The Vacuum in Quantum Optics

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ABSTRACT. A heated enclosed cavity, often referred to as a black body is filled with radiation which depends only on temperature and has a characteristic spectrum first deduced by Max Planck on the hypothesis of quanta. The amount of energy in a small volume within the black body fluctuates; this fluctuation was computed by Albert Einstein from thermodynamic principles. It became possible to relate it to light quanta or photons from the discovery of the statistics of the photons by Satyendra Nath Bose. At the same time in the wave picture of light waves it became natural to ask to what extent the wavefronts at nearby points are steady or fluctuating. These fluctuations give rise to partial coherence of light. With the advent of lasers we can construct wavefronts which are steady and hence exhibit coherence over a wide area. This topic can be strictly treated by quantum theory even for feeble light using the optical equivalence theorem discovered by George Sudarshan. When light falls on a clean metal surface, photoelectrons are liberated; the statistics of these photocounts is a method of studying the intensity fluctuations of light. Here classical stochastic theory will show that photocounts come bunched together so that there is an excess fluctuation over a purely random (Poisson) process. This is called "photon bunching." But quantum theory allows less than Poisson fluctuations, called "antibunching" which demonstrate conclusively the quantum nature of photo-counting. The propagation of coherence with rigorous details was formulated by Emil Wolf. The photocounting distributions were given by Leonard Mandel and Roy Glauber.

All these questions require a number of ingenious experiments to demonstrate and measure. Beginning with the work of Hanbury-Brown and Twiss and significantly advanced by Leonard Mandel, these are important contributions to be discussed in the article.

The quantum state of the photon and more generally of the electromagnetic field require careful discussion. C.A. Mead[14] had shown how the photon vacuum changes from empty space to a transparent dielectric. The work on "squeezed states of light" has shown that the photon vacuum is still more complex than we imagined; details are given in the full article.

Vacuum as a State of the Electromagnetic Field.

An enclosed cavity from which all air has been exhausted is empty space, what we usually call a vacuum. As the empty container is heated to a high temperature the cavity gets filled with light, the composition of which is independent of the walls of the cavity. This "blackbody radiation" is a continuous spectrum with the energy density going as the fourth power of the absolute temperature. The spectrum of this radiation is continuous with the maximum at a frequency which is proportional to the temperature. One can say that the emission spectrum of a blackbody is a single very broad line, with the shape in terms of the reduced frequency (ν/T) being universal. Earlier attempts based on classical thermadynamics gave absurd results. The first successful attempt to derive this universal shape was made by Planck[1)] at the turn of the century, and he had to introduce the quanta of energy and a universal constant of action. This was the birth of quantum theory. We may also recall that the blackbody radiation was the first place where classical statistical mechanics gave the absurd answer,[2)] that the specific heat capacity of the blackbody was infinite. There was thus "Much ado about Nothing." Vacuum was no longer the vacuum but the theatre for the play of electromagnetic radiation of all frequencies. The only case the empty cavity is literally a vacuum is at absolute zero when there is no radiation in the cavity. Thus we could say the vacuum is also a state of the electromagnetic field; it is the ground state of the electromagnetic field.

Planck considered, following Rayleigh and Jeans, that the standing modes of the electromagnetic field could be considered as oscillators with frequencies $\nu, 0 < \nu < \infty$ and that the energy levels of the oscillator could only be

$$E_n = nh\nu$$

The probability for the n^{th} state is

$$P_n = Z^{-1}\exp(-\beta nh\nu);\ \beta = \frac{1}{kT}$$

$$Z = \sum_0^\infty \exp(-\beta nh\nu)$$

Then the mean value of the energy is

$$\langle E\rangle = \frac{h\nu}{e^{h\nu\beta}-1} \; .$$

The number of modes with frequency between ν and $\nu+\Delta\nu$, taking account of the two polarizations, is given by

$$2.V.4\pi k^2 dk = V 8\pi \frac{\nu^2 \Delta\nu}{c^3} \; ,$$

where V is the volume of the cavity and $K=\nu/c$ is the wave number of the radiation. Hence the energy density is

$$u(\nu) = \frac{8\pi h\nu^3}{c^3(e^{\beta h\nu}-1)} \; .$$

This is the energy density that Planck derived for blackbody radiation at inverse temperature β. Given this mean energy, there is some fluctuation.[3)] It maybe instructive why classical physics gave an absurd answer when we calculate the energy density of the vacuum when the temperature is raised and how Planck's quantum hypothesis cured it. In classical thermodynamics each independent mode of vibration, technically called "degrees of freedom", must on an average have the same energy, proportional to the temperature. So the "spectral energy density," that is the energy in a finite interval of frequencies is proportional to the number of independent modes per unit volume; and is therefore finite. This was what Rayleigh and Jeans had predicted. Unfortunately there are an infinite number of modes of oscillation of the electromagnetic field if we count them over all frequencies. So we find the absurd answer for the full energy density integrated over all frequencies. This is patently absurd and at variance with experience. We could heat a fireplace to red heat with a finite amount of fuel.

So somehow we must make the number of degrees of freedom finite at finite temperatures. What Planck chose to do is very similar to the usual custom of allowing only those who can show a preassigned amount of money on deposit. His requirement is that only temperatures (for which the equivalent energy β^{-1} is above the quantum of energy h_ν are able to excite the mode with frequency ν. So eventhough the number of potentially available modes is infinite at any temperature only a finite number (growing as the cube of the temperature) can effectively participate. The energy density is this number multiplied by β^{-1} and therefore it goes as β^{-4}. For frequencies higher than β^{-1}/L the contribution is not zero, but falls to zero very rapidly.

When we think of a vacuum with zero energy density, there should be no photons in it. Classically this would mean zero electric and magnetic fields. But the situation in quantum theory of the electromagnetic field is more subtle. Even in the state of no photons, the electromagnetic fields have zero mean value but they have fluctuations around this zero mean. These vacuum fluctuations can be measured, for example in the precision spectrum of the hydrogen atom.

Classically the blackbody electromagnetic field is contributed by adding together very many contributions with varying amplitudes and phases. Consequently the ensemble of real and imaginary parts (of electric and magnetic fields) for each mode would be Gaussian with zero mean. This implies that the variance in the energy density per mode is the square of the mean energy density per mode:

$$\langle \Delta E_\nu^2\rangle = \langle (E_\nu - \langle E_\nu\rangle)^2\rangle = \langle E_\nu\rangle^2 .$$

Since independent modes fluctuate independently, we can compute the fluctuations of the energy for any volume:

$$\begin{aligned}(\Delta u_\nu)^2 &= \frac{8\pi^2 V h^2 \nu^4}{c^3(e^{\beta h\nu}-1)^2} \\ (\Delta u)^2 &= \int d\nu (\Delta u_\nu)^2 = \frac{\delta\pi^2 V h^2}{c^3}\int \frac{\nu^4 d\nu}{(e^{\beta h\nu}-1)^2} \; .\end{aligned}$$

All these considerations are for the energy density and energy fluctuations in empty space at temperature β.

Statistical Mechanics of Photons.

¿From the study of the photoelectric effect and the Bohr model of the atom one concludes that the energy differences $h\nu$ of the oscillator or the atom has its own existence as a physical object, the photon. Compton effect in the scattering of X-rays confirmed that the photons carry an energy of $h\nu$ and a momentum of $\frac{h\nu}{c}$. According to the theory of special relativity these correspond to a particle of zero rest mass which moves with the constant speed c in all inertial frames in empty space.

The statistical mechanics of an assembly of photons required however the discovery of a new statistics of identical particles by Bose.[4)] The essence of Bose's motivation was to say that permutations of identical particles does not change the state. Therefore a state of n_1 photons of energy $h\nu_1$ each, n_2 photons of energy $h\nu_2$ each, etc. maybe viewed as a single state with energy $n_1 h\nu_1 + n_2 h\nu_2 \ldots$. It is therefore identical with the state of the first oscillator excited to the n_1^{th} level, the second to n_2^{th} level, etc. This together with the fact that photons can be created or absorbed singly by the cavity so that the photon number is not fixed in the ensemble (zero chemical potential) enabled Bose to derive Planck's law as well as the extended grand canonical (Bose) distribution for photons, the Bose statistics. Thus the blackbody at finite temperature becomes a collection of photons distributed statistically. Equally well it could be considered as being filled with an ensemble of waves.

Partial Coherence.

In the propagation of natural light[5] we can have two equivalent pictures. One of these is to see it as a jumble of waves, the wavefronts and phases being distributed at random or according to some law. Because of this the light from two distinct locations in the field are in general only poorly correlated. An equally satisfactory (though less familiar) way is to think of an ensemble of locally plane waves with rays normal to their planewave surfaces.[6] So we have a collection of light rays simulating the motion of classical photons. In the special case when the light vibrations are in good correlation with each other we say that there is a patch of coherence; in this case there is an extended wavefront with uniformly converging, diverging or parallel rays over the region. In general, natural light has a finite correlation length beyond which the "partial coherence" decreases to zero very rapidly. From a laser we get an intense beam of light with a large "coherence length". The reciprocal relation between rays and wavefronts explains why the coherence patch increases with the distance of propagation, the van Cittert-Zemike theorem.[5]

Photocount Statistics and Intensity Correlations.

When light falls on a clean metal surface, photoelectrons are liberated; the statistics of these photocounts is a method of studying the intensity fluctuations of light.[7] Given an intensity of light I, the possibility that there will be a photocount in the interval Δt is proportional to I and Δt and we may write it as $\alpha I \Delta t$ where α is a suitable efficiency factor. The probability of a count in any small interval is independent of what happend before. The counter's response is stochastic, and hence the counts in a finite time interval from $t = t_1$ to $t = t_1 + T$ is independent of t_1 and has the Poisson form:

$$p(n, T) = \frac{\mu^n e^{-\mu}}{n!}$$

where

$$\mu = \alpha I T$$

is the mean number of counts in the interval. For this distribution the variance is

$$\sum n^2 p(n) - \Big(\sum_n np(n)\Big)^2 = \mu = \sum np(n)$$

which is characteristic of a Poisson distribution.

If the intensity is not unique but is a time independent ensemble with probability $P(I)dI$ the photocount distribution would have the effective distribution

$$\pi(n, T) = \int p(n, T, I) P(I) dI.$$

More generally, if the intensities fluctuate according to a correlation

$$\langle I(t_1) I(t_2) \rangle = \langle I^2 \rangle C(t_1 - t_2)$$

we can express $\pi(n, T)$ in terms of $C(t_1 - t_2)$. Here it is assumed that the ensembles are "stationary." The intensity correlation function $C(t_1 - t_2)$ now becomes a quantity that influences the computation of the effective counting distribution and hence of the variance. Then photocount data can be used to determine the intensity correlations. Classically we would expect a bivariate Gaussian distribution for the electric and magnetic fields or equally well the complex wave amplitudes. In this case all correlations and moments of counts can be computed, as Mandel and Glauber have shown, in terms of the variance of the Gaussian. The existence of intensity correlations was discovered by Hanbury-Brown and Twiss, first in radio astronomy and then in laboratory optical sources. In the meantime Wolf[8] has shown that the various correlation functions obey simple differential equations and he developed a rigorous theory of propagation of the two-point correlation functions.

Quantum Optical Fields and Coherent States.

But the electromagnetic field is quantized. For each oscillator mode we have a pair of conjugate variables $\varphi(\nu)$ and $\pi(\nu)$ which behave like generalized canonical quantum coordinate and momentum operators which do not commute. From them we can construct creation and annihilation operators $a^\dagger(\nu), a(\nu)$ which also fail to commute:

$$[a(\nu), a^\dagger(\nu')] = \delta(\nu - \nu') \cdot 1$$

The vacuum with no excitation of the electromagnetic field has no photons. Classically we would think of the vacuum as having no electric and magnetic fields. But since the fields are noncommuting quantities in quantum theory, the quantum vacuum cannot correspond to zero value for both the fields $\varphi(\nu)$ and $\pi(\nu)$. Yet much of our understanding of wave optics would be appropriate for quantum optics if we could make the vacuum state correspond to zero value for "the field", say $\varphi(\nu) + i\pi(\nu)$. This is the method of coherent states outlined below: it makes all classical waveoptical phenomena like reflection, refraction, image formation, interference, diffraction and polarization "already quantized". Yet it is no approximation, but an exact result; and it is capable of dealing with genuine quantum effects like photon antibunching. For this purpose we must make use, in addition to the familiar Schrödinger and Fock representation a very useful construction which was discovered by Schrödinger[9] and developed by Bargman. It is very useful for quantum optics.[10] This is over all entire functions of a complex variable z with a scalar product

$$(f(z), g(z) = \int (f(z))^* g(z) e^{-|z|^2} \frac{d^2 z}{\pi}.$$

On this vector space define $a, a^\dagger$ by

$$(af)(z) = z f(z)$$
$$(a^\dagger f)(z) = \frac{\partial}{\partial z} f(z)$$

The state corresponding to the function 1 is annihilated by a and is the vacuum. The n-excited state is represented by the function $\frac{1}{\sqrt{n!}} z^n$. These states are very useful for describing coherent radiation. The state having states $z(\nu)$ for the modes ν with mode functions $v(\nu, x)$ then the (positive frequency or annihilation part of the) electromagnetic field (with the appropriate helicity) has the eigenvalue

$$\int z(\nu) v(\nu, x) d\nu \ .$$

The excited modes of any particular state of the quantum electromagnetic field are those that arise in the eigenmode decomposition of the two-point correlation that we have already talked about:

$$\Gamma(x_1 t_1, x_2 t_2) = \langle \varphi(x_1, t_1) \varphi(x_2 t_2) \rangle \ .$$

This leads to the possibility of the display of the states of a quantum field as a quasi-classical ensemble of fields. This is true not only for intense fields but even feeble fields: and it is not an approximation but an exact "equivalence theorem" discovered by Sudarshan.[11] If the coherent states are represented by $|z \gg$, then the statistical density matrix for the quantized field may be realized as

$$\rho = \int \phi(z) |z \gg \ll z| d^2 z$$

where $\phi(z)$, the "diagonal weight" acts as density of the wave field.

Squeezing and Antibunching.

In classical stochastic theory the photocounts will show excess fluctuations that show "photon bunching." For the Gaussian quantum fields the excess fluctuation per mode is

$$\langle n^2 \rangle - \langle n \rangle^2 = \langle n \rangle^2 + \langle n \rangle \ .$$

Here the first term on the right hand side is the classical wave noise that Einstein had calculated; and the second one the classical Poisson noise. Such photon bunching[7] can therefore arise in suitable classical field ensembles. But in all such cases the variance is bounded below by the Poisson variance. But in a quantum wave ensemble the weight function $\phi(z)$ can be negative and these could be "photon antibunching" with less than Poisson variance. In the extreme case of a state of a fixed number of photons, the variance can vanish. Experimentally such antibunching[12] has been demonstrated.

Whenever we have canonical variables we can make linear canonical transformations. This leads to a Bogoliubov-Valatin transformation on the creation and annihilation operators:

$$q \to e^{-\lambda} q, p \to e^{\lambda} p$$
$$a \to \cos\text{h}\ \lambda a - \sin\text{h} \lambda a^\dagger$$
$$a^\dagger \to -\sin\text{h}\ \lambda a + \cos\text{h} \lambda a^\dagger$$

The state which is the vacuum for a and $a^\dagger$ is no longer the vacuum for the transformed operator. These new operators and the new vacuum are called the "squeezed operators" and "squeezed vacuum."[13] Such a situation naturally obtains for light in a transparent dielectric. Mead[14] has worked out the theory of light propagating in such media. But such squeezed states can even be in empty space; this has also been experimentally verified.[15]

Concluding Remarks.

All the above discussion is for the free electromagnetic field. But the electromagnetic field is in interaction with the quantized electron field and other quantized fields. The original vacuum is no longer the minimum excitation state. In fact that there are photons in quantum electrodynamics and/or a vacuum state are very reasonable assumptions but not proved by any means. It is generally accepted that they do exist and that the treatment we have for free fields continues to be applicable in the cases which we have considered in this article. So the vacuum of quantum electrodynamics has a very rich structure. In curved spacetime appropriate for gravitational fields the vacuum for the incoming fields and for the outgoing fields are not the same leading to particle creation. Hawking has used this to deduce that a blackhole radiates like a blackbody at a temperature proportional to the area of the blackhole. If tachyon exist the vacuum depends on the Lorentz frame in which it is viewed.

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