Wave propagation in a generalized Minkowski space and superluminal signals

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ABSTRACT. Tunnelling of wavepackets at velocities higher than the light speed in vacuum finds a natural explanation in the framework of a generalization of special relativity, in which nonlocal interactions are described in an effective way by means of a deformation of the usual Minkowski metric. This picture can be considered as an effective description of "virtual" particles (propagating with imaginary "classical" wavevector in the usual spacetime) as objects which propagate in the deformed Minkowski space with a "new", but real, wavevector. Our formalism is explicity applied to the superluminal propagation of electromagnetic evanescent waves in waveguides. We propose also an experimental setup, based on light propagation in optical fibers, which may provide a new optical test of e.m. superluminal tunneling.

1 - Introduction

The problem of the range of validity of the usual special Relativity (SR), based on the pseudo - Euclidean Minkowski space and its related Lorentz group of transformations, is, as well known, a much debated question since a long time.

Although generalizations of SR have been considered also at a large-scale level (in order e.g. to account for the existence of the preferred reference frame represented by the relic background radiation), the most interesting issue seems to be the modification of SR at high energies and/or at small distances, in order to overcome the difficulties encountered in recounciling SR and quantum mechanics[(1)].

In this connection, in the last years an extension of special Relativity has been developed[(2,3)], essentially aimed to describe, in an effective way, interactions structurally more general than the usual (local and derivable from a potential) ones. Such a formalism was named "deformed Special Relativity" (DSR), because basically based on a "deformation" of the usual Minkowski metric. Among the others, the DSR predicts a deviation of the particle lifetime from the usual Einsteinian behaviour[(3,4)], and permits to accomodate in a natural way the existence of superluminal causal speeds. Moreover, it is also able to account for possible nonlocal effects in electromagnetic interactions.

In this paper, we want to investigate the problem of wave propagation in the deformed Minkowski spacetime of DSR. As we shall see, the tunnelling of wavepackets at speeds higher than the light speed in vacum finds, in this framework, a natural explanation. This allows one, among the others, to give an effective description of "virtual" particles (propagating with imaginary "classical" wave vector in the standard spacetime) as objects propagating in the deformed Minkowski space with a "new", but real, wavevector.

The paper is organized as follows.In section 2, we provide the basic elements of DSR, as its axiomatic foundations and the generalization of the Lorentz transformations. The wave propagation in the deformed Minkowski space is discussed in sect. 3. Sect. 4 contains the physical analysis of the results obtained.

2 - Special Relativity in a deformed Minkowski space

Let us briefly review the main aspects of DSR. Its very foundation starts from an axiomatic formulation of the standard special relativity, whose basic postulates can be stated as follows[(5)]:

1 - *Space-time properties*: Space and time are homogeneous and space is isotropic.

2 - *Principle of Relativity*: All physical laws must be covariant when passing from an inertial reference frame K to another frame K', moving with constant velocity relative to K.

Let us notice that in the second postulate it is clearly understood that, for a correct formulation of SR, it is *necessary* to specify the total class, C_T, of the physical phenomena to which the relativity principle applies[(*)].[1] Depending on the explicit choice of C_T, one gets *a priori different* realizations of the theory of

[1] The importance of such a specification is easily seen if one thinks that, from an axiomatic viewpoint, the only difference between galileian and einsteinian relativities just consists in the choice of C_T (i.e. the class of mechanical phenomena in the former case, and of mechanical and electromagnetic phenomena in the latter).

relativity (in its abstract sense), each one embedded in the previous. Of course, the principle of relativity, together with the specification of the total class of phenomena considered, necessarily implies, for consistency, the unicity of the transformation equations connecting inertial reference frames.

It is possible to show that, from the above two postulates, there follow - without any additional hypothesis - all the usual "principles" of SR, i.e. the "principle of reciprocity", the linearity of transformations between inertial frames, and the invariance of light speed in vacuum(5).

Concerning this last point, it can be shown in general that postulates 1 and 2 above imply the existence of an invariant, real quantity, having the dimensions of the square of a speed(5), whose value must be experimentally determined in the framework of the total class $C_\mathcal{T}$ of the physical phenomena(**).[2] Such an invariant speed depends on the interactions involved in the physical procees considered. Therefore, *there is, a priori, an invariant speed for every interaction*, namely, a maximal causal speed for every interaction. In the following, we shall denote by u this invariant, maximal causal speed, without any reference to the interaction concerned.

All the formal machinery of SR in the Einsteinian sense (including Lorentz transformations and their implications, and the metric structure of space-time) is simply a consequence of the above two postulates and of the choice, for the total class of phenomena C_T, of the class of mechanical and electromagnetic phenomena.

The basic assumption of DSR is that, in order to include the class of nuclear and subnuclear phenomena in the total class of phenomena for which special relativity holds true, it is necessary to consider a generalization of the usual Minkovski metric $g_{\mu\nu} = diag(1, -1, -1, -1)$ (analogously to the generalization from the euclidean to the Minkowski metric in going from mechanics to electrodynamics). The generalization of $g_{\mu\nu}$ assumed in DSR(2) is the following "deformation" of the Minkowski metric

$$g \to \eta;$$

$$\eta = diag(b_0^2, -b_1^2, -b_2^2, -b_3^2), \tag{1}$$

where the parameters $b_\mu(\mu = 0, 1, 2, 3)$ are, in general, real and positive functions of the observables characterizing the system (in particular, of its total energy).We shall denote by $\tilde{M}$ the deformed Minkowski space endowed with the metric structure (1).

Metric (1) is assumed to provide a local representation, in average, of the effects of the interactions involved in the physical system considered(2,3). In general, such interactions can be structurally very general, including e.g. nonlocal (and/or nonpotential) terms (weak and strong forces, as is well known, do not admit a potential).

Due to their physical meaning, the parameters b_μ play a *dynamical* rôle, and represent, for nonlocal (nonpotential) interactions, what the Hamiltonian represents for a local (potential) interaction. In particular, the $b'_\mu s$ as well are *not* an input of the theory, but must be built up from the experimental knowledge of the physical data of the system (concerned in analogy with the specification of the Hamiltonian of a potential system). Moreover, it is clear that there exist infinitely many deformations $\{\tilde{\mathcal{M}}\}$ of the Minkovski space $\mathcal{M}$, corresponding to the different possible choices of the parameters b_μ.

The generalized interval in $\tilde{\mathcal{M}}$, corresponding to the metric tensor (1), is therefore given by

$$\begin{aligned}\tilde{x}^2 &= x * x = x_\mu\ \eta^{\mu\nu}(E)\ x_\nu \\ &= b_0^2(E)c^2t^2 - b_1^2(E)x^2 - b_2^2(E)y^2 - b_3^2(E)z^2.\end{aligned} \tag{2}$$

where E is the energy of the process considered.

Let us notice that *actually the deformed Minkowski space $\tilde{M}$ has zero curvature*, as it is easily seen remembering that, in a Riemann space, the scalar curvature is constructed from the derivatives, with respect to space-time coordinates, of the metric tensor. In others words, the space $\tilde{M}$ is *intrinsically flat* - at least in a mathematical sense. Namely, it would be possible, in principle, to find a change of coordinates, or a rescaling of the lengths, so as to recover the usual Minkowski space. However, such a possibility is only a mathematical, and not a physical one. This is related to the fact that *the energy of the process is fixed, and cannot be changed at will.* For that value of the energy, the metric coefficients do possess values different from unity, so that the corresponding space $\tilde{M}$, for the given energy value, is actually different from the Minkowski one. The usual spacetime M is recovered for a special value E_0 of the energy (characteristic of any interaction), such that indeed $\eta(E_0) = g = (1, -1, -1, -1)$ (see the phenomenological analysis of ref.[3]).

Actually, the deformed Minkowski space $\tilde{M}$ can be regarded as a subspace of a five-dimensional, genuine Riemannian manifold, with energy as fifth dimension (see refs. [6]). The deformation (1) of the spacetime metric requires a modification of the postulates 1-2 of SR. So, the new basic postulates of DSR read as follows (2,3):

1) deformed "inertial" frame is a reference frame in which space-time is homogeneous, but space is not necessarily isotropic;

[2] The invariant speed is obviously ∞ for Galilei's relativity, and c for Einstein's relativity.

2) *"generalized principle of relativity"*, or *principle of metric invariance"* all physical measurements within every deformed "inertial" frame must be carried out via the *same* metric

The first kinematical consequence of the generalized interval (2) just concerns the maximal causal speed in $\tilde{M}$. Let us consider the infinitesimal interval

$$d\tilde{s}^2 = b_0^2 c^2 dt^2 - b_k^2 (dx_k)^2 \tag{3}$$

($k = 1, 2, 3$). Assuming, for simplicity, an isotropic tridimensional space (i.e. $b_k = b \forall k$), we get, for null separation $d\tilde{s}^2 = 0$:

$$b^2 (dx^2 + dy^2 + dz^2) = b_0^2 c^2 dt^2 \tag{4}$$

or

$$\Big(\frac{1}{dt^2}\Big)(dx^2 + dy^2 + dz^2) = \Big(\frac{b_0}{b}\Big)^2 c^2 \tag{5}$$

From. eq. (5) it easily follows that

$$u = \Big(\frac{b_0}{b}\Big) c \tag{6}$$

i.e. a maximal causal speed u (whose value is parametrized by c) which depends on the physical system (and its interactions). Moreover, it is

$$u \overset{\geq}{\underset{<}{=}} c \quad \text{according to} \quad \frac{b_0}{b} \overset{\geq}{\underset{<}{=}} 1. \tag{7}$$

In other words, there may be maximal causal speeds *superluminal*, depending on the interaction considered. The maximal causal speed u can be interpreted, from a physical standpoint, as the speed of the quanta of the interaction which requires a representation in terms of a generalized Minkowski space. Therefore, such quanta must be zero-mass particles, in analogy with photons in the usual SR (at least inside the deformed spacetime region).

In order to check that u is actually an invariant speed, we need the explicit expression of the generalized Lorentz transformations, i.e. the transformations leaving the interval (2) invariant.

The new boosts (for motion, say, along the x-axis) can be expressed as[2]

$$\begin{aligned} x' &= \tilde{\gamma}(x - \beta x_0); \\ y' &= y; z' = z; \\ x_0' &= \tilde{\gamma}(x_0 - \frac{\tilde{\beta}^2}{\beta} x), \end{aligned} \tag{8}$$

where $\beta = v/c$ (v being the relative speed of the reference frames) is the usual speed parameter, and (*) 3

$$\tilde{\beta} = \beta \frac{b_1}{b_0}; \tag{9}$$

$$\tilde{\gamma} = (1 - \tilde{\beta}^2)^{-1/2}. \tag{10}$$

Let us notice, that assuming an isotropic three-space, and recalling expression (6) for the maximal causal speed in the deformed Minkowski space, the generalized speed parameter $\tilde{\beta}$ can be written in the more perspicuous form[2]

$$\tilde{\beta} = \frac{v}{u}. \tag{11}$$

As already stressed above, it can be $u > c$ and therefore speeds v such that $c < v < u$ are allowed. In other words, we can have tachionic motion (at least within suitable space-time regions), without any need of formal extensions of special relativity (like that considered in [5]). Indeed, on account of (11), the function $\tilde{\gamma}(v)$ can *never* assume imaginary values, and therefore, in this framework, the possibility of superluminal velocities is achieved without any recourse to neither imaginary quantities nor singularities in the transformation laws. In other words, when $u > c$, the interaction quanta (which, as already noted, are massless particles travelling at speed u) can transfer to particles with rest mass $m_0 \neq 0$ an impulse such to give them a speed higher than c, without at the same time giving them an imaginary mass $m_0 \tilde{\gamma}$. Let us also notice that such a superluminal propagation occurs only *locally* (in the space-time region corresponding to the physical system or process considered), and therefore no problem of global causality violation is expected to arise.

From eqs. (8) there easily follow the transformation laws for velocity:

$$v_x' = \frac{v_x - v}{1 - \frac{v v_x}{u^2}}; \quad v_y' = \frac{v_y}{\tilde{\gamma}(1 - \frac{v v_x}{u^2})}; \quad v_z' = \frac{v_z}{\tilde{\gamma}(1 - \frac{v v_x}{u^2})} \tag{12}$$

and, therefore, the generalized velocity composition law, that reads (for an isotropic 3-space):

$$V = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{u^2}} \tag{13}$$

The invariant character of the maximal speed u then easily follows by putting one of the speeds (say, v_1) equal to u:

$$V = \frac{u + v_2}{1 + \frac{u v_2}{u^2}} = \frac{u + v_2}{1 + \frac{v_2}{u}} = u \tag{14}$$

If we now give up the condition of spatial isotropy, the composition law for motion, say, along the x_k-axis, becomes

$$V = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{u_k^2}}; \quad u_k = \frac{c b_0}{b_k} \tag{15}$$

3 Let us notice that the transformations (8) do formally coincide with the "isotopic Lorents transformations" introduced by Santilli in [7]. However, in the context of DSR their physical meaning is rather different; in particular, no reference at all is made, in such a framework, to the existence of an underlying "medium". See refs.[2,3] for a more detailed discussion.

and, therefore, the speed that has as invariant character is

$$u_k = \frac{cb_0}{b_k}. \tag{16}$$

It follows that, in a given deformed Minkowski space, there exist infinitely many different, maximal causal speeds, corresponding to the different possible directions of motion (although, of course, only three of them are independent). Clearly, this result is a strict consequence of the spatial anisotropy of the space-time region considered. Let us notice that there is indeed a phenomenon - the Bose-Einstein correlation - which can be fully described in the framework of a deformed Minkovski space, but with the consequence of a local loss of space isotropy(8).

Further details and implications of the deformed theory of relativity can be found in refs. [2,3, 8]. To our present aims, let us notice that, although the formalism illustrated above has been essentially derived in order to deal with interactions structurally more general than the standard ones, it can be applied to all those processes in which nonlocal effects are expected to play a role. Then, according to the above discussion (see eqs. (6), (7)), it is expected that, *in all physical phenomena where nonlocal effects occur, field wavepackets may propagate faster than light in vacuum.*

3 - Wave propagation in DSR

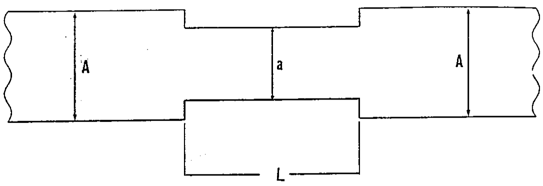

Figure 1. Experimental setup of the Cologne experiment. L = length of the smaller waveguide; a = width of the smaller waveguide; A = width of the larger waveguide (after ref.[9]).

We want now to discuss wave propagation in a deformed Minkowski spacetime. In order to elucidate the connection between the formalism of DSR and the tunnelling through a barrier, we shall consider, in analogy with the electromagnetic case, the propagation of a field inside a waveguide.

We consider, in particular, a rectangular waveguide with variable section, schematically pictured in Fig.1. We have taken the axis of the waveguide as the z-axis of our spatial frame.

In the DSR formalism, the field propagating inside the smaller waveguide, given by(3)

$$E_j(x^\mu) = A_j(x^i) e^{i\tilde{k}x} \tag{17}$$

($i, j = 1, 2, 3$, with $\tilde{k}^\mu = (\tilde{k}_x, \tilde{k}_y, \tilde{k}_z, \omega/c)$ being the wavevector inside the smaller waveguide), satisfies the generalized Helmholtz D'Alembert wave equation

$$\tilde{\Box} E_j = 0 \tag{18}$$

where the generalized d'Alambertian operator explicitly reads

$$\tilde{\Box} \equiv \partial_* \partial = b_1^2 \partial_x^2 + b_2^2 \partial_y^2 + b_3^2 \partial_z^2 - \frac{b_0^2}{c^2} \tag{19}$$

For simplicity, we consider a TE_{10} mode. Then, it is $E_x = E_z = 0$ and E_y can be written as

$$E_y(x, z, t) = g(x) e^{(i\tilde{k}_z\ z - \omega t)}\ . \tag{20}$$

Therefore the generalized Helmholtz wave equation (18, 19) becomes

$$b_1^2 \partial_x^2 E_y + b_3^2 \partial_z^2 E_y = \frac{1}{c^2} \partial_t^2 E_y. \tag{21}$$

Replacing (20) in (21) we get the following equation for $g(x)$:

$$b_1^2 \partial_x^2 g + \frac{b_4 \omega^2}{c^2} g - b_3^2\, \tilde{k}_z^2 g = 0 \tag{22}$$

whose solution is

$$g(x) = Acos(\tilde{k}_x x) + Bsin(\tilde{k}_x x) \tag{23}$$

with

$$\tilde{k}_x^2 = \frac{b_0^2\, \omega^2}{c^2} - b_3^2\, \tilde{k}_z^2. \tag{24}$$

Imposing the boundary condition $E_y = 0$ for $x = 0, a$ (where a is the width of the smaller waveguide: see Fig.1), we find

$$\tilde{k}_x = \frac{m\pi}{a} \tag{25}$$

(m integer). In our case $m = 1$, so that $\tilde{k}_x = \pi/a$. Moreover, on account of the mode involved, we can assume partial spatial isotropy inside the smaller waveguide, i.e. $b_1^2 = b_3^2$. Then, eq.(24) yields the following expression of $\tilde{k}_z^2$:

$$\tilde{k}_z^2 = (2\pi/c)^2 [(\frac{b_0^2}{b_3^2}) \nu^2 - \nu_c^2] \tag{26}$$

where $\omega = 2\pi\nu$ and

$$\omega_c = \frac{\pi c}{a} = 2\pi \nu_c. \tag{27}$$

We can also assume (without loss of generality) that the barrier crossing does not affect time, so that $b_0^2 = 1$ (isochronism hypothesis). Therefore we get

$$\tilde{k}_z^2 = (2\pi/c)^2 [(\frac{1}{b_3})^2 \nu^2 - \nu_c^2] \tag{28}$$

.

From eq.(28) it immediately follows that we obtain a *real* wavevector inside the barrier region (smaller waveguide), $\tilde{k}_z^2 > 0$, provided that

$$(\frac{1}{b_3^2}) > (\frac{\nu_c}{\nu})^2 \quad . \tag{29}$$

4 - Physical discussion

Let us analyze the results of the previous section from a physical point of view. Two points must be stressed. First, if we introduce the speed along z, u_z, as

$$u_z = \frac{b_0}{b_3}c = \frac{c}{b_3} \tag{30}$$

(in the isochronism hypothesis $b_0^2 = 1$), it can be shown that the generalized wavevector $\tilde{k}_z$ is imaginary for nonlocal subluminal velocities ($u_z < c$) and real for superluminal velocities ($u_z > c$) (for $u_z = c$, the deformed metric η reduces to the usual Minkowskian one). In other words, *an evanescent mode in the usual Minkowski space is described as a non-evanescent one, with a real wavevector, propagating at superluminal speed in the nonlocal deformed Minkowski space.* This is why we can refer to condition (29) as *the superluminality condition.*

Secondly, the same formalism, used in the case of barrier crossing, can be exploited to describe "virtual" objects, i.e. fields whose propagation wavevector is imaginary (like mesons, photons, and, in general, the virtual boson particles which are the carriers of fundamental interactions).

Then, the violation of energy conservation, expressed by the time-energy uncertainty principle for a time interval $\Delta t \gtrsim \hbar/\Delta E$, can be interpreted, in such a framework, as the time during which the Minkowski space is locally deformed, in order to permit the field propagation at superluminal speeds under the conditions whereby the field is regarded as a virtual object. Indeed, it is possible to see that the superluminal effects are connected to the square of the field component along the wavevector direction, as we found by the analysis of the statical equilibrium of an elementary charge producing a Columb - like field.

Lastly, let us notice that our formalism of wave propagation in DSR can be also exploited in order to provide an effective description of the recent experimental findings on propagation of electromagnetic wavepackets at superluminal group velocities[9−11]. Indeed, although such results already find a natural explanation in the framework of the standard electromagnetic theory[12], their interpretation involves a mechanism of pulse reshaping which can be regarded as due to nonlocal effects. On the basis of our discussion above, such effects can be described in terms of an effective deformation of the Minkowski spacetime.

A detailed discussion of the superluminal propagation of electromagnetic wavepackets in terms of a spacetime deformation inside a waveguide can be found in ref. [13]. It has been shown, among others, that such a propagation can be described by means of a deformation tensor (analogous to the stress tensor in a continuous medium). Here, we want only to stress some points concerning the results of the previous section and directly connected to the data of the Cologne experiments[9].

First of all, let us notice that the superluminal condition (29) is indeed satified by the parameter values of the Cologne experiments (with $(\nu_c/\nu)^2 = 1.2$).

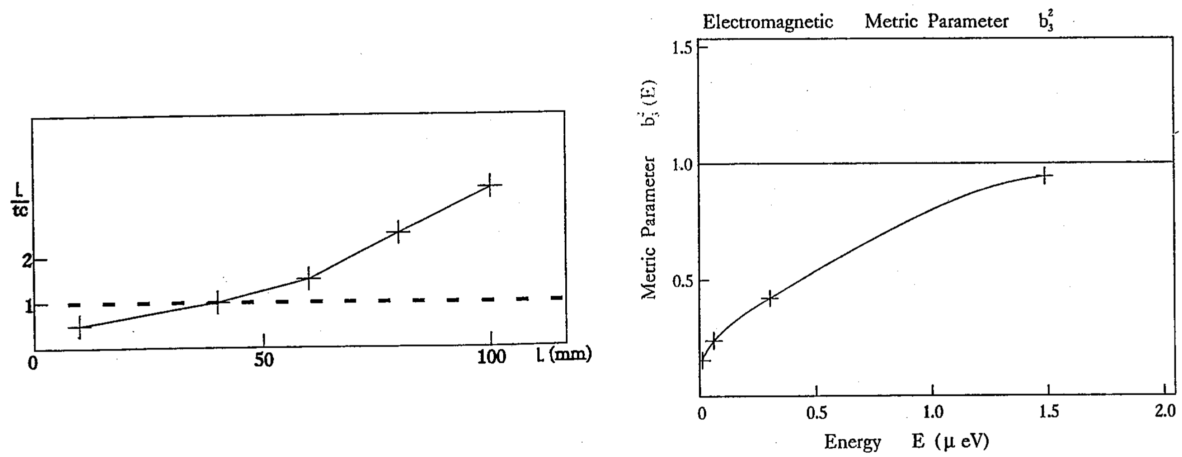

Figure 2. a) Plot of the trasmission velocity L/tc vs the length L of the smaller waveguide (after ref. [9]). b) Plot of the electromagnetic metric parameter $b_3^2(E)$ vs. the energy of the output signal.

Moreover, the analysis of the experimental data [9] allows us to get the functional dependence of the metric parameter b_3^2 on the energy. This can be done by using the expression of the energy, in the TE_{10} - like mode, of the output signal from the smaller waveguide of length L:

$$E = h\nu \, e^{-L/L_o} \tag{31}$$

where

$$L_o = \left(\frac{c}{2\pi}\right)(\nu_c^2 - \nu^2)^{-1/2} \tag{32}$$

($\nu < \nu_c$). Then, by using the experimental value of the relevant parameters of ref.[9] (cf. Fig. 2a), we get the plot of $b_3^2(E)$ pictured in Fig. 2b.

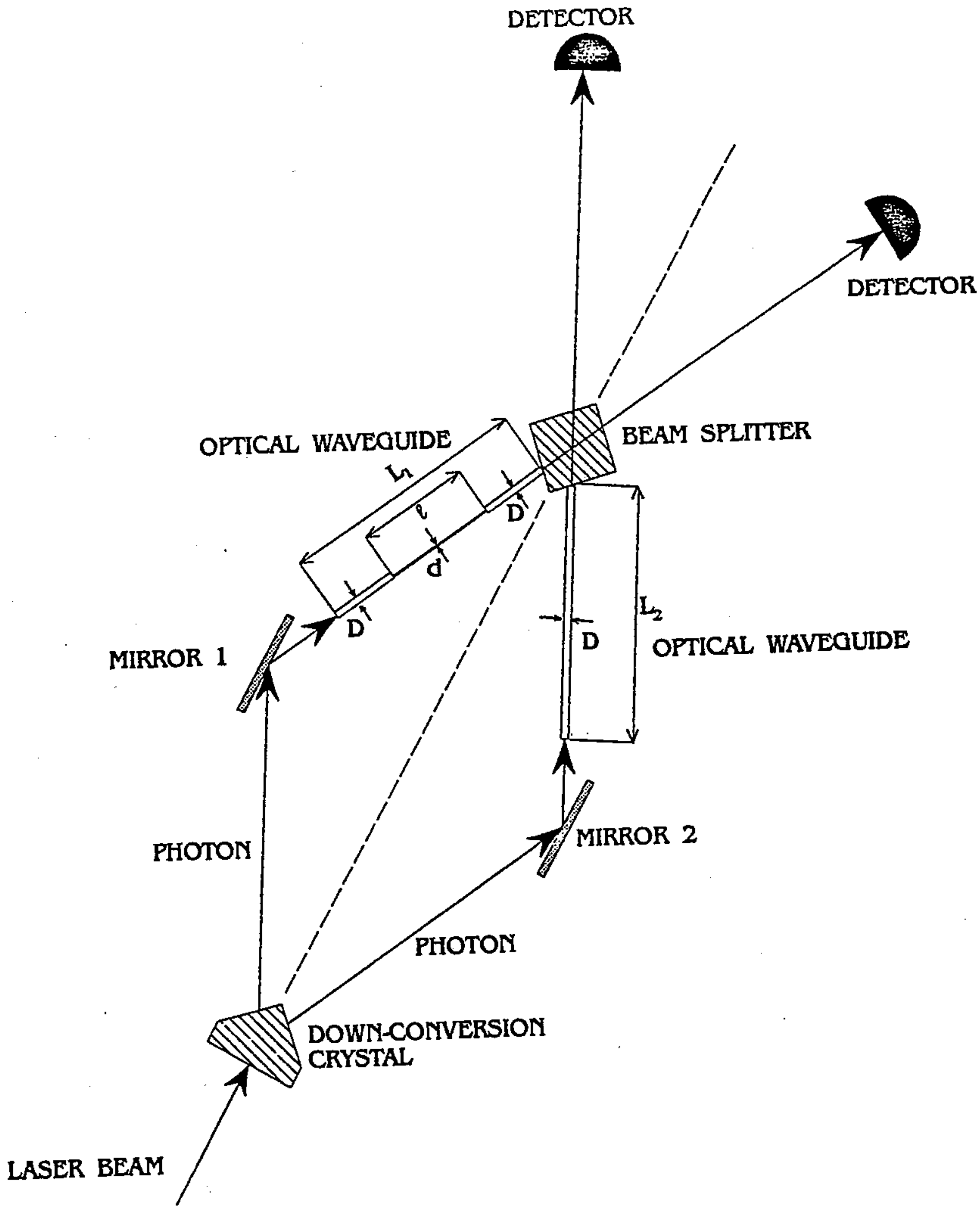

Figure 3. Twin photon interferometer with optical waveguides. The two mirrors have the same features. The optical waveguides 1 and 2 have the same length $L_1 = L_2 = L$, while the former one presents a central part of variable length ℓ, whith reduced section $d < D$. It is easily seen that the waveguide 1 is the optical analogous of the electromagnetic waveguide of the Cologne setup.

Let us stress that the behaviour of $b_3^2(E)$ with the energy *has exactly the same form of that obtained in the leptonic case*, from the analysis of the lifetime of the K_s^0 - meson$^{(1)}$. This is an intriguing result, on account of the well-known mixing between electromagnetic and weak interactions in the standard model.

A similar analysis of the Berkeley experiment [10] cannot be carried out, because it is difficul to estimate the dimensions of the region with higher transmission coefficient (the analogous of the reduced waveguide in the Cologne experiment). However, since in principle a translucid mirror - like that used by the Berkeley group - can be regarded as a high-frequency cutoff filter, we propose an optical experimental setup, in which all the parameters are known. The corresponding setup, a twin-photon interferometer with optical waveguides, is schematically shown in Fig.3. Such an optical device unifies, from an experimental point of view, the two experiments [9] and [10], which, admit - in our opinion - a similar interpretation in the formalism of DSR. Such an experimental setup may provide an independent, optical test of superluminal tunneling of e.m. signals.

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