Can the 6-component Weinberg-Tucker-Hammer Equations describe the Electromagnetic Field?

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ABSTRACT. It is shown that the massless $j = 1$ Weinberg-Tucker-Hammer equations reduce to the Maxwell's equations for electromagnetic field under the definite choice of field functions and initial and boundary conditions. Thus, the former appear to be of use in describing some physical processes for which that could be necessitated or be convenient. Possible consequences are discussed.

The attractive Weinberg's $2(2j+1)$ component formalism for describing higher spin particles [1] recently developed considerably in connection with the recent works of Dr. D.V. Ahluwalia *et al.*, ref. [2, 3, 4, 5]. Some attempts have also been undertaken in attaching interpretations of their ideas in my papers [6, 7, 8, 9].

The aim of the present paper is to find connections between the Weinberg-Ahluwalia equations for spin-1 fields in the massless limit and the equations for the classical electromagnetic field, thus generalizing the Maxwell's electromagnetic theory. This paper is continuation of the previous research in the papers cited above.

The main equation of the Weinberg formalism has been proposed in ref. [1]

$$\left[\gamma_{\mu_1\mu_2\ldots\mu_{2j}} p_{\mu_1} p_{\mu_2}\ldots p_{\mu_{2j}} + m^{2j}\right]\psi = 0 \quad . \qquad (1)$$

One can see that it is of the "$2j$" order in the momentum, $p_{\mu_i} = -i\partial/\partial x_{\mu_i}$, m is the particle mass. Analogs of the Dirac γ- matrices are the $2(2j+1)\otimes 2(2j+1)$ matrices which have also "$2j$" vectorial indices, ref. [10]. The possibility of slight modifications of the theory on the basis of the introduction of *two* signs in the mass term has been discussed in refs. [4, 5, 6, 7, 8], see also below.

At the moment I take a liberty to repeat the previous results. Equations (4.19,4.20), or equivalent to them Eqs. (4.21,4.22), presented in ref. [1b,p.B888] and in many other publications:

$$\nabla\times[\mathbf{E}-i\mathbf{B}] + i(\partial/\partial t)[\mathbf{E}-i\mathbf{B}] = 0 \quad , \qquad (4.21)$$
$$\nabla\times[\mathbf{E}+i\mathbf{B}] - i(\partial/\partial t)[\mathbf{E}+i\mathbf{B}] = 0 \quad , \qquad (4.22)$$

are found in ref. [2] to have acausal solutions. Weinberg claimed directly that the above equations are "just Maxwell's free-space equations for left- and right- circularly polarized radiation". Apart from the dispersion relations $E = \pm|\mathbf{p}|$ compatible with the relativity theory we have a "wrong" dispersion relation $E = 0$. The origin of this fact appears to be the same with the problem of the "relativistic cockroach nest" of Moshinsky and Del Sol, ref. [11], because both problems can be treated on an equal footing on the basis of the spinorial analysis.[1] On the other side, *"the $m \to 0$ limit of Joos-Weinberg finite-mass wave equations, Eq. (1), satisfied by $(j,0)\oplus(0,j)$ covariant spinors, ref. [3], are free from all kinematic acausality."*

The same authors (D. V. Ahluwalia and his collaborators) proposed the Foldy-Nigam-Bargmann-Wightman-Wigner-type (FNBWW) quantum field theory, *"in which bosons and antibosons have opposite relative intrinsic parities"*, ref. [4]. This *Dirac*-like modification of the Weinberg theory is an excellent example of combining the Lorentz and the dual transformations. Its recent development, ref. [5], could be relevant for describing neutrino oscillations and be useful in realizing the role of space-time symmetries for all types of interactions.

In ref. [6] I concern with finding connections between antisymmetric tensor fields [13, 14, 15] and field functions satisfied the equations considered by Weinberg (and by Hammer and Tucker [16] in a slightly different form). In the case of the choice[2]

$$\psi = \begin{pmatrix}\mathbf{E}+i\mathbf{B}\\ \mathbf{E}-i\mathbf{B}\end{pmatrix} \qquad (2)$$

the equivalence of the Weinberg-Tucker-Hammer approach and the Proca approach has been found. I

[1] See further work in this direction, ref. [12], which was written recently. It is still required detailed comment.
[2] My earlier attempts to give an interpretation for ψ in terms of potentials were unsuccessful in a certain manner, ref. [17].

mean that the equations for the antisymmetric field tensor $F_{\mu\nu}$

$$m^2 F_{\mu\nu} = \partial_\mu \partial_\alpha F_{\alpha\nu} - \partial_\nu \partial_\alpha F_{\alpha\mu}\,, \qquad (3a)$$

$$\partial_\lambda^2 F_{\mu\nu} = m^2 F_{\mu\nu} \qquad (3b)$$

are equivalent in the physical content to the Tucker-Hammer equation

$$(\gamma_{\alpha\beta}\partial_\alpha\partial_\beta + p_\alpha p_\alpha + 2m^2)\psi = 0\,, \qquad (4)$$

see Eq. (3,4,8) in [6] or Eq. (A4) in [18b].

Furthermore, the possibility of consideration of another equation (the Weinberg "double") [3] was point out. In fact, it is the equation for the antisymmetric tensor dual to $F_{\mu\nu}$, which had also been considered earlier, *e.g.* ref. [21]. In the paper [7] the Weinberg fields were shown to be able to describe a particle with transversal components (*i.e.*, spin $j = 1$) as opposed to the conclusions of refs. [14, 15] and of the previous ones [13]. Origins of contradictions with the Weinberg theorem ($B - A = \lambda$),[4] which have been encountered in the old works (of both mine and others), have been partly clarified. The propagators for the Weinberg theory have been proposed in ref. [8]. The remarkable feature is the presence of four terms. This fact will be explained in my forthcoming papers.

We continue with the goal of the present paper: one should consider the question, under which conditions the Weinberg-Tucker-Hammer $j = 1$ equations (4) can be transformed to Eqs. (4.21) and (4.22) of ref. [1b] ? On using the interpretation of ψ in the chiral representation, Eq. (2), and the explicit form of the Barut-Muzinich-Williams matrices, ref. [10], I am able to recast the $j = 1$ Tucker-Hammer equation, which is free of tachyonic solutions, or the Proca equation (3a) to the form[5,6]

$$m^2 E_i = -\frac{1}{c^2}\frac{\partial^2 E_i}{\partial t^2} + \epsilon_{ijk}\frac{\partial}{\partial x_j}\frac{\partial B_k}{\partial t} + \frac{\partial}{\partial x_i}\frac{\partial E_j}{\partial x_j} \qquad (7)$$

$$m^2 B_i = \frac{1}{c^2}\epsilon_{ijk}\frac{\partial}{\partial x_j}\frac{\partial E_k}{\partial t} + \frac{\partial^2 B_i}{\partial x_j^2} - \frac{\partial}{\partial x_i}\frac{\partial B_j}{\partial x_j}\quad . \qquad (8)$$

The Klein-Gordon equation (the Einstein dispersion relations in the momentum representation, indeed)

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x_i^2}\right) F_{\mu\nu} = -m^2 F_{\mu\nu} \qquad (9)$$

is implied.

Restricting ourselves by the consideration of the $j = 1$ massless case one can re-write (7,8) in the following vector form:

$$\frac{\partial}{\partial t}\,\text{curl}\,\mathbf{B} + \text{grad div}\,\mathbf{E} - \frac{1}{c^2}\frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \quad , \qquad (10)$$

$$\nabla^2 \mathbf{B} - \text{grad div}\,\mathbf{B} + \frac{1}{c^2}\frac{\partial}{\partial t}\,\text{curl}\,\mathbf{E} = 0 \quad . \qquad (11)$$

Let us consider the first equation (10). We can satisfy it provided that (e.m.u. system is used)

$$\tilde{\rho}_e = \frac{1}{4\pi c^2}\,\text{div}\,\mathbf{E} = const_x \quad , \qquad \mathbf{J}_e = \frac{1}{4\pi}\,\text{curl}\,\mathbf{B} - \frac{1}{4\pi c^2}\frac{\partial \mathbf{E}}{\partial t} = const_t \quad . \qquad (12)$$

However, this is a particular case only. Let me mention that the equation

$$\frac{1}{c^2}\frac{\partial \mathbf{J}_e}{\partial t} = -\,\text{grad}\,\tilde{\rho}_e \qquad (13)$$

follows from (10) provided that $\mathbf{J}_e$ and $\tilde{\rho}_e$ are *defined* as in Eq. (12).

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[3] It is useful to compare the method applied in the papers [2, 6] with the Dirac's way of deriving the famous equation for $j = 1/2$ particles, ref. [20]. Namely, his aim was to obtain the linear differential equation; the coefficients in derivative terms and in the mass term were not known *ab initio* and they turn out to be matrices. The second requirement which he imposed is: the equation should be compatible with the Klein-Gordon equation, *i.e.*, with relativistic dispersion relations. In our approach the orders of the equations are defined by the spin of a considered particle.

[4] The Weinberg theorem reads: "*... a massless particle operator of helicity λ can only be used to construct fields which transform according to [the Lorentz group] representation (A, B) such that $B - A = \lambda$.*" It is a consequence of the general kinematical structure of the theory based on the definite representation of the Lorentz group. If one of various "gauge" constraints one places on the dynamics leads to the results which contradict with the underlying kinematical structures this can signify the only thing: one is not allowed to use the dynamical constraint in such a way. Namely, the Weinberg theorem permits two values of the helicity $\lambda = \pm 1$ for a massless $j = 1$ Weinberg-Tucker-Hammer field. Setting the generalized Lorentz condition (i. e. two Maxwell's free-space equations, see for a discussion the footnote # 11 in ref. [7]) yields the physical excitation of the very puzzled nature, $\lambda = 0$. Therefore, imposing the generalized Lorentz condition in such a form $q_j a_{ij}(q)|\Psi>= 0\ q_0 = |\mathbf{q}|$, the formulas (18) of ref. [13b], on the 'quantal' physical states may be misleading in the case of the quantum field consideration and it may be incompatible with specific properties of the antisymmetric tensor field.

[5] I restored c, the light velocity, in the terms.

[6] We used additional equations to construct propagators for the Weinberg theory [8]. From the dual equation

$$(\gamma_{\alpha\beta}p_\alpha p_\beta - p_\alpha p_\alpha - 2m^2)\psi_2 = 0\,, \qquad (5)$$

(see Eqs. (10) or (12) of ref. [6]) and from the $\gamma^5 P$-conjugated equations

$$(\gamma^T_{\alpha\beta}p_\alpha p_\beta - p_\alpha p_\alpha - 2m^2)\tilde{\psi}_1 = 0\,, \qquad (6a)$$

$$(\gamma^T_{\alpha\beta}p_\alpha p_\beta + p_\alpha p_\alpha + 2m^2)\tilde{\psi}_2 = 0\,, \qquad (6b)$$

(see Eqs. (18,19) of ref. [6]) the reader can derive corresponding equations in the vector form without any problems.

Now we need to take relations of vector algebra in mind:

$$\text{curl curl } \mathbf{X} = \text{grad div } \mathbf{X} - \nabla^2 \mathbf{X} \quad , \tag{14}$$

where $\mathbf{X}$ is an arbitrary vector. Recasting Eq. (10) and taking the D'Alembert equation (9, $m \to 0$) in mind one can come in the general case to

$$\mathbf{J}_m = -\frac{\partial \mathbf{B}}{\partial t} - \text{curl } \mathbf{E} = \text{grad } \chi_m \quad , \tag{15}$$

in order to satisfy the re-written equation (10):

$$\text{curl } \mathbf{J}_m = 0 \quad . \tag{16}$$

The second equation (11) yields

$$\mathbf{J}_e = \text{curl } \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \text{grad } \chi_e \tag{17}$$

(in order to satisfy curl $\mathbf{J}_e = 0$). After adding and subtracting $\frac{1}{c^2}\partial^2\mathbf{B}/\partial t^2$ one obtains

$$\tilde{\rho}_m = \text{div } \mathbf{B} = const_x \quad , \qquad \frac{\partial \mathbf{B}}{\partial t} + \text{curl } \mathbf{E} = const_t \quad , \tag{18}$$

provided that

$$\nabla^2 \mathbf{B} - \frac{1}{c^2}\frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \tag{19}$$

(*i.e.*, again the D'Alembert equation taken into account). The set of equations (18), with the constants are chosen to be zero, is *"an identity satisfied by certain space-time derivatives of $F_{\mu\nu}\ldots$, namely*

$$\frac{\partial F_{\mu\nu}}{\partial x^\sigma} + \frac{\partial F_{\nu\sigma}}{\partial x^\mu} + \frac{\partial F_{\sigma\mu}}{\partial x^\nu} = 0\,," \tag{20}$$

refs. [22, 23]. However, it is also a particular case. Again, the general solution is found

$$\frac{1}{c^2}\frac{\partial \mathbf{J}_m}{\partial t} = -\text{ grad } \tilde{\rho}_m \quad . \tag{21}$$

We must pay attention to the general case. What are the chi-functions? How should we name them? From Eqs. (13) and (17) we conclude

$$\tilde{\rho}_e = -\frac{1}{c^2}\frac{\partial \chi_e}{\partial t} + const \quad , \tag{22}$$

and from (15) and (21),

$$\tilde{\rho}_m = -\frac{1}{c^2}\frac{\partial \chi_m}{\partial t} + const \quad , \tag{23}$$

what tells us that $\tilde{\rho}_e$ and $\tilde{\rho}_m$ are constants provided that primary functions χ are linear functions in time (decreasing or increasing?). It is useful to compare definitions $\tilde{\rho}_e$ and $\mathbf{J}_e$ and the fact of appearance of the functions χ with the 5-potential formulation of electromagnetic theory [23], see also ref. [18, 19, 24, 25, 26].

At last, I would like to note the following. We can obtain

$$\text{div } \mathbf{E} = 0 \quad , \quad \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} - \text{curl } \mathbf{B} = 0 \quad , \tag{24}$$

$$\text{div } \mathbf{B} = 0 \quad , \quad \frac{\partial \mathbf{B}}{\partial t} + \text{curl } \mathbf{E} = 0 \quad , \tag{25}$$

which are just the Maxwell's free-space equations, in the definite choice of the χ_e and χ_m, namely, in the case they are constants. In ref. [18] it was mentioned that solutions of Eqs. (4.21,4.22) of ref. [1b] satisfy the equations of the type (7,8), *"but not always vice versa".* In other words, the equations considered by Weinberg, Gersten, Ahluwalia and me contain additional solutions comparing with the ordinary Maxwell's theory and, thus, give us more information. Interpretations of this statement and investigations of Eq. (1) with other choice of initial and boundary conditions (or, of the functions χ) deserve further elaboration (both theoretical and experimental).

Next, if I use the bi-vector formula (2) as the field function, of course, the question arises on its transformation from one to another frame. I would like to draw your attention to the remarkable fact which follows from the consideration of the problem in the momentum representation. For the first sight, one could conclude that under a transfer from one to another frame one has to describe the field by the Lorentz transformed function $\psi'(\mathbf{k}) = \Lambda\psi(\mathbf{p})$. However, let us take into account the possibility of combining the Lorentz, dual (chiral) and parity transformations in the case of higher spin equations.[7] This possibility has been discovered and investigated in [4, 7]. The four bispinors

$$u_1^{\sigma\,(1)}(\mathbf{p}) = \frac{1}{\sqrt{2}}\begin{pmatrix}\left[m + (\mathbf{J}\cdot\mathbf{p}) + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{(E+m)}\right]\xi_\sigma \\ \left[m - (\mathbf{J}\cdot\mathbf{p}) + \frac{(\mathbf{J}\cdot\mathbf{p})^2}{(E+m)}\right]\xi_\sigma\end{pmatrix} \quad , \tag{26}$$

$u_2^{\sigma\,(1)}(\mathbf{p}) = \gamma^5 u_1^{\sigma\,(1)}(\mathbf{p})$, $u_1^{\sigma\,(2)}(\mathbf{p}) = \gamma^5\gamma^{44}u_1^{\sigma\,(1)}(\mathbf{p})$ and $u_2^{\sigma\,(2)}(\mathbf{p}) = -\gamma^{44}u_1^{\sigma\,(1)}(\mathbf{p})$, cf. Eqs. (10), (11), (12) and (13) of ref. [8], form the complete set (as

[7] Equations for the four functions $\psi_i^{(k)}$, Eqs. (8), (10), (18) and (19) of ref. [6] or the equations of the footnote # 6, reduce to the equations for $\mathbf{E}$ and $\mathbf{B}$, which appear to be the same for every case in the massless limit.

[8] After completing the preliminary version of this article I learnt that conditions of the closure for the second-type $j = 1/2$ and $j = 1$ bispinors similar to Eq. (27) have been obtained in ref. [5b,Eqs.(24,25)]. The equations (22a-23c) of the above-mentioned reference (i. e., the explicit forms of the 2-spinors and 3-vectors in the zero-momentum frame) could also be relevant in future discussions, see also ref. [9].

well as $\Lambda u_i^{\sigma\,(k)}(\mathbf{p})$). Namely,[8]

$$a_1 u_1^{\sigma\,(1)}(\mathbf{p})\overline{u}_1^{\sigma\,(1)}(\mathbf{p}) + a_2 u_2^{\sigma\,(1)}(\mathbf{p})\overline{u}_2^{\sigma\,(1)}(\mathbf{p}) + a_3 u_1^{\sigma\,(2)}(\mathbf{p})\overline{u}_1^{\sigma\,(2)}(\mathbf{p}) + a_4 u_2^{\sigma\,(2)}(\mathbf{p})\overline{u}_2^{\sigma\,(2)}(\mathbf{p}) = \mathbb{1} \quad (27)$$

Constants a_i are defined by the choice of the normalization of bispinors. In any other frame we are able to obtain the primary wave function by choosing appropriate coefficients c_i^k of the expansion of the wave function (in fact, using appropriate dual rotations and inversions)

$$\Psi = \sum_{i,k=1,2} c_i^k \psi_i^{(k)} \quad . \quad (28)$$

Of course, the same statement is valid for negative-energy solutions, since they may be chosen to be the same as the positive-energy ones in the case of the Hammer-Tucker formulation for a $j = 1$ boson, ref. [7, 16]. Using the plane-wave expansion it is easy to prove the validity of the conclusion in the coordinate representation. Thus, the question of fixing the relative phase factors by appropriate physical conditions (if exist) in each point of the space-time appears to have physical significance for both massive (charged) and massless particles in the framework of relativistic quantum electrodynamics.[9] Finally, let me mention that in the nonrelativistic limit $c \to \infty$ one obtains the dual Levi-Leblond's 'Galilean Electrodynamics', refs. [27, 28].

The conclusion is:[10] The Weinberg-Tucker-Hammer massless equations (or the Proca equations for $F_{\mu\nu}$ in the massless limit), see also (7) and (8), are equivalent to the Maxwell's equations in the definite choice of the initial and boundary conditions, what proves their consistency. They (Eq. (1) for spin j) were shown in ref. [2] to be free from all kinematical acausality as opposed to Eqs. (4.21) and (4.22) of [1b]. Therefore, at least, we have to think, why did S. Weinberg speak out about the Maxwell's equations (4.21) and (4.22): *"The fact that these field equations are of first order for any spin seems to me to be of no great significance. . . "* [1b, p. B888]. The necessity of generalizations of the Maxwell's formalism on the basis of the consideration of dual and parity-conjugate solutions, and various field configurations, including, possibly, 'longitudinal' modes and imaginary parts of the field functions, perpetually becomes obvious.

In the meantime, I would not like to shadow theories based on the use of the vector potentials, *i.e.*, of the representation $D(1/2, 1/2)$ of the Lorentz group. While the description of the $j = 1$ massless field using this representation contradicts with the Weinberg theorem $B - A = \lambda$, what signifies that we do *not* have well-defined creation and annihilation operators in the beginning of the quantization procedure, one cannot forget about significant achievements of these theories. The formalism proposed here and in my previous papers [6]-[9] could be helpful only if we should necessitate to go beyond the framework of the standard model, *i.e.* if we should come across the reliable experimental results which cannot have satisfactory explanation on the ground of the concept of a minimal coupling introduced in the usual manner (see, *e.g.*, ref. [5] for a discussion of the theoretical model for neutrinos, which forbids such a form of the interaction).

This is the last paper of the series [9,6-8] and it provides fresh glances at the Maxwell's electromagnetic theory and points out problems which are required adequate explanation. The necessity of building the relativistic classical/quantum theory on the first principles, i.e., on using the extended Lorentz group symmetries and the principle of causality, is obvious.

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In fact, this paper is the *Addendum* to the previous ones [6, 7, 8]. It has been thought on September 3-4, 1994 as a result of discussions at the IFUNAM seminar (México, D. F., 2/IX/94).

Note Added (Feb. 2, 1999). Thanks to suggestions of Prof. A. Gersten I would like here to discuss a question related to the Gauss law and the conservation laws.

The source Maxwell equations have been generalized in our work (see the first equations of (12), (18) and (22-23)). This generalization is related to the Majorana-Oppenheimer and Imaeda-Ohmura generalizations [Phys. Rev. 38 (1931) 725; Prog. Theor. Phys. 5 (1950) 133; ibid. 16 (1956) 684, ibid. 685]. Such generalizations as pointed out by R. A. Lyttleton and H. Bondi [Proc. Roy. Soc. A252 (1959) 313] and Ll. G. Chambers [J. Math. Phys. 4 (1963) 1373] may lead to small local non-conservation of the electric (magnetic) charge and to cosmological models of the expanding Universe.

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[9]The paper which is devoted to the important experimental consequences of this fact (*e.g.*, the Aharonov-Bohm effect and some others) is in progress.

[10]This conclusion also follows from the results of the paper [2, 6, 17] and ref. [1b] provided that the fact that $(\mathbf{Jp})$ has no the inverse matrix has been taken into account.

Nevertheless, in ref. [19] Gersten presented us a new conserved current and related it to the total helicity of the electromagnetic field (the difference in the number of left- and right- polarized photons). Due to the fact that both the Gersten formalism and the formalism presented here have common ideas and intersect each other we agree with the interpretation of ref. [19].

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