter which is used instead of the parameter k_1. Inside $\mathcal{R}_s$ the following approximate relations take place

$$C_1 = A\exp\left[-m^2(X+vT)^2/2 + O(\epsilon^{1/2})\right], \tag{4.23}$$

$$C_2 = A(1+\alpha)\exp[-m^2(X-vT)^2/2 + O(\epsilon^{1/2})], \tag{4.24}$$

$$v = q_1(m^2+q_1^2)^{-1/2}, \quad \Gamma = (m^2+q_1^2)^{1/2}m^{-1} \tag{4.25}$$

$$k_0 = \sqrt{m^2+k_1^2} = m\Gamma + O(\epsilon^2), \quad k_1 = q_1 + O(\epsilon^2) \tag{4.26}$$

The region $\mathcal{R}_c :\mid t \mid< L, \quad \mid x \mid< L$ can be considered approximately as a region, where the wave packets overlap.

The state ψ of the system $\mathcal{S}_{KG}$ can be described by means of world lines tangent to the vector j^l for the state ψ. The schematic picture of world lines associated with the vector j^l for the wave function (4.22) with small enough $\alpha > 0$ is shown in Fig.4a. In reality the world lines oscillate rather strongly in the region, where $C_1 \simeq C_2$. This oscillations are not shown in Fig.4a.

It is possible to construct schematic picture of world lines in the case, when the stationary wave function ψ_s is substituted in Eq.(4.18) by the wave function ψ_u of an uniform state which is defined by the relation

$$\psi = \psi_u(t,x,W) = (C_1e^{ik_0t+i\varphi_1} + C_2e^{-ik_0t+i\varphi_2})e^{ik_1x} \tag{4.27}$$

where W are the same parameters, as in Eq.(4.14). In this case instead of Eqs.(4.16)-(4.20) one obtains

$$j_0 = -k_0(C_1^2 - C_2^2), \quad j_1 = -k_1[C_1^2 + C_2^2 + 2C_1C_2\cos\nu], \tag{4.28}$$

$$T^{00} = k_0^2(C_1^2+C_2^2), \quad T^{01} = k_0k_1(C_1^2-C_2^2),$$
$$T^{11} = k_1^2(C_1^2+C_2^2) - 2m^2C_1C_2\cos\nu, \quad \nu = 2k_0t + \varphi_1 - \varphi_2 \tag{4.29}$$

Corresponding picture of world lines associated with the current j^l for the wave function (4.27) with parameters (4.23)–(4.26) is shown in Fig. 4b.

According to the definition of the world lines in Fig. 4 are lines tangent to corresponding currents. They can be interpreted as mean

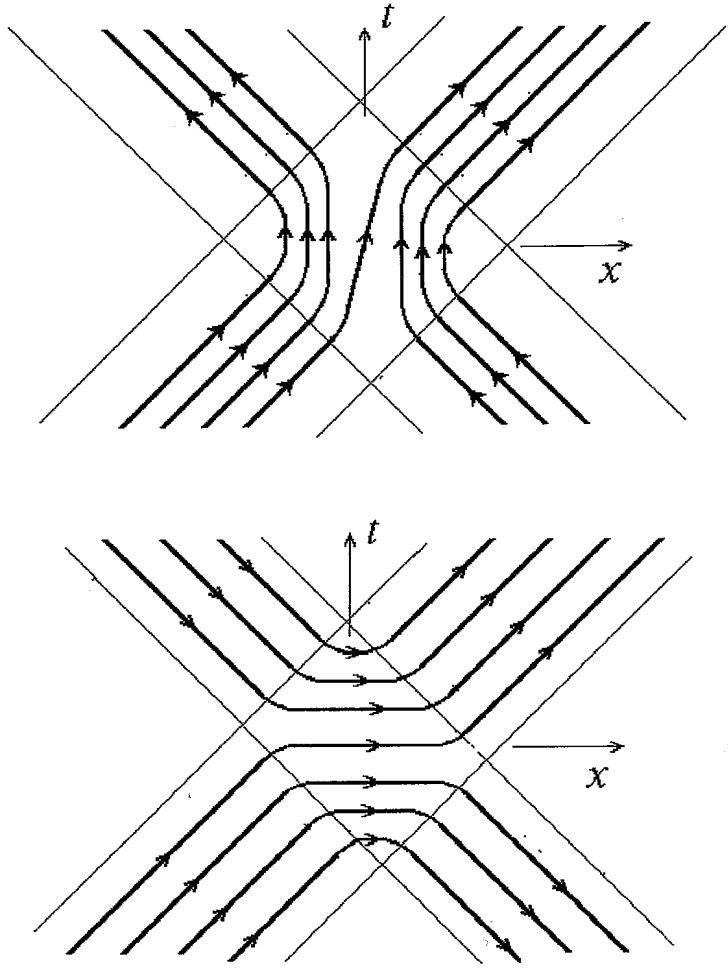

Figure 4: Mean world lines of quantum particles and antiparticles: a) the case of only particles, b) the case of particles and antiparticles.

*WL*s describing mean motion of *SWL*s. In both cases (4.15) and (4.27) the κ-field can be expressed by the formula (2.10)

$$\kappa_k = \frac{1}{2}\partial_k \log \rho, \quad \rho = C_1^2 + C_2^2 + 2C_1C_2\cos\nu \tag{4.30}$$

The main contribution is given by the rapidly oscillating last term. κ-field is very small outside the region $\mathcal{R}_c$, where one of $C_k, \quad k = 1, 2$ is very small (of the order ϵ). Inside the region $\mathcal{R}_c$ it is convenient to estimate the κ-field, calculating the mass ratio Kdefined by Eqs.(2.2), (4.30)

$$K^2 - 1 = m^{-2}\rho^{-1/2}\partial_l\partial^l\rho^{1/2} \tag{4.31}$$

Estimation of K^2 inside $\mathcal{R}_c$ gives for the stationary state ψ_s and the uniform state ψ_u respectively

$$K_s^2 \simeq 1 + 2(k_1/m)^2 > 1, \quad K_u^2 \simeq 1 - 2(k_0/m)^2 < 0 \tag{4.32}$$

Let us compare classical *WL*s in Fig. 1b, 2b with the quantum *WL*s in Fig. 4a, 4b. In Fig. 1b, 2b one can see interacting mean *WL*s. The reason of the interaction is a reconnection effect which cannot be described dynamically. The reason of interaction of quantum *WL* in Fig. 4 is the κ-field. One concludes that the κ-field describes the reconnection, or at any rate imitates it. The independence of deterministic *WL*s in the ensembles $\mathcal{E}_1$ and $\mathcal{E}_2$ associates with the independence of the wave packets, passing one through another "without interaction". The last expression "without interaction" means that outside the overlapping region each of the wave packets evolves in such a way, as though another wave packet were absent (or a sum of solutions is a solution of the dynamic equation, written in terms of the wave function).

All this can be interpreted in the sense that the κ-field describes the reconnection (exchange) effect, or at least imitates it dynamically.

5 Quantum properties as dynamic effects

Let us try to explain quantum effects from point of view of the *SET*-scheme, i.e. without a reference to the *QM* principles. Of course, it is impossible to explain all quantum effects in one paper, and only the main quantum effects (uncertainty principle, interference, diffraction, appearance of discrete quantum numbers) will be considered.

Uncertainty principle. Let a beam of electrons with the momentum $\mathbf{p} = \{0, 0, p_3\}$ drop on a diaphragm placed in the plane of the

coordinate axis x^1, x^2. Let the diaphragm have a narrow slit of the width $\Delta x \ll \hbar/p_3$ along the axis x^2, and the width of the beam in the direction x^1 $L_1 \gg \Delta$x. Then only small part of electrons passes through the diaphragm. This passage of electrons through the slit can be considered as a measurement of the beam electron position in the direction x^1. The energy of the κ-field is very small before a passage through the slit, but it increases inside the slit and becomes of the order $U_\kappa = (2m)^{-1}(\hbar\partial_1\rho/\rho)^2\rho \simeq (2m)^{-1}(\hbar/2\Delta x)^2\rho$. After the passage of the beam through the slit the potential energy U_κ of the κ-field turns to the kinetic energy of electrons which passed through the slit. As it follows from dynamic equations only the momentum component p_1 is changed, and this energy is added only to the component $p_1^2/2m$ of the kinetic energy. The mean value of this energy component corresponds to $\langle p_1^2\rangle \simeq (\hbar/2\Delta x)^2$. It can be interpreted as a result of the measurement of the electron position in the x^1 direction. Conventionally this result is considered as a corollary of the uncertainty principle, but here it is simply a result of dynamic equations. One may put the question. From where do the passing electrons take the additional energy? The formal answer looks as follows. The result follows from the dynamic equations for the quantum fluid. The observed effect reminds of cumulative effect in the usual hydrodynamics. Another answer is as follows. Electrons of the beam move stochastically, and the slit selects only those of them which have the large enough energy. In other words, the electrons of the passed beam take additional energy from the electrons absorbed by the diaphragm. The more narrow slit, the more larger part of electrons is absorbed, and the more larger additional energy is added to the passed electrons.

Diffraction. Let there be a screen in the back of the diaphragm. The electron are distributed over the screen in the form of narrow parallel straightlinear zones, but not uniformly. This effect is known as a diffraction effect. It can be calculated on the base of dynamic equations and expression for the current j^k. As far as j^k is an attribute of dynamics, it means that the diffraction can be calculated *only* on the base of dynamics, i.e. without a usage of statistical propositions. Usually, the diffraction is calculated in terms of the wave function ψ, because dynamic equation in terms of ψ are most simple for solution (linear differential equation with constant coefficients). This is convenient, but it is not conceptual. The dynamic equations can be solved in any variables. The result is to be the same.

Interference. The interference effects are calculated on the base dynamic equations and expressions for the current j^k in the same way, as diffraction effects. They are also explained on the base of only dynamics.

Discrete quantum numbers. Conventionally the discrete quantum numbers are considered as eigenvalues of linear operators associated with physical quantities. It seems that it is impossible to explain their appearance without a reference to the statistical propositions (or QM principles) which describe correspondence between the physical quantities and corresponding linear operators.

However, let us note that the Hamiltonian $\hat{H}$, describing the evolution of the dynamic system, can be obtained directly from dynamic equations written in terms of the wave function ψ. Thus, the linear operator $\hat{H}$ can be derived from dynamics only. Eigenstates of the Hamiltonian are stationary states, that also follows from the dynamics (without a reference to the statistical propositions). Although QM dynamic equations admit existence of atoms at non-stationary states, nevertheless all atoms and molecules exist practically all the time at the stationary states, because only stationary states are stable. Indeed, being at the non-stationary state, an atom, or a molecule emits the electromagnetic radiation until it becomes at a stationary state [**?**]. As far as for the non-relativistic case the Hamiltonian coincides with the energy, one can measure eigenvalues of the Hamiltonian by spectrometric methods, measuring frequency ω (and the energy $\hbar\omega$) of the emitted electromagnetic radiation. In other words, measuring eigenvalues of the Hamiltonian, one uses only dynamic equations and the energy conservation law, which is also a dynamic relation.

Only position can be measured instantly. Other physical quantities such as the momentum and functions of the momentum need a rather long time for exact measurement. A physical quantity can be measured, provided its operator commutes with the Hamiltonian and its eigenvalues enable to label stationary states [**?**]. Thus, if some operator $\hat{S}$ commute with the Hamiltonian $\hat{H}$, the eigenvalues of $\hat{S}$ label the stationary states and can be measured by an identification of the stationary state, that can be made on the base of the dynamics only. If $\hat{S}$ does not commute with $\hat{H}$, the eigenvalues of $\hat{S}$ cannot be measured, and $\hat{S}$ cannot be considered as an observable quantity.

Let us analyze the Stern-Gerlach experiment [**?**]. Passing across a strongly non-uniform magnetic field, a beam of atoms is splitted into two discrete beams. A reason of such a splitting is explained in the

following way. Energy of nth stationary atom state depends on an external magnetic field by means of an additional term $\Delta E_n = \Delta E_n(\mathbf{H})$. The magnetic moment $\boldsymbol{\mu}_n$ of the nth stationary state is defined by the relation

$$\boldsymbol{\mu}_n = \frac{\partial \Delta E_n(\mathbf{H})}{\partial \mathbf{H}} \tag{5.1}$$

If $\boldsymbol{\mu}_n$ does not depend on $\mathbf{H}$, the force, acting on the atom has the form

$$\mathbf{F}_n = -\nabla \Delta E_n(\mathbf{H}) = \mu_{n\alpha} \nabla H_\alpha \tag{5.2}$$

Let a beam of atoms is found at such a state, where there are two stationary states, having different projections μ_H of the magnetic moment onto the magnetic field. In the non-uniform magnetic field the force (5.2) is different for different atoms, and the beam is splitted into discrete beams, provided stationary states are discrete. Using the relation (5.2), one can calculate projections $\mu_H = \boldsymbol{\mu}_n \mathbf{H}/H$ of the magnetic moment on the base of the measured split of the beam.

Thus, in the Stern-Gerlach experiment only the projection μ_H of the magnetic moment of the atom stationary state is measured directly (Note that μ_H can be determined also from spectrometric experiments of the atom in the magnetic field). The statement that in the Stern-Gerlach experiment one measures the spin projection σ_H onto the magnetic field uses the additional operator relation

$$\boldsymbol{\mu} = -\frac{e\hbar}{2mc}\boldsymbol{\sigma} \tag{5.3}$$

based on an usage of quantum principles.

One can consider this relation as a definition of the spin, or as a corollary of the statistical propositions. In any case the Stern-Gerlach experiment tests only discreteness of stationary states, but not the discreteness of such quantities as the magnetic moment, or spin. In other words, the Stern-Gerlach experiment tests only dynamics, but not the statistical propositions (QM principles).

6 Concluding Remarks

Formal results, concerning the properties and interpretation of the κ-field, are presented in the text of the paper and in the abstract. One should like to mention here about some informal results of the above

consideration. First, the SET approach demonstrates that the quantum mechanics can be constructed as a dynamic construction, where the statistical propositions (and QM principles) play a secondary role.

Second, the SET approach appears to be more physical and reasonable, than the conventional one which describes all quantum effects in terms of wave functions. The conventional approach cannot exist without a reference to a wave function, although from the physical point of view the wave function is something indefinite. In the SET approach the wave function is not a vehicle of quantum properties. It is simply a way of description of a continuous medium. The wave function (2.33) is constructed of potentials κ, $\boldsymbol{\xi}$. The Lagrangian coordinates $\boldsymbol{\xi}$ are determined to within a rather general transformation (2.23) that determines an indefiniteness of the wave function.

Third, the SET approach is a more general approach which permits to develop the quantum theory in an alternative direction. Especially it concerns the pair production problem. The statement of this problem in the up-to-date QFT seems rather artifical as compared with the statement of this problem in the SET approach, where the pair production is an intrinsic property of the κ-field, and one needs only to investigate and to develop properly this property.

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