Why we observe an almost classical spacetime

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Abstract. We argue that, in order to obtain decoherence of spacetime, we should consider quantum conformal metric fluctuations of spacetime. This seems to be the only required environment in the problem of selfmeasurement of spacetime in quantum gravity formalism.

Résumé. Nous justifierons que, pour obtenir la décohérence de l'espace-temps, il faut considerer des fluctuations quantiques conformes de la métrique spatio-temporelle. Il nous semble que c'est le seul environnement nécessaire en regardant le problème de l'auto-mesure de l'espace-temps dans le formalisme quantique de la gravitation.

1 Introduction

It has been recently suggested (see e.g.,[1] and [2]) that it would be possible to demonstrate, by means of a fully quantum treatment, that spacetime becomes classical by a process similar to that of the emergence of classical properties of a macroscopic system in standard decoherence models. The latter arises from the quantum mechanical entanglement of the states of the system with its environment: when introduced in the time evolution given by the Schroedinger equation, the environment *measures* certain properties of the system thereby destroying the off-diagonal terms of the density matrix in space representation.(see [3]).

On the other hand, the spacetime structure is classically obtained from the matter distribution in the universe through Einstein's field equations. Nevertheless, the field equations do not tell us, by themselves, which geometry is to be the geometry of spacetime, because they say nothing *a priori* about the matter distribution of the universe — Indeed, they just state which geometries can be possibly associated with a given matter distribution. We think this observation to be in order since it is a general belief that the environment in the quantum mechanical treatment of the problem of *classicity* of spacetime structure (i.e., the existence of correlations of the quantum states, in the WKB sense, over the classical allowed trajectories) should also involve some distribution of matter [4][5]. On the other hand, when spacetime is non-classical, we do not have necessarily to "believe" in Einstein's field equations [6], nor to think that the matter content of the universe "tells space to behave classically" in Joos' words [1].

In this paper a new line of thought will be followed. As long as the very nature of spacetime should take into account, in quantum theory, the properties of vacuum, [1] then there should exist, in this framework, a natural extension of what is considered as the environment. We think that the *classicity* of spacetime is a consequence of the existence of some allowed degrees of freedom in the evolution of the manifold representing the properties of gravitational empty solutions, which , without the need of any particular matter distribution, would lead to *classicity.* As a simple model we propose the existence of conformal metric fluctuations in the classical domain. The classical allowed solutions have, of course, no dynamics at all [2], yet the corresponding quantum dynamics will prove to be non trivial, giving rise to the expected entanglement of the states of such fluctuations with the non-quantized variable of the cosmological model —the scale factor.

2 Classical dynamics of the conformal field

¿From the hypothesis of the previous section, we are now going to develop the dynamics of a model of conformal fluctuations in spacetime. For the sake of mathematical simplicity, we will establish the hamiltonian formulation of gravity just in the isotropic cosmological model.

We are interested, for the time being, in the classical behaviour of spacetime, then, in order to derive the dynamics, we will use the Hilbert action corresponding to the following metric

$$ds^2 = l^2[N(t)^2 dt^2 - a(t)^2 d\sigma^2] \tag{1}$$

[1] When dealing with the theory of gravity there exists no vacuum at all, only *empty space solutions.*

[2] if it were not so, then, the initial value problem in general relativity would be physically inconsistent

where $d\sigma^2 = \gamma_{ij} dx^i dx^j$ is the metric of the space-like slices of the manifold and $l^2 = 2/3\pi m_p^2$

The Hilbert action (using natural units) is (see [7])

$$S = \frac{1}{16\pi} \int R(g)^{1/2} dx^4 = \frac{1}{2} \int (-\frac{a}{N} \dot{a}^2 + Na) dt \tag{2}$$

where we have integrated the lagrangian density over the three spheres of constant time. Hence, we obtain a lagrangian functional for the metric variables

$$L = \frac{1}{2}(-\frac{a}{N}\dot{a}^2 + Na) \tag{3}$$

Now consider the transformation

$$dt \to e^{\phi} dt = d\tilde{t} \tag{4}$$

for some unspecified scalar function ϕ. We are interested in those scale functions $a(t)$ with the conformal transformation property given by

$$a(t) \to a(t)e^{\phi} = \tilde{a}(\tilde{t}) \tag{5}$$

This, of course, is equivalent to studying the dynamics of the conformal functions ϕ

$$d\tilde{s}^2 = e^{2\phi} ds^2 \tag{6}$$

Then the transformed metric reads

$$d\tilde{s}^2 = l^2[\tilde{N}(\tilde{t})^2 d\tilde{t}^2 - \tilde{a}^2(\tilde{t}) d\sigma^2] \tag{7}$$

where $\tilde{N}(\tilde{t}) = N(t)$ and $\tilde{a}(\tilde{t}) = e^{\phi} a(t)$.

But taking into account also the general covariance of the theory and comparing (1) and (7), we obtain an identical expression for the transformed lagrangian in terms of its new metric quantities

$$\tilde{L} = \frac{1}{2}(-\frac{\tilde{a}}{\tilde{N}}(\frac{d}{d\tilde{t}}\tilde{a})^2 + \tilde{N}\tilde{a}) \tag{8}$$

Now, using the "old coordinates" we get, in terms of ϕ

$$\tilde{L} = \frac{1}{2}(-\frac{e^{\phi} a(t)}{N(t)} e^{-2\phi} [\frac{d}{dt}(e^{\phi} a(t))]^2 + a(t)N(t)e^{\phi}) \tag{9}$$

We could consider $N(t)$, $a(t)$ and $\dot{a}(t)$ as known functions of time; therefore, as we are just interested in the classical dynamics which corresponds to the ϕ field, we can define, without any loss of generality, a particular set of functions. Let us consider a time reparametrization such that $N(t) = 1$. Upon doing this we finally get

$$\tilde{L_a}(\phi, \dot{\phi}) = \frac{1}{2} a\{1 - (\dot{\phi} a + \dot{a})^2\} e^{\phi} \tag{10}$$

We must now develop the theoretical consequences involved in the transformation properties of this lagrangian. Thus, as we have defined ϕ to be a generic scalar function of time, we can explicitly state its transformation rule under a time reparametrization, i.e.,

$$\begin{aligned} a \to \tilde{a} &= a + \alpha(a) & (11) \\ \phi(a) \to \tilde{\phi} &= \phi + \delta\phi & \\ \delta\phi &= \dot{\phi}\alpha(a) & (12) \end{aligned}$$

where we have taken into account that $\tilde{\phi} = \phi(\tilde{a})$.

In addition, the lagrangian transforms as a density, i.e.,

$$\tilde{L_a}(\tilde{\phi}, \dot{\tilde{\phi}}) = L_a(\phi, \dot{\phi}) + \delta L \tag{13}$$

where,

$$\delta L = \frac{d}{da}(L_a \alpha(a)) \tag{14}$$

On the other hand, using the transformation of ϕ (see (12))

$$\begin{aligned} \delta L &= \frac{\partial L_a}{\partial \phi}(\dot{\phi}\alpha(a)) + \frac{\partial L_a}{\partial \dot{\phi}} \frac{d}{da}(\dot{\phi}\alpha(a)) + \alpha(a)\frac{\partial L_a}{\partial a} = \qquad (15) \\ &= (\frac{\partial L_a}{\partial a} + \frac{\partial L_a}{\partial \phi}\dot{\phi} + \frac{\partial L_a}{\partial \dot{\phi}}\ddot{\phi})\alpha(a) + \frac{\partial L_a}{\partial \dot{\phi}}\dot{\phi}\dot{\alpha}(a) = \\ &= \frac{dL_a}{da}\alpha(a) + \frac{\partial L_a}{\partial \dot{\phi}}\dot{\phi}\dot{\alpha}(a) = \\ &= \frac{d(\alpha(a)L_a)}{da} + (\frac{\partial L_a}{\partial \dot{\phi}}\dot{\phi} - L_a)\dot{\alpha}(a) = \frac{d(\alpha(a)L_a)}{dt} \end{aligned}$$

implying the typical hamiltonian constraint

$$H_a \equiv \frac{\partial L_a}{\partial \dot{\phi}}\dot{\phi} - L_a = 0 \tag{16}$$

It is now straightforward to obtain this hamiltonian function by means of the Legendre transformation (in terms of its coordinates and canonical momenta)

$$H_a(\phi, p_\phi) = p_\phi \dot{\phi} - L_a(\phi, \dot{\phi} \to p_\phi) \tag{17}$$

where

$$p_\phi = \frac{\partial L_a}{\partial \dot{\phi}} = -a^2 e^\phi (\dot{a} + \dot{\phi} a) \tag{18}$$

which can also be inverted ($\dot{\phi} \to p_\phi$)

$$\dot{\phi} = -\frac{\dot{a}}{a} - \frac{1}{a^3} p_\phi e^{-\phi} \tag{19}$$

Finally

$$H_a(\phi, p_\phi) = -\{\frac{1}{2} a e^\phi + \frac{p_\phi \dot{a}}{a} + \frac{p_\phi^2 e^{-\phi}}{2a^3}\} \tag{20}$$

Notice that we can cast this expression in a simpler, suggestive way

$$H_a(\phi, p_\phi) = -[p_\phi - p_0(\phi, a^2)]^2 \frac{e^{-\phi}}{2a^3} + (\dot{a}^2 - 1)\frac{e^\phi a}{2} \tag{21}$$

where

$$p_0(\phi, a^2) = -\dot{a} a^2 e^\phi \tag{22}$$

Taking into account the negativeness of the energy for the gravitational field, we have to consider those cosmological models satisfying $\dot{a}^2 \leq 1$. Therefore, upon assuming this, the global negative sign of (21) is a typical *footprint* of the fact that ϕ is indeed a gravitational field.

For the sake of mathematical simplicity let us consider the particular ansatz $\dot{a}(t)^2 = 1$, and $a(t)^2 = t^2$; hence, we will identify cosmological time and scale factor hereafter. Now, the constraint $H_a = 0$ implies

$$[p_\phi - p_0(\phi, a^2)]^2 e^{-\phi} = 0 \tag{23}$$

and, using again (18) ($p_\phi \to \dot{\phi}$) we obtain the classical solutions of the conformal field

$$p_\phi \quad = \quad p_0(\phi, a) \tag{24}$$

or,

$$a^2 e^\phi (1 + \dot{\phi} a) \quad = \quad a^2 e^\phi$$

that is $\dot{\phi} = 0$. But this is the condition required in the classical theory since, in that case, $d\tilde{t} = e^{\phi} dt = d(e^{\phi} t)$. This is now integrated to get $\tilde{t} = e^{\phi} t$, which is also the prescription for having $\tilde{a}(\tilde{t}) = e^{\phi} a(t)$ if and only if $a(t) = t$; indeed this has been our choice of the scale factor function.

3 Wheeler-DeWitt formalism in minisuperspace

In spite of the trivial classical dynamics associated to the conformal function ϕ, equation (21) could be treated quantum mechanically as a hamiltonian system. Thus, the quantum dynamics of this system could, in principle, be expressed by means of the hamiltonian constraint (16) together with the standard rule for the canonical momentum in the operator formalism, (i.e., $p_\phi \to \hat{p}_\phi = i\frac{\partial}{\partial\phi}$)

$$H_a(\phi, i\frac{\partial}{\partial\phi})\tilde{\Psi}(a^2;\phi) = 0 \tag{25}$$

The above equation is just the Wheeler-DeWitt equation

$$\{\{e^{-p\phi}[i\frac{\partial}{\partial\phi} + \dot{a}a^2 e^{\phi}]^2 e^{(p-1)\}\phi} + (\dot{a}^2 - 1)e^{\phi}a^4\}\tilde{\Psi}(a^2;\phi) = 0 \tag{26}$$

p denotes the factor ordering ambiguity of the theory.

Notice that the effect of the background metric is just a shift in the momentum of the conformal field. On the other hand, any selection for the factor ordering leads to a complex time-dependent wave equation for pure gravity whose solutions (for $\dot{a}^2 = 1$) are given by

$$\tilde{\Psi}(a^2;\phi) = \Psi(a)B_p(\phi)e^{ia^2 e^{\phi} - 1} \tag{27}$$

Here, a is considered a *c-number*, $B_p(\phi)$ is a polynomial and $\Psi(a)$ is again a constant with respect to the ϕ-field; the latter should be identified with the wave amplitude of the scale factor in its configuration space (i.e., when it were not considered as a classical variable). These wave functions have not a natural normalization. On the other hand, if we take $B_p = 1$, then, we could obtain a set of normalized wave functions in the sense of the Dirac delta function. To see this, we can study the classical equation corresponding to the momentum constraint when $\dot{a}^2 = 1$ (see (24)). Upon making the standard replacement $p_\phi \to \hat{p}_\phi = i\frac{\partial}{\partial\phi}$ we have

$$[ie^{-\beta\phi}\frac{\partial}{\partial\phi}e^{(\beta-1)\phi} + a^2]\chi_{a^2}(\phi) = 0 \tag{28}$$

$\chi_{a^2}(\phi)$ being the solutions of the momentum constraint and β representing again the uncertainty in the factor ordering. Thus, $\beta = 1$ is the natural choice since, in this case, we can write the momentum constraint in terms of the relevant gravitational quantity, i.e., the conformal field $\gamma \equiv e^{\phi}$; this assumption being done, (29) takes the simpler form

$$i\frac{\partial}{\partial\gamma}\chi_{a^2}(\gamma) = -a^2\chi_{a^2}(\gamma) \tag{29}$$

where we have put $e^{-\phi}\frac{\partial}{\partial\phi} = \frac{\partial}{\partial\gamma}$. But, as far as we are interested, in this particular case, in the continuum spectrum of γ, we can try and normalize the wave functions using the standard quantum mechanical prescription

$$\int \chi_{a^2}(\gamma)\chi^*_{\tilde{a}^2}(\gamma)d\gamma = \delta(a^2 - \tilde{a}^2) \tag{30}$$

or

$$\chi_{a^2}(\gamma) = (\frac{1}{2\pi})^{1/2}e^{ia^2\gamma} \tag{31}$$

Then, by using this eigenfunction basis we can also construct the operator whose eigenvalues coincide with the square of the scale factor values corresponding to the background.

$$\hat{a^2_\gamma} \equiv -\hat{p}_\gamma = -[i\frac{\partial}{\partial\gamma}] \tag{32}$$

Hence, the eigenstate equation for this operator reads

$$\hat{a^2_\gamma}\chi_{a^2}(\gamma) = a^2\chi_{a^2}(\gamma) \tag{33}$$

We could then try and obtain the quantum evolution of the states of the conformal fluctuations in terms of the cosmological time.

On the other hand, a general solution of (26) is given by (upon defining $\gamma \equiv \frac{\dot{a}}{|\dot{a}|}e^{\phi}$)

$$\tilde{\Psi}_{\pm}(a;\gamma) = \tilde{\Psi}_L(a;\gamma) \pm \tilde{\Psi}_R(a;\gamma)$$

where the left ($\Psi_L(a;\gamma)$) and right ($\Psi_R(a;\gamma)$) solutions of the Wheeler-DeWitt equation are given by

$$\tilde{\Psi}_L(a;\gamma) = \begin{cases} = \Psi(a)e^{i|\dot{a}|a^2\gamma}e^{+a^2(1-\dot{a}^2)^{1/2}(\gamma+1)} \text{ if } \gamma < -1 \\ = \Psi(a)e^{i|\dot{a}|a^2\gamma}e^{-a^2(1-\dot{a}^2)^{1/2}(\gamma+1)} \text{if } \gamma > -1 \end{cases}$$

$$\tilde{\Psi}_R(a;\gamma) = \begin{cases} = \Psi(a) e^{i|\dot{a}|a^2\gamma} e^{-a^2(1-\dot{a}^2)^{1/2}(\gamma-1)} \text{ if } \gamma > 1 \\ = \Psi(a) e^{i|\dot{a}|a^2\gamma} e^{+a^2(1-\dot{a}^2)^{1/2}(\gamma-1)} \text{if } \gamma < 1 \end{cases}$$

The wave functions are peaked about the classical allowed solutions, i.e. $\gamma^2 = 1$; moreover, we can not single out any particular solution, thus leading to interference between the expanding ($\gamma = +1$) and collapsing ($\gamma = -1$) classical solutions (but for the case $\dot{a}^2 = 1$, i.e., a matter free cosmolgical model where efective decoherence could be obtained considering the conformal field as the environment).

4 Time evolution equation

Indeed, the problem of time in quantum gravity is that of giving sense to the Wheeler-DeWitt formalism. This is so since, in the context of quantum cosmology, waves have a trivial evolution. It comes from the fact that in quantum gravity the physical interest is represented by the scale factor itself (strictly speaking three geometries and their configuration space, i.e., *superspace*) which could be quantized in DeWitt sense of quantum gravity (see, for instance [7] -[9]), i.e., the squared of the wave function would lead to a probability for the quantized metric of spacetime. Then, in dealing with this problem, there is no evolution whatsoever; in fact, we are generally working in minisuperspace, that is the quantization is often done just for the parametric time corresponding to the scale factor. Hence, time lies out of the quantum formalism as a result of the hamiltonian constraint, i.e., of the general covariance of the theory.

Our problem is different. In spite of the fact that we have been dealing with the parametric time, $a(t)$, we did not consider the possible quantum states for $a(t)$ itself, i.e., we are not interested in defining a probability for wave functions $\Psi(a)$. Yet, we are concerned in the problem of the quantization of the conformal field (something analogous to the *breathing modes* of the cosmological model). This led us to studying wave functions defined, in principle, on the continuum spectrum; such states were denoted in the previous section by $\chi_{a^2}(\gamma)$. Moreover, $\{\chi_{a^2}(\gamma)\}$ could be used as a orthonormal basis in order to develop the cosmological time evolution of generic operators, for instance, that corresponding to the momentum (p_γ) or the one of the scale factor squared

($\hat{a^2_\gamma}$, see (32) and (33)).To see this, we observe the cosmological time-reversal invariance of our solutions in (31). This comes from the character of a time-evolution wave equation which possesses $\chi_{a^2}(\gamma)$:

$$-i\frac{\partial}{\partial a}\Psi(a,\gamma) = 2a\gamma\Psi(a,\gamma) \tag{34}$$

Here we have defined $\Psi(a,\gamma) \equiv \chi_{a^2}(\gamma)$.

Now applying the time reversal operator gives,

$$\hat{T}\{\Psi(a,\gamma)\} = \Psi(-a,\gamma) \tag{35}$$

while for the complex conjugate operator we obtain

$$\hat{C}\{\Psi(a,\gamma)\} = \Psi^*(a,\gamma) \tag{36}$$

Nevertheless, according to (34)

$$\hat{T}\{\Psi(a,\gamma)\} \neq \hat{C}\{\Psi(a,\gamma)\} \tag{37}$$

Then (34), though similar to the Schroedinger equation, is not to be regarded so in a very strict sense.

Now the physical meaningful cosmological solutions should not depend on the environment. Moreover, we have to take into account that we still do not know the properly normalized solutions of the whole Wheeler-DeWitt operator which, in general, should depend on the cosmological model (i.e., it would develop other solutions when the particular ansatz $\dot{a}^2 = 1$ were not made). In addition, $\chi_{a^2}(\gamma)$ is just one solution corresponding to the eigenstates of the momentum operator belonging to the kernel of the hamiltonian constraint; this lack of information should be considered in our model upon computing the reduced density matrix for the quantum states of the envinonment; the latter is being done upon tracing out our solutions over the internal degrees of freedom for γ.

$$\tilde{\rho}(a^2,\tilde{a}^2) = \int_{-\infty}^{\infty} \chi_{a^2,\dot{a}}(\gamma)\chi^*_{\tilde{a}^2,\dot{a}}(\gamma)D_{\dot{a}}\gamma\Psi(a)\Psi^*(\tilde{a}) \tag{38}$$

Here,$D_{\dot{a}}\gamma$ denotes a measure corresponding to the environment when a general solution is considered.

5 Quantum Correlations

We have obtained a time evolution equation for the states of the quantum conformal metric fluctuation corresponding to an isotropic background gravitational field in terms of the cosmological time a. Then, in order to obtain a generalization of the Schroedinger formalism, we could try developing the initial value problem for this equation.

Yet, as long as we do not know the internal freedom for γ, we should model this ambiguity upon asuming the initial state being correlated with the classical solution (i.e., $\gamma = 1$). Hence,let us consider an initial wave packet given, in terms of the γ-field, by

$$\Psi(0,\gamma) = \sigma^{-1/2}\pi^{-1/4}e^{\frac{(\gamma-1)^2}{\sigma^2}} \tag{39}$$

where σ is a constant which characterizes the minimal uncertainty of the conformal field γ. Then, at a different time, say a, by Fourier-transforming we get from (34)

$$\Psi(a,p_\gamma) = \frac{1}{(2\pi)^{1/2}}\int_{-\infty}^{+\infty} e^{i(p_\gamma\gamma+a^2(\gamma-1)+a^2)}\cdot\Psi(0,\gamma)d\gamma \tag{40}$$

or

$$\Psi(a,p_\gamma) = (\frac{\sigma}{\pi^{1/2}})^{1/2}e^{i(a^2+p_\gamma)}e^{-\frac{\sigma^2}{2}(p_\gamma+a^2)^2} \tag{41}$$

Now, following the usual prescription for the probability amplitude, we get

$$\rho_\sigma(a,p_\gamma) = |\Psi(a,p_\gamma)|^2 = \frac{\sigma}{\pi^{1/2}}e^{-\sigma^2(p_\gamma+a^2)^2} \tag{42}$$

On the other hand, we can also obtain a probability measure not only for the momentum p_γ but also for any function $F(p_\gamma)$; for instance, we may obtain the probability measure for the operator corresponding to the scale factor squared (see (32)),

$$\rho_\sigma(a,\tilde{a}^2) = |\frac{\partial p_\gamma}{\partial a^2}|\rho_\sigma(a,p_\gamma\to\tilde{a}^2) \tag{43}$$

where we have made use of the jacobian of the transformation. That is

$$\rho_\sigma(a,\tilde{a}^2) = \frac{\sigma}{\pi^{1/2}}e^{-\sigma^2(a^2-\tilde{a}^2)^2} \tag{44}$$

Here, we have denoted by $\tilde{a}^2$ the eigenvalues of $\hat{a^2}$.

On the other hand, if we take the limit $\sigma \to \infty$ in (45),

$$\rho(a, \tilde{a}^2) = \lim_{\sigma\to\infty} \rho_\sigma(a, \tilde{a}^2) = \lim_{\sigma\to\infty} \frac{\sigma}{\pi^{1/2}} e^{-\sigma^2(a^2-\tilde{a}^2)^2} = \delta(a^2 - \tilde{a}^2) = \frac{1}{2\tilde{a}}\{\delta(a+\tilde{a}) + \delta(a-\tilde{a})\} \quad (45)$$

6 Discussion

Let us now try and put the above result into context. We have seen that the almost classical character of spacetime could be a consequence of the initial conditions of conformal metric fluctuations. This restricts the set of quantum states to the *expanding* (forward time solution) and *collapsing* ones (backward time solution), therefore, there would exist possible tunneling between both physical allowed states, i.e., *interference*. Moreover, the Weyl tensor is the only geometrical invariant under the conformal field; it implies that since upon considering conformal type fluctuations, we have obtained two possible quantum states, then the way spacetime would become classical (i.e., the selection of one of these two branches of the solution) should only be a consequence of the initial conditions in this quantity. On the other hand, this tensor is precisely that part of the Riemannian curvature which is source free, a fact that strengthens our previous believe that classical properties of spacetime should not be a consequence of any particular model corresponding to a matter field environment (It is also somehow in accordance with Penrose proposal [10].)

We have seen that spacetime may become classical when the density matrix (see (45)) is peaked about the classical allowed configurations. Moreover, the feature of exponential behaviour depends on the initial conditions for the environment (in order to see this, recall that the coherence width of the geometrical scale parameter satisfies $\delta a^2 \sim \sigma^{-1}$). The latter agrees with Hartle's conjecture[11][12] (see also [13]); from this point of view the initial conditions control the extent to which macroscopic states decohere.

A remarkable fact is that the scale factor becomes sharply peaked about the classical solution when the limit $\sigma \to \infty$ is considered, that is, when the initial quantum fluctuations of spacetime are very large; it seems to be a typical behaviour of a phase transition process (i.e. large scale fluctuations in a system also develop far from the equilibrium correlations). The question of the arrow of time in the recollapsing quantum universe has been recently examined by Kiefer and Zeh [14].

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