Probing the Electric Dipole Moment of the Gauge Bosons Using Discrete Time Spin Polarization Precession

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ABSTRACT. By considering the composite structure of a gauge boson we demonstrate the if the gauge boson is allowed to precess in a static electric and magnetic field the precession frequency can be used to place limits on both the electric dipole moment of the gauge bosons and the discrete time interval in discrete time quantum mechanics.

RÉSUMÉ. En considérant la structure composite d'un boson de jauge, nous démontrons que si le boson de jauge peut effectuer un mouvement de précession dans un champ électromagnétique statique, la fréquence de précession peut être utilisée pour assigner des limites à la fois au moment dipolaire électrique des boson de jauge et à l'intervalle de temps discret de la mécanique quantique à temps discret.

1. Introduction

It is generally agreed amongst most students of elementary particle theory that the standard model $(SU(3)_C \times SU(2)_L \times U(1)_y)$ of strong, weak and electromagnetic interactions will be modified at high energy because of its asymmetric electroweak structure and the need for so many input parameters (about 20) necessary to specify the quark and lepton masses, the mixing angles, the coupling constants, and the magnitude of strong CP violation along with the details of symmetry breaking in the Higgs sector[1,2,3]. Left right symmetric models have been proposed to better understand the left-handedness of quarks and leptons in terms of a hierarchy of symmetry breaking,[4] Grand-unification models have been proposed in order to encompass quarks and leptons into the same multiplet and necessitate just one coupling constant at high energy as well as giving a dynamical mechanism for proton decay[5]. Technicolor finds its origin in trying to understand the composite structure of the Higgs sector in terms of the binding of techniquarks[6] and supersymmetry has been proposed as a mechanism to solve the hierarchy problem in not allowing the particles of the electroweak sector to gain arbitrarily large radiative corrections to their masses[7]. Superstring theory offers us a wonderful avenue of unification since it reduces to the Standard Model at lower energies and its embryonic stage requires only two parameters (the sting tension and Reggi slope) to define it[8]. The other attempt at a deeper level of understanding of elementary particles is found in the notion of compositeness[9]. This idea is partly inspired by the observation that all previous systems have admitted to a composite structure (atoms, nuclei, hadrons) and it only seems natural that quarks and leptons in turn should reveal a composite structure. Numerous composite models have been proposed in the past with varying degrees of complexity with the three fermion model of Harari[10] and the fermion-boson model of Fritzsch-Mandelbaum[11] being representative of how the quarks and leptons can be built from sub-quarks with hypercolor providing the binding mechanism in much the same way that color binds the quarks together in hadrons. To probe this composite structure, the conventional path is to study the effect that compositeness has on form factors, anomalous moments and rare decays[12,13,14]. In previous works we have advocated a different probe to compositeness[15], namely if time attains a grainy or discrete like structure at some small scale, the quantum dynamics of the elementary preons would be effected by such discreteness and would give rise to observational consequences in phenomena such as spin-polarization precession and electron-spin resonance. In this regard we have applied discrete time Q.M. to study the structure of leptons using spin polarization precession[16], and the structure of gauge bosons using gauge boson spin precession[17,18]. We have also applied discrete time Q.M. in searching for hidden internal quantum numbers of elementary particles using spin-flip frequencies of particles in an external magnetic field (ref. 15). In another investigation we have shown that these ideas lead to an upper limit for the mass of elementary particles[19]. In ref (18) we briefly discussed the characteristics of spin polarization precession when composite particles precess both in a magnetic field and an electric field. The purpose of this note is to further elaborate on these discussions by demonstrating that if a particle (lepton, gauge boson) has a small electric dipole moment in addition to a magnetic dipole moment it will have an altered precession frequency and there will be an angle of tilt between the axis specified by the magnetic field and the spin precession axis. If a beam of particles is initially polarized by a strong magnetic field and it is allowed to enter a region of crossed electric and magnetic fields we demonstrate the precise measurement of the precession frequency will provide us with information regarding the composite structure of the particle, its electric dipole moment and the associated discrete time effects on the precession frequency.

2. Spin Precession of Gauge Bosons in Crossed Electric and Magnetic Fields as a Probe to Compositeness and Discrete Time Quantum Mechanic

We begin our analysis by considering a 2 preon (spin 1/2) composite gauge boson of charge -e.[20] For the Hamiltonian we have a two preon system with internal spin-spin coupling interacting with an external electric and magnetic field

$$H = M_0c^2 + \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2} + \frac{e_1}{m_1}S_{Z_1}B + \frac{e_2}{m_2}S_{Z_2}B + P(S_{x_1} + S_{x_2})E + g\vec{S}_1.\vec{S}_2 \quad (2.1)$$

Here
$B = z$ component magnetic field,
$E = x$ component electric field,
M_0C^2 = rest mass parameter,
m_1, m_2 = heavy preon masses,
$-e_1, -e_2$ = preon electric charges,
$-P$ = electric dipole moment of composite w^- in units of $\hbar$,
$gS_1\ S_1$ = spin-spin coupling of preons,
in the spirit of (Ref. 17) we look for eigenstates of the Hamiltonian in Eq. (2.1), one result of Ref. (17) was that for two negatively charged preons we must have

$$\frac{e_1}{m_1} = \frac{e_2}{m_2} = \frac{e}{m}$$

in order that the composite gauge boson precesses in a s component magnetic field. For the total wave function we have the product function of the eigenstates

$$\Psi = \left[a_1, \alpha\alpha + a_2\beta\beta + a_3\left(\tfrac{\alpha\beta+\beta\alpha}{\sqrt{2}}\right)\right] U(x_1, x_2)T(t)$$

Here

$$a_1(\alpha\alpha) + a_2\beta\beta + a_3\left(\tfrac{\alpha\beta+\beta\alpha}{\sqrt{2}}\right) = \quad \text{spin function,} \quad (2.2)$$

$U(x_1, x_2)$ = spatial function,

$T(t)$ = temporal part of wave function.

According to Ref. (15, 16, 17) we write the discrete time difference Schrodinger Equation as

$$H\Psi = i\hbar\left[\frac{\Psi\left(t+\frac{\tau}{2}\right) - \Psi\left(t+\frac{\tau}{2}\right)}{\tau}\right] \quad (2.3)$$

(τ = discrete time interval),

which was first investigated by Caldirola[21] and whose mathematical structure was later studied by Santilli[22] et. al. Combining Eq. (2.1), Eq (2.2) and Eq. (2.3) wa have for the eigenstages

$$\left[\frac{e}{m}(S_{z_1} + S_{z_2})B + P(S_{x_1} + S_{x_2})E\right]\left(a_1, \alpha\alpha + a_2\beta\beta + a_3\left(\tfrac{\alpha\beta+\beta\alpha}{\sqrt{2}}\right)\right) = E_2\left(a_1\alpha\alpha + a_2\beta\beta + a_3\left(\tfrac{\alpha\beta+\beta\alpha}{\sqrt{2}}\right)\right) \quad (2.4)$$

$$\left[M_OC^2 - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2}\right]U(x_1, x_2) = E_1U(x_1x_2) \quad (2.5)$$

$$(E_1 + E_2)T(t) = i\hbar\left[\frac{T\left(t+\frac{\tau}{2}\right) - T\left(t+\frac{\tau}{2}\right)}{\tau}\right] \quad (2.6)$$

If we consider the preons to be bound within a one dimensional potential well of length L, we find the following solution to Eq. (2.5) with

$$E_1 = M_0C^2 + \frac{n_1^2h^2}{8mL^2} + \frac{n_2^2h^2}{8mL^2} \quad (2.7)$$

$$U(x_1x_2) = \frac{1}{\sqrt{2}}\left(\tfrac{2}{L}\sin\tfrac{n_1\pi x_1}{L}\sin\tfrac{n_2\pi x_2}{L} - \tfrac{2}{L}\sin\tfrac{n_1\pi x_2}{2}\sin\tfrac{n_2\pi x_1}{L}\right) \quad (2.8)$$

(n_1, n_2 = integers, here and in Eq. (2.7) we assume identical fermionic preons with $m_1 = m_2, e_1 = e_2 = e$), we have also anti-symmetrized Eq. (2.8). the normalized eigenfunctions of Eq.(2.4) are (Ref. 17).

$$E_{2+} = \frac{g\hbar^2}{4} + \sqrt{(PE\hbar)^2 + \left(\tfrac{e\hbar B}{m}\right)^2}$$

$$\Psi_{2+} = \frac{1}{\sqrt{1 + (\frac{PE\hbar}{\sqrt{2}})^2 (\frac{1}{(K-S)^2} + \frac{1}{(K+S)^2})}} \begin{pmatrix} \frac{-PE\hbar}{\sqrt{2}(K-S)} \\ \frac{PE\hbar}{\sqrt{2}(K+S)} \\ 1 \end{pmatrix} \tag{2.9}$$

$$E_{2-} = \frac{g\hbar^2}{4} - \sqrt{(PE\hbar)^2 + (\frac{e\hbar B}{m})^2}$$

$$\Psi_{2-} = \frac{1}{\sqrt{1 + (\frac{PE\hbar}{\sqrt{2}})^2 (\frac{1}{(K-S)^2} + \frac{1}{(K+S)^2})}} \begin{pmatrix} \frac{-PE\hbar}{\sqrt{2}(K+S)} \\ \frac{PE\hbar}{\sqrt{2}(K-S)} \\ 1 \end{pmatrix} \tag{2.10}$$

$$E_{2_0} = \frac{g\hbar^2}{4}$$

$$\Psi_{20} = \frac{1}{\sqrt{1 + 2(\frac{PE\hbar}{\sqrt{2}K})^2}} \begin{pmatrix} \frac{-PE\hbar}{\sqrt{2}K} \\ \frac{PE\hbar}{\sqrt{2}K} \\ 1 \end{pmatrix} \tag{2.11}$$

here

$$\begin{pmatrix} K = \frac{e\hbar B}{m} \\ \\ S = \sqrt{(PE\hbar)^2 + K^2} \end{pmatrix}$$

For each combination of spatial state and spin state with $E = E_1 + E_2$ we have the temporal solution of Eq. (2.2)

$$T(t) = e^{-\frac{2}{\tau}\sin^{-1}(\frac{(E_1+E_2)\tau}{2\hbar})it} \tag{2.12}$$

We now consider that each of three spin states in Eq. (2.9), Eq. (2.10) and Eq. (2.11) has the same spatial function as expressed in Eq. (2.8) with the energy specified by the two integers n_1, n_2. We now consider a linear combination of Eq. (2.9), Eq. (2.10) and Eq. (2.11) with their associated temporal factors as given by Eq. (2.12) multiplied by the spatial state in Eq. (2.8). To calculate the spin polarization precession frequency we note that since the magnetic field is in the z direction and the electric field is in the x direction the spin will precess about a line in the xz plane, actually any initial polarization that has a component of Ψ_{2_0} with one of the other components of Ψ will give a time warying spin polarization along with y axis. We will consider the general case when all three eigencomponents $\Psi_{2(0,\pm)}$ are present.

We now define the following constants

$$A = \frac{PE\hbar}{\sqrt{2}}\ ,\ D = \frac{1}{\sqrt{1 + A^2 (\frac{1}{K-S)^2} + \frac{1}{(K+S)^2})}}\ ,\ E = \frac{1}{\sqrt{1 + \frac{2A^2}{K^2}}}$$

with

$$K = \frac{e\hbar B}{m}, S = \sqrt{(PE\hbar)^2 + K^2} \tag{2.13}$$

Here

$$e_1 = \frac{2}{\tau}\sin^{-1}(\frac{(E_1+E_{2+})\tau}{2\hbar})$$
$$e_2 = \frac{2}{\tau}\sin^{-1}(\frac{(E_1+E_{2-})\tau}{2\hbar})$$
$$e_3 = \frac{3}{\tau}\sin^{-1}(\frac{(E_1+E_{2_0})\tau}{2\hbar}) \tag{2.14}$$

We also note that the eigenvectors of Eq. (2.9), Eq. (2.10) and Eq. (2.11) are in the S_z basis

$$\begin{pmatrix} +1 \\ -1 \\ 0 \end{pmatrix}$$

To evaluate the S_y spin polarization we now take the following linear combination of Eq. (2.9), Eq. (2.10) and Eq. (2.11).

$$\Psi = (a_1\Psi_{2+}e^{-ie_1t} + a_2\Psi_{2\_}e^{-ie_2t} + a_3\Psi_{2_0}e^{-ie_3t}) \times u(x_1, x_2) \tag{2.15}$$

where $U(x_1, x_2)$ is given by Eq. (2.8). When we write out the components of Eq. (2.15) in the $\alpha\alpha, \beta\beta, \frac{\alpha\beta+\beta\alpha}{\sqrt{2}}$ basis using Eq. (2.9), Eq. (2.10) and Eq. (2.11) we have

$$\begin{aligned}\Psi =& U(x_1, x_2)\left[a_1(\frac{-A}{(K-S)D})e^{-ie_1t}\frac{a_2A}{D(K+S)}e^{-ie_2t} - \frac{a_3A}{KE}e^{-ie_3t}\right]\alpha\alpha \\ &+ U(x_1, x_2)[(----)]\beta\beta + U(x_1, x_2)[(----)]\frac{\alpha\beta+\beta\alpha}{\sqrt{2}}\end{aligned} \tag{2.16}$$

We now evaluate the y spin polarization

$$< S_{y_1} + S_{y_2} >= \int_0^L \int_0^L \Psi^+(S_{y_1} + S_{y_2})\Psi dx_1 dx_2\ , \tag{2.17}$$

since the spatial part of Eq. (2.16) us normalized and $S_y\alpha = \frac{i\hbar}{2}\beta, S_y\beta = -\frac{i\hbar}{2}\alpha$, we find the following result of evaluating Eq. (2.17) after a long calculation

$$< S_{y_1} + S_{y_2} >= \frac{i\hbar}{\sqrt{2}} B a_1{}^* a_3 \left[e^{i(e_1-e_3)t} - e^{-(e_1-e_3)t} \right]$$

$$+ \frac{i\hbar}{\sqrt{2}} B a_2{}^* a_3 \left[e^{i(e_2-e_3)t} - e^{-(e_2-e_3)t} \right] \quad (2.18)$$

Here we have imposed the condition

$$a_1{}^* a_3 = a_1 a_3{}^*, a_2{}^* a_3 = a_2 a_3{}^* \quad (2.19)$$

Eq.(2.18) then gives

$$< S_{y_1} + S_{y_2} >= -\sqrt{2} B\hbar a_3 a_1{}^* \sin(e_1 - e_3)t$$

$$-\sqrt{2} B\hbar a_2{}^* a_3 \sin(e_2 - e_3)t \quad (2.20)$$

here

$$B = \frac{2AK}{DE(K^2 - S^2)} - \frac{2A}{KED}$$

We see from Eq. (2.20) that $a_3\ a_1{}^*$ and $a_3 a_2{}^*$ determine the amplitude of the two sinusodial components of (S_y). In Ref. (17) we have derived a formula for gauge boson spin precession in a magnetic field, but Eq. (2.20) applies also to the situation when the electric-dipole interaction is present. We also note that a_1, a_2, a_3 in Eq. (2.15) must fulfill the normalization condition. The two precessions frequencies in Eq. (2.20) are

$$\omega_1 = \frac{2}{\tau} \left[\sin^{-1} \frac{(E_1 + E_{2+})\tau}{2\hbar} - \sin^{-1} \frac{(E_1 + E_{2_0})\tau}{2\hbar} \right] \quad (2.21)$$

$$\omega_2 = \frac{2}{\tau} \left[\sin^{-1} \frac{(E_1 + E_{2_0})\tau}{2\hbar} - \sin^{-1} \frac{(E_1 + E_{2_-})\tau}{2\hbar} \right] \quad (2.22)$$

In Eq. (2.22) we have reversed the sign in the $\sin^{-1}$ function since $e_3 > e_2$, also this will only change the sign of the amplitude but not effect the frequency. If we expand the function in Eq. (2.21) and Eq. (2.22) using $\sin^{-1}(x) \simeq x + \frac{x^3}{3!}$ and Eq. (2.7), Eq. (2.9), Eq. (2.10) and Eq. (2.11) we find

$$\omega_{\substack{1(+)\\2(-)}} = \sqrt{(PE)^2 + \left(\frac{eB}{m}\right)^2} + \frac{\tau^2}{24\hbar^3} \left(\begin{array}{c} 3E_1^2 \left(\sqrt{(PE\hbar)^2 + \left(\frac{e\hbar B}{m}\right)^2} \right) \\ +3E_1 \left(\frac{g\hbar^2}{2} \sqrt{(PE\hbar)^2 + \left(\frac{e\hbar B}{m}\right)^2} \pm \left((PE\hbar)^2 + \left(\frac{e\hbar B}{m}\right)^2 \right) \right) \\ +3 \left(\frac{g\hbar^2}{4}\right)^2 \sqrt{(PE\hbar)^2 + \left(\frac{e\hbar B}{m}\right)^2} \\ \pm 3 \left(\frac{g\hbar^2}{4}\right) \left((PE\hbar)^2 + \left(\frac{e\hbar B}{m}\right)^2 \right) \\ + \left((PE\hbar)^2 + \left(\frac{e\hbar B}{m}\right)^2 \right)^{\frac{3}{2}} \end{array} \right) \quad (2.23)$$

where the +refers to ω_1 and - to ω_2. If the two amplitudes in Eq. (2.20) are adjusted to be equal we find for the beat frequency for the two closely calculated frequencies in Eq. (2.23).

$$f_B = \frac{(\omega_1 - \omega_2)}{2\pi}$$

$$= \frac{\tau^2}{2\pi} \left(\frac{1}{24\hbar^3} \right) \left(\begin{array}{c} 6E_1 \left((PE\hbar)^2 + \left(\frac{e\hbar B}{m}\right)^2 \right) \\ +6 \left(\frac{g\hbar^2}{4}\right) \left((PE\hbar)^2 + \left(\frac{e\hbar B}{m}\right)^2 \right) \end{array} \right) \quad (2.24)$$

Eq. (2.24) provides us with both a sensitive probe to the electric dipole moment and internal energies E_1 and $\frac{g\hbar}{4}$ as well as a probe to the discrete time interval τ.

3. Conclusion

The above analysis has demonstrated that a spin 1 gauge bonson ($q = -e$) in both an magnetic and electric field will precess about an axis that lies in the plane determined by B and E. Any Doppler like effects in the spin precession frequency will be a clear signal of discrete time effects since when $\tau = 0$ both frequencies in Eq. (2.23) are identical. If order of magnetude estimates can be made on E_1 (internal spatial energy) and g (internal spin-spin coupling) then estimates of τ can be made from a measurement of Eq. (2.24). The central result of this investigation is that "any Doppler-like peaks in the spin precession frequency" will be a distinct signature of discrete time quantum effects. The above ideas might also be applied to hadrons where estimates of the internal structure (E_1 and g) would be more reliable, and then estimates of τ could be better trusted. it is also fascinating that if discrete time effects were discovered they could be used as a very delicate probe to electric dipole moments of particles (e^-, N^-) that presently test our theory of strong CP violation[23]. We also note that the result of this note is independent of the simple one dimensional model of internal structure and that any model of a spin one gauge boson with internal structure will predict the Doppler frequency for the $< S_y >$ espressed in Eq. (2.24).

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