Quantum Mechanics From General Relativity : Particle Probability from Interaction Weighting

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ABSTRACT. Discussion is given to the conceptual and mathematical change from the probability calculus of quantum mechanics to a weighting formalism, when the paradigm change takes place from linear quantum mechanics to the nonlinear, holistic field theory that accompanies general relativity, as a fundamental theory of matter. This is a change from a nondeterministic, linear theory of an open system of 'particles' to a deterministic, nonlinear, holistic field theory of the matter of a closed system.

RÉSUMÉ. On discute le changement conceptuel et mathématique qui fait passer du calcul des probabilités de la mécanique quantique à un formalisme de pondération, quand on effectue le changement de paradigme substituant à la mécanique quantique linéaire la théorie de champ holistique non linéaire qui est associée à la théorie de la relativité générale, prise comme théorie fondamentale de la matière. Ce changement mène d'une théorie non déterministe et linéaire décrivant un système ouvert de 'particules' à une théorie de champ holistique, déterministe et non linéaire, de la matière constituant un système fermé.

1. Introduction.

In my view, one of the three most important experimental discoveries of 20th century physics was the wave nature of matter. [The other two were 1) blackbody radiation and 2) the bending of a beam of starlight as it propagates past the vicinity of the sun]. The wave nature of matter was predicted in the pioneering theoretical studies of L. de Broglie in 1924 [1]. It was subsequently verified in the experimental observations of electron diffraction from a crystal lattice. These were the studies of Davisson and Germer [2] in the US and simultaneously by G. P. Thomson [3] in the UK.

The significance of the discovery of the wave nature of the electron, to me, was that it was an indication of a paradigm change in our view of matter - away from the

atomistic model in terms of singular particles. These are the elements of an open system of 'things', that are then allowed to interact with each other (or not). The change is to the view of a closed system, characterized by the continuous field concept and holism. In the latter view, the apparent 'things' become the distinguishable manifestations of a single continuum, analogous to the ripples of a disturbed pond.

These discoveries in the 1920s introduced contemporary physics to the subject of quantum mechanics. Aside from the spectacular successes of this theory in matching the empirical features of matter in the microdomain, controversy remains on the interpretation of the wave nature of the things of matter. The standard Copenhagen interpretation, in contrast with that of de Broglie, is that while the particles of matter do have a wave nature, they are nevertheless singular, discrete entities wherein the observed 'waves' have to do with the probability distribution for measuring these particles with a given set of physical properties at the different space-time points.

The probability function follows in quantum mechanics from an equation of continuity, that is incorporated into wave mechanics by virtue of the gauge invariance of the theory, in the relativistic or non-relativistic versions. When this equation is integrated over all of 3-space, within any chosen frame of reference (a Lorentz frame in special relativity or a generally covariant frame in general relativity) we come to the sum of a time derivative of an integral of the squared wave function over the chosen 3-space plus a surface integral that covers this 3-space. With the condition that the matter field vanishes on the latter surface, the result is that the time derivative of the integral over the squared wave function in 3-space is zero - it is conserved in the time frame of the chosen reference frame. Thus, the latter constant integral is a 'constant of the motion'; when the matter field is normalized, it may then be interpreted as a probability amplitude. It is the squared matter field, depending on the 4n coordinates for the n-body system in relativity theory, that is then the probability distribution in the quantum mechanical theory.

My research program, aimed at deriving the formal structure of quantum mechanics from general relativity [4], has led to some questions that are aside from the empirical predictions of the theory. One of these questions relates to the shift from the probability paradigm of the quantum theory to that of a weighting formalism for a closed system, in the holistic, continuum view of general relativity theory. In the former view, the laws of matter in the microdomain (and not the macrodomain) are based on a discrete particle model that is expressed in terms of a probability calculus. In contrast, the paradigm that entails holism and continuity of general relativity theory, implies that the laws of matter are the same in all domains. In this view, probability, per se, plays no fundamental role. The purpose of this note is to further clarify this conceptual shift and to identify the new paradigm in the continuum field theory that replaces the probability approach of the quantum theory.

2. Transition from elementary particle to elementary interaction.

The mathematical basis of the quantum theory is in the form of a probability calculus expressed in terms of a Hilbert function space. This is more general than earlier classical probability theories because not only does it yield the probabilities for matter to be in the(infinite multitude of) states of a microsystem, it also yields the probabilities of transitions between these states. The probabilities of the material system to be 1) in particular states and 2) making transitions between them are primary in the very definition of the elementary particle of matter, in the quantum paradigm.

With the continuum, holistic approach of general relativity theory, as a fundamental theory of matter, there are no discrete, singular particles of matter at the outset. There are only distinguishable modes of a continuum. In the quantum theory, the probability wave, called a 'probability amplitude', is represented by the complex function $\psi(x)$. This relates to the (continuously distributed) chance of measuring the properties of a particle at a particular space-time point, x. The following question must then be asked : If the law of this 'wave function', in the Hilbert space formalism, leads to predictions of measured properties of elementary matter that match the empirical facts in the microdomain, and if this law in the continuum, holistic approach *is only an approximation*, where probabilities play no fundamental role, then what is the interpretation of this *global extension* of the probability wave in the new paradigm ? This is the central question addressed here.

The answer to this question must lie in the change of ontology that accompanies the paradigm shift from the quantum theory to general relativity. Then, what is the entity that replaces the elementarity of the particle of the quantum view ? It is my contention that the answer to this question is that the new paradigm based on general relativity, as a fundamental theory of matter, implies that, rather than the 'elementary particle', it is the *interaction relation*, holistically, that is elementary here. I have called it : 'elementary interaction'.

3. The interaction field amplitude.

In the quantum theory for a many-body system, the absolute square of the amplitude$|\Psi(x_1, x_2, \ldots, x_n)|^2$ is interpreted as the probability density (probability/volume) for the location of each of the n particles to be at the respective space-time points $(x_1, x_2, \ldots, x_n)$, in a 4n-dimensional space-time. In the continuum field theory, the *global extension* of this probability density is the *weighting function* $|\Psi(\psi_1(x), \psi_2(x), \ldots, \psi_n(x)|^2$. This is the weighting /volume for the interaction of a n-component, closed system, at each point of a single space-time x. This is then a nonlocal field theory because the trajectories of individual, separable particles of matter are not specified.

Instead, the distinguishable field amplitudes $\psi_i(x)$ of the component modes of the continuum, $\{\psi_1(x), \psi_2(x), \ldots, \psi_n(x)\}$ are each a solution of a matter field equation, for a distinguishable mode of the continuum. This view is somewhat akin to Schrödinger's interpretation of the many-body wave function is terms of the normal modes of vibration of an ensemble, rather than relating to individual particles. [5]

The differential operator for the matter field equation, whose solution is $\psi_i(x)$, entails a functional that depends on all of the matter fields except the ith one, $I_i(\psi_1, \psi_2, \ldots, \psi_{i-1}, \psi_{i+1}, \ldots, \psi_n)$, representing the coupling of all other matter fields of the closed system to ψ_i. Since the latter matter fields similarly entail their own interaction functionals, I_1, I_2, $\ldots I_{i-1}$, I_{i+1},$\ldots$, each depending on ψ_i, the functional I_i must depend implicitly on ψ_i. It then follows that the equation that yields the solutions of the ith matter field is automatically nonlinear, as well as nonlocal. Such a formalism then cannot relate to a probability calculus, which is based on linearity, in *its definition* .

The matter field equation in the ith mode of the continuum, ψ_i, asymptotically approaches a wave amplitude of the quantum theory, as 1) the respective coupling term, I_i, either approaches zero (this is the 'free field limit' - a limit that may be approached as closely as one pleases, but cannot in principle be reached) or 2) it can be approximated by an average background potential field, independent of the individual field amplitudes of the closed system. (The latter is similar to the approximation in the Dirac bispinor equation for wave mechanics with a potential present). This limit corresponds to energy-momentum transfers within the closed system that are small enough to neglect (that is, without the need to take account of 'recoil' of the other matter fields of the closed system). In this case the closed system 'looks like' an open system wherein each matter field amplitude depends on its own set of space-time coordinates. One may then use the linear approximation for its form.

In the latter limit of 'uncoupling' of each mode of a closed system from all of the other modes, each field amplitude may be mapped in its own space-time. The entire set of modes become a localized set, $\psi_1(x_1), \ldots, \psi_n(x_n)$. The linear equations that this approximation for the individual modes now solve allows their description with a Hilbert space formalism.

We see, then, that in the limit of sufficiently small energy-momentum transfer between the modes of a continuum, one reaches a linear, local limit of the equations in ψ_i.

But precisely how do we come to the Schrödinger form of wave mechanics (or Dirac's form of wave mechanics) in particular, in this limit ? The answer lies in our implementation of the *principle of correspondence*. This is one of the three axioms of the field theoretic approach to matter of general relativity in my research program. [The other two are : the *principle of general covariance* and the *generalized Mach principle*. (In the latter it is concluded that all previously considered intrinsic qualities of matter, not only inertial mass, are measures of coupling within a closed system).[6]]

What was shown previously was that the formal Hilbert space expression of quantum mechanics may be taken as a linear approximation for a generally covariant field theory of inertia. The reason that the solutions of the fundamental equations are complex variables is that the symmetry group of relativity theory (in its general or special form) entails *irreducible representations* that behave algebraically as quaternions ; the basis functions of the latter are two-component spinors. [It has been shown by Pontrjagin[7] that the most general sort of associative algebra, subject to very general topological constraints must be in terms of quaternions, which reduce in special cases to complex number systems and real number systems]. The components of the spinor basis functions of the quaternions are complex variables. This relates to a question that Schrödinger asked Einstein : How can quantum mechanics emerge from a unified field theory in general relativity since the former necessarily entails complex variables whereas general relativity (in the tensor form that Einstein discovered) seems to entail only real number-valued variables ? The answer is that in its most general (*irreducible*) form, Einstein's general relativity must indeed be expressed in terms of quaternions and spinors - which are necessarily formed from complex variables. Thus it is its algebra that indicates that general relativity must incorporate the complex variable expression that is natural for quantum mechanics. Finally, the *principle of correspondence*, in turn, implies that the most simple form of the inertial field equations, if they are to match the equations of quantum mechanics in a linear limit, are first order (quaternion-valued) differential equations that are homogeneous in their spinor solutions.

These restrictions, then, are enough to yield a generally covariant spinor field theory of inertia whose asymptotic, linear limit is precisely in the form of the Hilbert space probability calculus - the form of quantum mechanics [4]. Further discussion of the new paradigm that shifts the *law of conservation of probability* to a law of *conservation of interaction* in terms of interaction weighting is now in order.

4. Conservation of interaction.

Taking the interaction field amplitude $\Psi(\psi_1(x), \psi_2(x), \ldots, \psi_n(x))$ for an n-mode coupled closed system to transform as a spinor variable, as a basis function of the irreducible representations of the symmetry group of relativity theory, the differential expression of the postulated law of conservation of interaction has the form of a continuity equation. Using Dirac's bispinor notation, it is :

$$\partial^\mu(\bar{\Psi}\gamma_\mu\Psi) = 0 \qquad (\bar{\Psi} = \Psi^+\gamma_0)$$

where γ_μ are the Dirac matrices.[6] With the condition that Ψ vanishes at the boundaries of the closed system, this equation then implies that in any local observer's frame of reference, the quantity represented by the integral in three-dimensional space :

$$\int \Psi^+ \Psi dr$$

is a constant with respect to the time measure in this reference frame.

With the required imposition of normalization on the interaction field amplitude, if it is to be interpreted in terms of a *weighting function*, the integral of the positive-definite function $\Psi^+\Psi$ over all space is unity. It is interpreted here as relating to the weighting of the total interaction within a closed system, described in a single space-time. Note that the conservation of interaction does not imply that it is necessarily uniform throughout space and time. It does mean that, given a closed material system, the intrinsic mutual interaction generally has a flexible mapping in space and time that persists for all times with respect to any local observer's measurements. Any alteration of the environmental conditions in a local region that may be made in some experimental investigation would then give rise to a redistribution of this weighting within the entire system. But any such alteration *within the closed system* cannot cause the weighting function to vanish anywhere, at any time, even though it may become arbitrarily weak in particular regions of space and time.

Pair Annihilation and Creation

To exemplify the role of the interaction field and its physical implications, consider the commonly referred to events of pair annihilation and creation. If matter should indeed be annihilated and created at arbitrary times and places, as it is assumed to happen in quantum field theory, then the weighting function relating to the density of interaction conservation would no longer be a conserved quantity. In this case it would no longer be true that $\int \Psi^+\Psi dr$ is constant in time. Thus the field theory discussed must predict all of the experimental results that are conventionally interpreted as pair annihilation and creation - but without actually creating or annihilating matter at any time. These results were indeed proved in this theoretical program [8].

The Hydrogen Atom

It is interesting to examine the interpretation of the conventional description of the hydrogen atom in quantum mechanics (or, more generally, many-electron atoms) within the framework of the idea of interaction weighting. While the nonlinear field equations for the *e-p* system in the context of this theory do approach the exact Schrödinger form of wave mechanics in nonrelativistic quantum mechanics, or the Dirac relativistic version, the features of hydrogen must still be interpreted differently in this view. The important quantity here is the weighting of the interaction between the electron and the proton, as field modes of a closed system, rather than considering hydrogen as two singular particles of matter, perturbing each other at a distance. In the holistic view considered in field theory, the presence of the electron and the proton in the universe must be accounted for in terms of a *continuous field* that *weights* their mutual interaction. It follows from the solutions of the matter field equations for the coupled electron and proton matter fields, that relate explicitly to the inertial feature of matter, that the electron-proton interaction is weighted most heavily in the region of space that is a sphere with a radius called the 'first Bohr orbit'. [9] With this interpretation, no reference need be made to the electron and proton as isolated entities. In this way, the 'atom' can be represented with a formalism that is based entirely on the continuous field concept, and in strict accord with the axioms of this theory : *the principle of covariance , the generalized Mach principle and the principle of correspondence.*

It should be emphasized at this point that that the discreteness of physical observables, such as atomic energy levels, within the context of this theory, is only an *apparent discreteness*. For it is only within an approximation for the exact nonlinear equations of the theory that one arrives at the linearized eigenvalue equations for the atomic system - thereby leading to the predicted (apparent) discreteness of atomic energy levels, *asymptotically*. This is in accordance *with the correspondence principle* - the third axiom at the basis of this holistic field theory of matter. Thus, this theory predicts that atomic energy levels are not in principle discrete, but they do indeed have finite width, arising from the physical coupling within the remainder of the closed system. Since, according to the *generalized Mach principle*, this coupling can never be totally 'off', the line widths for spectral distributions of atomic energy levels, for example, can never go to zero. That is to say, the properties of matter (of any quantity, in *any domain*) have a continuous spread of values, though they can be peaked under special physical circumstances, when it appears that there are individual, discrete particles of matter, in interaction.

Radioactivity

To exemplify further the contrast between the aspects of continuity and discreteness in physically measured properties, consider the operation of a Geiger counter, in detecting radioactive emissions. At first sight this device functions in a way that appears to entail the occurrence of *discrete energy bundles* that enter the window of the counter at *random intervals of time*. One then associates the 'clicks' of the counter with the existence of discrete things that appear in an acausal fashion. But it is clear that the data does not compel one to assert the idea that randomness and discreteness are ingredients that must underlie these data, as a fundamental explanation. After all, the Geiger counter is not more than an electronic instrument that has a voltage bias which is set by the experimenter at a convenient level, in order to discharge electrical energy whenever the interaction with some signal exceeds some predetermined threshold voltage. As the voltage bias (and therefore the threshold for a 'click' to happen) is lowered, more clicks would be heard in any fixed amount of time. In the limit where there would be no voltage bias, the discrete clicks would wash out into a steady background 'noise'. That is, in this limit the 'signal-to-noise ratio' would be reduced to unity. To interpret this 'noise' as a random set of effects of uncoupled things would be to assume an ideal limit that cannot be directly verified in an experiment ; it can only be postulated ! Indeed, this is a postulate of indeterministic atomism that is implicit in the ontology of the quantum theory. Still, the actual data does not compel one to adopt this model as providing a unique explanation. The 'noise' in this experiment could also represent the peaks of a continuously connected curve, totally predetermined, though there is not enough resolution to see it in this particular experimentation.

What I am saying is that the property of discreteness in the atomic domain is a model-dependent conclusion, *abstracted from* the measurements of continuous (though peaked) values for the conserved properties of the matter. It is continuous in the sense that between any two measured values of some physical property, no matter how close they may be, one can always measure another value of this property. [In mathematics, such a continuous set of numbers is called 'dense'].

In the quantum theory one must say that the latter statement that the measured values of a quantum system is "dense" is not true in the ideal limit. There is a residual quasi-continuity in the set of physical values of any property of micromatter, according to the quantum theory. But this is not an empirical fact based on the actual experimentation ! The reason given in the quantum theory for this conclusion is that there is a finite, irreducible line width associated with all measured (asserted) discrete values of the properties of micromatter - because of the Heisenberg uncertainty principle. In the theory of matter following from general relativity, the discrete limit postulated in the quantum theory to be an elementary feature of micromatter, does not exist. The finite, irreducible line width here has to do with the nonlinear coupling of all of the components of a closed system. The main point made here is that whether the underlying abstract idealization that matter is fundamentally discrete with discrete eigenvalues for its physical properties, or the nature of matter is based on the continuous field concept and holism, where in principle there are no discrete eigenvalues for its properties, is something that can only be tested indirectly. That is, these are theoretical abstractions that can only be postulated and then logically and mathematically tested ; they are not directly observable assertions.

To sum up, the clicks of a Geiger counter, the optical spectrum of a radiating gas, etc., clearly indicate a peaked nature of the *interaction weighting* of the corresponding coupled systems. But the results of these experiments do not necessarily require the conceptual basis of the quantum theory for an explanation. Indeed, the nonlinear field theory of inertia that fully exploits the idea of the elementarity of interaction (the closed system) rather than the elementarity of the particle (thing-in-itself) does describe the same data - in terms of continuous, though peaked sets of values. [4]

Scattering Cross Section

Since the free particle does not exist within the relativistic interaction theory of matter that we discuss here, the definition of the experimentally observed entity called 'scattering cross section' must be redefined since it is usually interpreted in term of *free particles* that scatter from other *free particles.*

In this theory, the 'cross section' is defined in terms of the weighting of the interaction between the projectile field, ψ^P, the target field ψ^T and the rest of the environment of the scattering entities. The terms 'target' and 'projectile' are used here, not to signify separate, singular entities, but rather they refer to *apparently separate* entities that asymptotically look that way, when the intrinsic coupling is sufficiently weak. This would be analogous to the mutual scattering of moving ripples of a pond from each other - these ripples are not in reality separate entities ; rather, they are all modes of the same continuum.

The 'cross section' is formally defined as the ratio of the total flux flowing out of some domain of interaction (per second) to the incident flux (per cm2 per second) flowing into it. But 'flux'in this view refers to the flow of interaction weighting, rather than to the flow of free particles of matter. Defining the interaction current density as $\boldsymbol{j} = \bar{\Psi}\boldsymbol{\gamma}\Psi$, where Ψ is the interaction field amplitude and $\boldsymbol{\gamma}$ are the spatial components of the Dirac matrices, the mathematical expression for the cross section is as follows :

$$\sigma = \frac{\int_\Sigma \boldsymbol{j}.\boldsymbol{n}\, d\Sigma}{j_{inc}}$$

where the integration is over a surface integral of domain Σ that encloses some volume V. The field current of incident flux is :

$$j_{inc} = f^T \bar{\Psi}_0^P \gamma_k \psi_0^P$$

where k denotes the spatial direction of the incident beam of interacting matter, ψ_0^P is the asymptotic limit of the projectile matter field, when it is very weakly coupled to the target matter field ψ^T, which is defined to be outside of an interaction volume V, surrounded by the surface Σ. f^T are the asymptotic target variables. It then follows that the scattering cross section may be expressed in the following form :

$$\sigma = \left| \frac{\partial_t \int \Psi^+ \Psi dr}{f^T(\bar{\psi}_0^P \gamma_k \psi_0^P} \right|$$

The numerator in this relation followed from the use of Gauss'theorem, applied to the continuity equation, i.e.

$$-\partial_t \int_V \Psi^+ \Psi dr = \int_V \partial^j (\bar{\Psi}\gamma_j \Psi) dr = \int_\Sigma (\bar{\Psi}\gamma_j \Psi n^j d\Sigma$$

The general expression for the cross section is then given in the formula above for σ.

It followed in this holistic field theory of matter in general relativity that proof was shown for the physical equivalent of the *Pauli exclusion principle*. It was derived from first principles, without approximation methods, based on the full nonlocality and nonlinearity of this field theory of matter. [10]. Applied to the scattering problem, this follows whenever the two scattering particle fields would be such that they satisfy the three conditions : 1) they are in the same state of motion,

2) they have a mutually repulsive interaction and 3) their inertial masses are equal. In this case, the interaction field amplitude Ψ in the numerator of the equation above for the cross section would be replaced by the interaction field amplitude Ψ_{uv} . The latter represents the total field amplitude for the closed system excluding the interaction of the component uth and vth fields that obey the restrictions of the Paul exclusion principle.

In the case in which the nonlinear coupling between the projectile and target components would be sufficiently weak, the target variables in the numerator of the cross section expression would factor out, canceling the target variable f^T in the denominator. The remaining rate of transition from the unscattered beam to the scattered beam would then only entail the coupling of the projectile matter and the environment of the target.

It was found, within the context of this theory, that the inertial manifestation of matter entails a background 'physical vacuum' of pairs (electron-positron and proton-antiproton), that are a countable set of matter fields, as in the case of a gas of molecules. [8]. It is a model that is in contrast with the model in quantum electrodynamics, where the 'physical vacuum' is a *noncountable* set of pairs, supposedly annihilating and being created spontaneously from the vacuum in a random fashion.

The problem of analyzing the scattering of fermions from other fermions may then also entail the interaction of the projectile matter field and a countable set of pair fields that are components of the 'physical vacuum', in the background of the target fermion field. The source of the interaction, here, is a predetermined, nonlinear coupling. In this regard, an important problem for future investigation concerns the implications of the Pauli exclusion principle in high energy, charged matter scattering, within the context of this deterministic, nonlinear field theory of matter and inertia.

5. Summary.

The purpose of this note has been to further clarify the probability aspect of the paradigm shift from the quantum theory to that of general relativity, as underlying

theories of matter. What was stressed here was how we go smoothly from the elementarity of the particle of matter and its probability calculus to the elementarity of the interaction within a holistic, continuous field theory, and the role therein of a weighting formalism. The latter may be viewed as a global extension of the law of conservation of probability for an open system of particles, to a law of conservation of interaction within a closed system of modes.

An example affected by this paradigm shift had to do with the concept of the scattering cross section in particle physics, and its global extension to the holistic theory in general relativity wherein there are no singular particles in the first place. The view indicates a change from scattered and incident fluxes of singular particles, that defines the measured cross section, to the fluxes of interaction within a continuum.

The general aim has been to emphasize the idea that in the continuum, holistic field theory of matter in general relativity, all remnants of the singular particle of matter are *exorcised*. They are replaced with the distinguishable modes of a nonsingular, continuous matter field, without separable parts. This is an extension of an earlier view of the matter field, to a later one when passing from the quantum paradigm to that of general relativity theory.

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