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The quantum state and the doublets*

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It is the dissymetry which creates the phenomena.
Pierre Curie [1]

ABSTRACT. The study of the quantum state is revisited to put in view the possibility to interpret all the quantum state in the framework of Sommerfeld model. Taking support of the relativity of the motion which impose to have the same causes responsible of it in the space of the electron than in that of the proton, the variations of the mass of the electron are supposed to be responsible of the variations of the speed. As a result there are variations of the radial but also of the angular speed independently of the angular quantification. It is the possibility of constant or variable angular speed which originates the relativist doublets. The interaction is attributed to exchanges of matter between the electron and the potential. This approach of the motion leads to regard the rotation as the result of two orthogonal motions of rotation. As a result only half of the action associated with rotation leads to an observable magnetic moment. This property allows to understand the spatial quantification for all the quantum states. It Appears that the spin notion escapes to Sommerfeld's theory as well to Dirac's theory.

1 INTRODUCTION

The experimental study of the spectral lines emitted by an atom reveals that they are classified in series and that the lines of some of these series are double, called regular doublets. The separation of double lines is very weak, the traditional example is that of the line D of the sodium which is double,

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the respective wavelengths being $\lambda_1 = 5890$Å and $\lambda_2 = 5896$Å. The set of the lines thus observed for various atoms forms the experimental base of the quantum state. To interpret the spectral lines Sommerfeld was brought to quantify in the motion study of the electron around the proton the angular action and the radial action [2]. This manner of making leads to a great number of remarkable results but leaves without answer the origin of the regular doublets and the existence of the half integer numbers [3]. Only up to now the introduction of the wave functions and the theoretical model of Dirac made it possible to find the set of the quantum states and the energy levels associated with the regular doublets. However, it remains in the current state of search a fact very surprising: these two theories lead to the same expression of the energy of the levels of the various quantum states whereas the interpretation of the regular doublets escapes the corpuscular traditional approach from Sommerfeld.

Putting besides these difficulties the model of Sommerfeld with the concept of trajectory has a remarkable explanatory force that have not Dirac's model. For example it makes it possible to understand the attraction between atoms of which the most external electrons are in a state "s" [4]. The trajectory indeed gives an electric dipolar character to the atoms and thus allows to understand for example attraction between alkaline atoms. On the other hand the trajectory of electron has allowed to propose an interpretation of the mechanism of conductivity and superconductivity in the superconducting oxides [5]. Furthermore the assumption of trajectory is suggested by the magnetic properties of the matter: indeed magnetism is before all the reflection of the motion of electrical charges. These various aspects suggest that there exists a connexion between the model of Sommerfeld and of Dirac and that it is possible to consider the trajectory of electron in its different quantum states. Thus the purpose of this work is to put in view this aspect of the quantum state.

To find the connection between the theory of Sommerfeld and that of Dirac it is necessary to notice that in the approach of Dirac the doublets are related to two groups of very close states which differ by relativistic corrections of the mass of the electron. But what is remarkable it is that these two groups of states have radial wave functions very close [6], a point which in the approach of Sommerfeld means that they gravitate on ellipses of very close eccentricity. In particular on circles of practically identical radii for the circular states. As the variations of mass are associated to variations of speed, that means that it can exist variations of mass other that that associated to the variations of radial speed. This result leads to reconsider the origin of the orbital motion. In the study of the phenomena the causes of the physical laws

must be independent of the place of the observation. Let consider the space of the electron supposed very small, the question is which are the variables able to determine the quantification of the system electron-proton. Let us consider the radial speed of the electron. When it varies, in the space of the electron, just the variations of the energy, thus of its mass can be associated with it. As a result it is necessary to consider that in fact the variations of the mass of the electron are at the origin of the variations of its radial speed. It results from it that the interaction between the proton and the electron is related to exchanges of mass between them which lead to a motion with fixed or variable mass. This approach of the interaction leads to consider the potential and the electron itself as fluid matter. Let us suppose then that the density of mass which describe the interaction is inversely proportional to the distance which separates the centres of gravity of the electron and of the proton and proportional to the charge of the proton. The classical characteristics of the potential are preserved but they are widened. Indeed the motion with variable mass are not related to only the variations of the radial distance. We will see that this conception clarifies the comprehension of the quantum state.

By studying the concept of spin we already used exchanges of matter between the potential and the electron to interpret the wave function. It is supposed to determine by exchanges of matter, the mechanical action to which is subjected the electron and which guides it along its trajectory upon an element of length and time {dl, dt} [6,7]. Thus the exchanges of mater determine the quantity of energy, mass and momentum which are exchanged between the potential and the electron, considered as a fluid mass [6,7]. They are the exchanges of matter which determine displacements of the electron compared to the proton and consequently the trajectory. This approach of the motion by exchanges of matter reveals another difficulty in the traditional description of orbital rotation. Indeed if the orbital motion of rotation just has two degrees of freedom it remains that the exchanges which generate it take place in the three directions of space. It is this aspect of the phenomena which makes it possible to understand the half-integer angular momentums.

This approach of the motion consists to give to the mechanical action a fundamental role. This role is natural since any momentum and any quantity of kinetic energy are always related to a space displacement and an interval of time. It was in fact the idea of Sommerfeld [2] to extend the assumption of Bohr concerning the angular momentum [8] to elliptic orbits. With Louis de Broglie the action also plays a fundamental role: it is guided by the idea of a major identity between the principle of less action and that of Fermat that he was led to propose the hypothesis of a wavelength associated to the momentum of the electron by the quantum of action " h " [9]. It was also the idea of

Schrödinger which builds the differential equation whose wave function is solution by introducing a propagation velocity of surfaces of constant action [10]. We propose in this work to show how this conception of the action by exchange of matter between the proton and the electron makes it possible to understand the existence of half-integer angular momentum and to find the whole of the energy levels in the approach of Sommerfeld.

2 THE ACTION AND THE SPATIAL QUANTIFICATION

We wish to show how the mechanical action makes it possible to propose a link between quantum corpuscular mechanics and that introduced with the operators acting on the wave function. In classical mechanics the action is the product on an element of space and time {dt, dl}, either of the momentum by the element of length dl, or of the energy by the element of time. In the theory of Dirac the operators act on the wave function by first order derivation with respect of the variables of space and time. If these operators act on a function representative of the action they gives access to the various components of the momentum and energy. Then let us take a different position from the classical interpretation and suppose that the representative function of the action is precisely the wave function. Moreover we suppose that the action generated by the wave function takes place by exchanges of matter between the electric potential and the electron. The quantification of the wave function is then that of the action associated over one period with the different degrees of freedom. This assumption leads to suppose that the electron charge and the potential are made of elements extremely small compare with dimension of electron, having a mass and that we call grains. We suppose thus that the exchanges of matters result of exchanges of grains between the electron and the electric potential.

To describe the motion of the electron around the proton we consider Ra an atomic reference frame considered as fixed, made of a system of orthogonal axes, the centre of gravity P of the proton being at the origin. This centre P is also the centre of the potential which acts on the electron. The intensity of the potential in a A point is inversely proportional to the PA distance that separates it from P centre. As a result the density of matter allowing to describe the potential is itself inversely proportional to this distance. Let us then γ be the centre of gravity of the electron. As for the potential we suppose that in the space of the electron, the density of matter allowing to describe the electron charge is a function inversely proportional to the distance to the centre γ of gravity of the electron. The surface which delimits in the space of

the proton the volume of the electron is consequently that which corresponds at the minimum of density. It is through this surface that the exchanges of matter determine the action and the trajectory.

2.1 The action of rotation

Let us consider the motion of the electron in the atomic reference frame. To describe the motion the choice of the reference frame is arbitrary. As long as we consider only the central potential as source of the interactions there is no dissymmetry which makes it possible to privilege a reference frame more than another. Let us consider the " 1s " states which are the deepest and correspond to only one quantum of action h. In Sommerfeld's model the motion of the states " 1s " is circular and there are just two degrees of freedom which are independent. On the other hand the exchanges of matter which determine the momentum and generate the orbital rotation are distributed in a volume. They do not have any raison to be only distributed on the two degrees of freedom of the trajectory. Consequently the action associated with rotation cannot be correctly described by the product of two vectors, the momentum and dl displacement, all the two contained in the plan of the trajectory. These two vectors must necessarily have two components in orthogonal plans so that the action results from exchanges in volume. In other words the plane rotation is the result of two orthogonal rotations. We will see how this property makes it possible to put in evidence the half-integer quantum numbers.

We then have to consider two aspects of the motion : the rotation which only makes it possible the electron to turn around the proton and a possible supplement of momentum which without the rotation will not make it possible to the electron to turn around the proton. This supplement of momentum causes to decrease the binding energy of the electron compared to the proton. The deepest levels are those of the states " 1s " which have the characteristics of the rotation. We will start by studying them.

Let us then consider the spherical reference frame : r, θ and φ, with $0 \leq \varphi \leq 2\pi$ and $0 \leq \theta \leq \pi/2$. This choice of frame of reference result supposing that the motion is contained in a plane and that the action also. The hypothesis of exchanges of matter, leads to consider that the mechanical action results of exchanges in all the directions of the space. To respect this hypothesis we suppose that the action of rotation, which is the alone present in the states " 1s ", is the result for each element of action pdl and whatever be the considered time, of two components of action working parallel to two orthogonal planes. In other word one can consider that the motion is the sum of two orthogonal motions each one having a variable φ with $0 \leq \varphi \leq 2\pi$. Further-

more there is no reason to favour one variable more than the other. As a result the corresponding actions must be equal.

2.2 The two components of the action of rotation

To study this property let us determine how the action of rotation can be split up into orthogonal components. Let us consider to this end a differential element dA of action. It is the product of a momentum p by a differential element of length dl. The projection of the action is thus the product of projections of p and of dl. In view to have equivalent components to the starting action, we will split up dA into two components equal in the tangent plane to the sphere on which revolves the electron at the point where it is.

Figure 1. The motion of the electron.

Let G be the plane of gravitation of the electron which contains the axis Px. Let us consider the equatorial plane E which passes by the centre P of the potential and which contains the axes Px and Py (figure 1) and the plane N containing the axes Px and Pz which is normal to it. They are these two planes that we choose to split up the action associated with the rotation. The plane of gravitation cuts these two planes along the axis Px. Let B be one of the two points common to the trajectory and to these three planes. Let us first consider the electron at the time where it passes in B (figure 1). The lines of intersection of the tangent plane out of B with the two planes of projection are the perpendiculars out of B with each one of these planes. For an interval of time and of length {dt, dl}, which bring the electron from B to C, if p is the momentum, the corresponding action is pdl. To split up the action of motion of rotation into two orthogonal actions the corresponding actions must be the product of the projection of momentun p and of the element of length dl. Let α be the angle between the planes G and E, the projection of pdl on E is $dA_E = \cos^2\alpha pdl$ and the projection of pdl on N is $dA_N = \sin^2\alpha pdl$. The sum of these projections gives the total action. For the action of rotation that of the states " 1s " we saw that these actions are equal, which implies that $\alpha = 45°$.

Now let us suppose that the electron moved from B to B_1, and let us still consider an interval of time and length {dt, dl} which brings B_1 in C_1 (figure 2). Let D_E be the straight line of intersection of the tangent plane in B_1 with the equatorial plane and D_G the straight line of intersection of the plane of gravitation with the tangent plane in B_1. Let us call also D_N the straight line of intersection of the planes G and N. In the tangent plane in B_1 we can still

consider the decomposition in dA_1 and dA_2 to 45° here and there of D_G. We have:

$$dA_1 = dA_2 = \_pdl \tag{2.1}$$

Figure 2. Study of the projection of the action.

Now let us suppose that the electron moved from B to B_1, and let us still consider an interval of time and length {dt, dl} which brings B_1 in C_1 (figure 2). Let D_E be the straight line of intersection of the tangent plane in B_1 with the equatorial plane and D_G the straight line of intersection of the plane of gravitation with the tangent plane in B_1. Let us call also D_N the straight line of intersection of the planes G and N. In the tangent plane in B_1 we can still consider the decomposition in dA_1 and dA_2 to 45° apart here and there of D_G. We have:

$$dA_1 = dA_2 = \_pdl \tag{2.1}$$

These components of action are not parallel with the planes E and N. To determine the projections from these points it is enough to project the two components dA_1 and dA_2 on D_E and D_N. Let us consider the projections $P_E(dA_1)$ of dA_1 and $P_E(dA_2)$ of dA_2 upon D_E. Let β be the angle between the line carrying the component dA_1 and the straight line D_E. We have:

$$P_E(dA_1) = dA_1\cos^2\beta \quad \text{et} \quad P_E(dA_2) = dA_2\sin^2\beta \tag{2.2}$$

Taking into account (2.1) we have :

$$dA_E = P_E(dA_1) + P_E(dA_2) = \_pdl \tag{2.3}$$

Let us consider now D_N the straight line of intersection of the planes G and N. The same reasoning shows that we also have:

$$dA_N = P_N(dA_1) + P_N(dA_2) = \_pdl \tag{2.4}$$

2.3 The half-integer angular momentum

Let us then consider a homogeneous magnetic field H. It is generated by a solenoid. As a result the modifications of density of grains which it involves are described with only one angular variable that of orthogonal planes to the field H. Consequently the field H modifies the action only in this direction of

planes. These modifications act thus only on the projection of the two components of action of the motion of rotation on the direction of planes perpendicular with H. Let S_E and S_N be these components all of them equal to $\_pdl$ for the element of length dl and let δ be the angle between the planes perpendicular with H and the plane E. The projection of components of the action of rotation are $S_E\cos^2\delta$ and $S_N\sin^2\delta$. We have :

$$S_E\cos^2\delta + S_N\sin^2\delta = \_pdl \tag{2.5}$$

Let us consider the states " 1s ". These states correspond to just one quantum *h*. For these states as for the others there is rotation. The action of rotation is thus *h*. For the states " 1s " the trajectory is a circle. Let r be its radius, over one period the cover length is $2\pi r$. Thus the corresponding action $\_pdl$ for the segment dl is :

$$\_\,\mathrm{pdl} = \_\,\mathrm{h}\frac{dl}{2\pi r} \tag{2.6}$$

As a result for the observable angular momentum upon an element of length *dl* we have :

$$\_\,rpdl = \mathrm{M}dl = \_\hbar dl \tag{2.7}$$

Thus we have :

$$\mathrm{M} = \_\hbar \tag{2.8}$$

Thus has for the states " 1s " whatever be the orientation of the magnetic field H one can observed only the magnetic moment which corresponds to the kinetic moment $\_\hbar$.

It is interesting to notice that the relations (2.6), (2.7) and (2.8) show us that the conservation of the angular momentum results from the quantification of the action.

If now in addition to the quantum of rotation, the considered quantum state have several additional quanta of action associated with an additional momentum, these quanta distribute between the two components of rotation and those associated to the variations of mass as the study of doublet will show it.

Thus the remarkable property of the different quantum states to exhibit, through the associated magnetic moment, an observable angular momentum half-integer, comes of that the rotation have necessarily two equivalent orthogonal components of action while the magnetic field of have only one.

Let us underlined that this property is that of the orbital rotation since the study which just have been done concern only the movement of centre of gravity. It is possible that the orbital rotation is related to that of the own rotation but it is an open question out of frame of this work. Finally it is interesting to stress that in all quantum state one must distinguish the quantum of rotation which has different properties from other quanta.

3 THE MODEL OF SOMMERFELD

In the study of Sommerfeld which takes into account the relativistic variations of the mass, the electron is assimilated to a point. There are thus two independent degrees of freedom. Let us consider the trajectory of the electron and let be, in the plane of the trajectory, r and Ψ the radial and angular coordinates with the centre P of the potential at the origin. Let us then consider an elliptic trajectory for example a state 2s. The momentum which are supposed to be quantified, are p_Ψ and p_r. The calculation of the energy of a level starts from the definition of the kinetic energy T and the speed v of the electron in relativity. If m_0 is the rest mass of the electron and c the speed of the light, we have :

$$T = m_0 c^2 \left[\frac{1}{\sqrt{1-\beta^2}} - 1 \right] \tag{3.1}$$

The speed v of the electron at the considered instant is contained in the expression of β:

$$\beta = \frac{v}{c} = \frac{1}{c}\sqrt{\dot{r}^2 + r^2\dot{\psi}^2} \tag{3.2}$$

In this expression $\dot{r}$ and $\dot{\psi}$ represent the derivative with respect to the time of the variables r and Ψ. If e^2/r is the potential energy, a quantum state is defined by its total energy W which is a constant of the motion. We have :

$$W = \frac{m_0 c^2}{\sqrt{1-\beta^2}} - \frac{e^2}{r} \tag{3.3}$$

The search of the stable quantum states consists in determining the various possible values of W. To carry out this quantification Sommerfeld uses the relativistic Lagrange function:

$$L = m_0 c^2 \sqrt{1-\beta^2} \quad -U \tag{3.4}$$

where U is the potential energy, that is to say for the electron compared to the proton $U = e^2/r$. The moments of Lagrange of the variable r and Ψ are :

$$p_r = \frac{\partial L}{\partial \dot{r}} \quad \text{and} \quad p_\psi = \frac{\partial L}{\partial \dot{\psi}} \tag{3.5}$$

While supposing that the mass which determines these two momentums is the mass m_0 of the electron at rest, it comes

$$p_r = \frac{m_0 c^2}{2\sqrt{1-\beta^2}} \frac{\partial \beta^2}{\partial \dot{r}} = \frac{m_0 \dot{r}}{\sqrt{1-\beta^2}} \tag{3.6}$$

$$p_\psi = \frac{m_0 c^2}{2\sqrt{1-\beta^2}} \frac{\partial \beta^2}{\partial \dot{\psi}} = \frac{m_0 r^2 \dot{\psi}}{\sqrt{1-\beta^2}} \tag{3.7}$$

The quantum conditions are:

$$\int \frac{m_0 r^2 \dot{\psi}}{\sqrt{1-\beta^2}} d\psi = kh \tag{3.8}$$

$$\oint \frac{m_0 \dot{r}}{\sqrt{1-\beta^2}} dr = ph \tag{3.9}$$

The number k and p are by assumption positive integer numbers. In Sommerfeld approach the number p can be null but not k.

These two equations define, during the period of the motion, the action associated with the considered degree of freedom. The equation (3.9) because one integrate on a period the product of linear momentum p_r by the element of length dr. The equation (3.8) because one integrate on a period the product $r\dot{\psi}$ of speed associated to the variable Ψ, by the corresponding mass $m_0\left(1-\beta^2\right)^{1/2}$ and by the element of length $rd\Psi$ associate with Ψ. Consequently the equation (3.8) definite an action on all the period of motion. This

point is significant because we know that the observable angular momentum is not equal to the action which generates it. As a result it is preferable of regarding p_Ψ as action per unit of angle that we will call the angular action, more than like the angular momentum. Let us take notice that the angular action always include the quantum of rotation that Sommerfeld theory's does not allows to apprehend.

Let us consider the energy levels W. They depend of the quantum numbers k and p. Let us introduce the quantum principal number $n = k + p$. Let E be the energy defined by the relation:

$$E = \mathrm{W} - m_0\mathrm{c}^2 = \mathrm{T} + m_0\mathrm{c}^2 - \frac{e^2}{r} \tag{3.10}$$

For a couple of numbers n and k the calculation of the integrals (3.8) and (3.9) allows that of energy $E_{n,k}$ of a level which is given by the relation:

$$\frac{E_{n,k}}{m_0c^2} = \left[1 + \frac{\alpha^2}{\left[p + \sqrt{k^2 - \alpha^2}\right]^2}\right]^{-\frac{1}{2}} - 1 \tag{3.11}$$

α is the fine structure constant $\alpha = \mathrm{e}^2/\hbar\mathrm{c}$, k characterises the number of quanta of actions associated with p_Ψ by the relation (3.8). As k occurs by its square it can take any positive or negative integer values but not zero values. However if it is supposed that k represents the angular momentum only the positive values are to be considered. It is this interpretation which prevailed at the time of Sommerfeld. The comparison with the model of Dirac where the positive and negative values are retained leads to suppose that another choice is possible.

Then let us consider the layer L which corresponds to $n = 2$. The experiment shows that this layer contains three levels : $2s_{1/2}$, $2p_{3/2}$ and $2p_{1/2}$. On the other hand the expression (3.11) and the quantification of p_Ψ and p_r put in view only two levels $E_{n,k}$ for the layer L. A level with $k = 2$ and $p = 0$, whose motion is circular, it corresponds to the level $2p_{3/2}$; the other with $k = 1$ and $p = 1$ whose motion is elliptic with a radial component of the linear momentum, it corresponds to the level $2s_{1/2}$. It misses in this approach the level $2p_{1/2}$. It is the fundamental difficulty on which the approach of Sommerfeld stumbled and which seems to indicate that only the method of Dirac which finds the three levels is correct.

3.1 Speed and mass

In the approach of Sommerfeld only the variations of the mass with the radial speed of the centre of gravity γ of the electron are considered. In the fluid model of electron where the grains are the support of the energy of motion and mass, when radial speed decreases or increases, the mass, that is the numbers of grains, increases or decreases. On the other hand for a constant angular speed, without radial speed, the exchanges of matter conserve the mass. They are these two cases that are determined by the quantum conditions (3.8) and (3.9). With the model of grains we can also consider variations of the mass associated with variations the angular speed of the centre of gravity of the electron while preserving the radial momentum. This property is already present with the variations of mass associated with the variations radial speed. Indeed these variations preserve p_Ψ and the two components of the angular momentum associated to the rotation. This property can also concerns p_r if the mass varies independently of $\dot{r}$.

Let then h be the quantum of action associated with rotation. This unit of action always contributes to the value of the quantum number k defined by the relation (3.8). The exchanges of matter associated with rotation can take place with constant mass, but there are other possibilities. For the states with constant mass, the corresponding units of action contribute to the quantum number k which characterises the angular action independent of the action associated with variations of mass. If the rotation takes place with variable mass in addition to the unit of action which generates the rotation it can have one or more units of action associated with variations of mass. These variations of mass generate variations of speed: either radial, or angular. The number of these units of action belongs to the quantum number p defined by the relation (3.9).

Let us consider then, for a same value of the principal quantum number n, a state with a unit of action generating variations of the angular velocity and another state where this unit of action is transferred on the quantum number k. For these two states the number of quanta associated at the radial speed remains the same one. Their trajectories are thus very close, their energies differ only by relativistic corrections. They correspond to two types of solutions which give rise to the regular doublets. We will discuss further the possibility of several units of action generating the variations of the angular velocity. Thus the two types of solutions, associated with the two lines of a doublet, are the reflection of : *" The capacity of the electron, for a same an-*

gular momentum, to have a constant or variable mass independently of the variations of the mass corresponding to those of the radial speed ". On the other hand, nothing indicates that they correspond to two orientations of the own rotation like the assumption of this rotation introduced by Uhlenbeck and Goudsmit [11,12] can let it believe, it is a question which remains open. They correspond in fact to two energy levels which differ one from the other by relativistic corrections and which preexist to the application of a magnetic field what, with George Lochak, we have already underlined [13].

3.2 The degrees of freedom and the calculation of the energy of a level

Thus a variation of mass can take place on a component of the linear momentum without modified the other. This property implies that the different components of the linear momentum associated with these variations of mass are independent degrees of freedom. For radial speed this property while bringing a particular enlightening, does not give access to new properties, for the variations of the radial speed are always connected to variations of the mass. On the other hand, the independence of the variations of mass compared to the quantification of the angular momentum associated with the angular speed is a property which has not yet been recognised up to now for the perpendicular speed to the radial speed. It results from it indeed that there are two degrees of freedom which quantify rotation and this leads to consider again the calculation of the energy.

Since there are two degrees of freedom which quantifies the rotation we can still to call p_ψ the angular momentum defines by the relation (3.7). It is independent of the variations of mass. Let then $\dot{\omega}$ be the linear component of speed, orthogonal with the radius vector *r* associated with the variations of mass related to the orbital motion of rotation and let p_ω be the linear momentum associated with these variations of mass. For a circular trajectory this component is tangential with the circle but distinct from p_ψ. The momentum p_ω affects the angular velocity, it is the reason for which we use the Greek letter omega, but it is not an angular momentum since it is generated by variations of mass independent of the angular velocity. We have:

$$p_\omega = \frac{\partial \mathrm{L}}{\partial \dot{\omega}} \tag{3.12}$$

it comes : $$p_\omega = \frac{m_0 c^2}{2\sqrt{1-\beta^2}} \frac{\partial \beta^2}{\partial \dot{\omega}} = \frac{m_0 \dot{\omega}}{\sqrt{1-\beta^2}} \tag{3.13}$$

Le us write : $$\oint P_\omega dl = ah \tag{3.14}$$

The variations of mass associated with p_ω involve variations of the angular speed $\dot{\omega}$ just like the variations of mass associated with the radial speed, but they are independent from the radial speed. It results from them that for a same total energy, the quantum conditions which determine the elliptic trajectories remain valid for the variations angular speed $\dot{\omega}$, as the experiment shows it for the states $2s_{1/2}$ and $2p_{1/2}$ their energy levels being practically equal [14].

To see it in the field of calculation let us pose:

$$\oint P_r d\ell = rh \tag{3.15}$$

let us equally write : $$p_m = p_r + p_\omega \tag{3.16}$$

It comes : $$\oint p_m d\ell = \oint (p_r + p_\varpi) dl = (r + a)h \tag{3.17}$$

let us write : $$p = a + r \tag{3.18}$$

Let us show that the quantum number p defined by the relation (3.18) is that of the relation (3.9) which determines the value of the energy of the level $E_{n,k}$.

The momentum p_r and p_ω are orthogonal, one thus has :

$$p_m^2 = p_r^2 + p_\omega^2 \tag{3.19}$$

The independence of p_ω and p_Ψ makes that one has:

$$p_\Psi^2 + p_\omega^2 + p_r^2 = p_\Psi^2 + p_m^2 \tag{3.20}$$

As in the calculation of E, the momentum occur by their squares, the leading of calculation remains the same if one replaces p_r defined by the relation (3.5) by p_m defined by the relation (3.16). The calculation is due to Sommerfeld [15] and it can be find with de Broglie [3].

In the model of Sommerfeld the calculation of the energy of a level starts from the definition of the kinetic energy T given by the relation (3.1) and the speed of the electron in relativity, relation (3.2). Taking into account the variations of mass associated with p_ω it is necessary to replace the expression of β relation (3.2) by :

$$\beta = \frac{\mathrm{v}}{\mathrm{c}} = \frac{1}{\mathrm{c}}\sqrt{\dot{r}^2 + \dot{\omega}^2 + r^2\dot{\psi}^2} \tag{3.21}$$

The work of Sommerfeld evidently does not refer to existence of speed $\dot{\omega}$ which characterises the speed associated with variations of mass in the direction perpendicular to the radius vector r. But as we will see the calculation is not modify by this additional degree of freedom.

The calculation of $E_{n,k}$ consists in calculating p_m starting from W given by the relation (3.3). By integrating p_m over one period, the quantum condition (3.17) with $a + r = p$ leads then to the expression of $E_{n,k}$. Let us consider then the momentum p_r, p_ω and p_Ψ. They are defined by the relations (3.5) and (3.12). Their expressions given by the relations (3.6), (3.7) and (3.13) allow to calculate p_m starting from the following sum :

$$p_\mathrm{r}{}^2 = \frac{m_0^2\dot{r}^2}{1-\beta^2}\,;\quad p_\omega{}^2 = \frac{m_0^2\dot{\omega}^2}{1-\beta^2}\,;\quad \frac{p_\psi^2}{r^2} = \frac{m_0^2 r^2\dot{\psi}^2}{1-\beta^2} \tag{3.22}$$

it comes :

$$p_\mathrm{r}{}^2 + p_\omega{}^2 + \frac{p_\psi^2}{r^2} = m_0^2\,\frac{\dot{r}^2 + \dot{\omega}^2 + r^2\dot{\psi}^2}{1-\beta^2} = \frac{m_0^2\mathrm{c}^2\beta^2}{1-\beta^2} \tag{3.23}$$

thus :

$$m_0^2\mathrm{c}^2 + p_\mathrm{r}{}^2 + p_\omega{}^2 + \frac{p_\psi^2}{r^2} = m_0^2\mathrm{c}^2\left[1 + \frac{\beta^2}{1-\beta^2}\right] = \frac{m_0^2\mathrm{c}^2}{1-\beta^2} \tag{3.24}$$

The expression of W given by (3.3) is transforms using (3.24) in :

$$\mathrm{W} = \mathrm{c}\sqrt{m_0^2\mathrm{c}^2 + p_r^2 + p_\omega^2 + \frac{p_\psi^2}{r^2}} - \frac{\mathrm{e}^2}{r} \tag{3.25}$$

A quantum state will be stable if there are constant values of W. As the energy of the rest mass is a constant, it is convenient to rewrite the relation (3.10) in the following way:

$$\mathrm{W} = \mathrm{E} + m_0 \mathrm{c}^2 \tag{3.26}$$

As we have seen p_Ψ is a constant of the motion different from the observable angular momentum. The relation (3.8) then makes it possible to write:

$$p_\Psi = k\hbar \tag{3.27}$$

Using the relations (3.3), (3.25) and (3.26) we can write:

$$p_\mathrm{m} = \pm\sqrt{A + \frac{2B}{r} + \frac{2C}{r}} \tag{3.28}$$

with the following notations :

$$A = \frac{E^2}{\mathrm{c}^2} 2m_0 \; = + \; m_0 \mathrm{c}^2 \left[\left(1 + \frac{E}{m_0 \mathrm{c}^2} \right)^2 - 1 \right] \tag{3.29}$$

$$B = \frac{E\mathrm{e}^2}{\mathrm{c}^2} + m_0 \mathrm{e}^2 + \; m_0 \, \mathrm{e}^2 \tag{3.30}$$

$$C = \; \frac{\mathrm{e}^4}{\mathrm{c}^2} - p_\psi^2 = \frac{\mathrm{e}^4}{\mathrm{c}^2} - k^2 \hbar^2 \tag{3.31}$$

By introducing the fine structure constant $\mathrm{e}^2/\hbar\mathrm{c}$ one obtains :

$$C = \; k^2 \hbar^2 \left[1 - \frac{\alpha^2}{k^2} \right] \tag{3.32}$$

By integrating p_m over one period by the method of the residues one obtains :

$$\oint \sqrt{A + \frac{2B}{r} + \frac{2C}{r^2}} dr \; = 2i \left[\sqrt{C} - \frac{B}{\sqrt{A}} \right] = ph \tag{3.33}$$

The equality with *ph* comes from the quantification by taking account of the relations (3.9), (3.17) and (3.18). By replacing A, B and C by their values one obtains after a simple calculation the expression (3.11) in which $p = a + r$.

4 THE MODELS OF SOMMERFELD AND DIRAC

Any quantum state associated with p_ω can seem to have the same characteristics as those associated with p_r. There is however a significant difference: let us suppose that there is a quantum of action associated with p_ω. It characterises the variations of the angular speed, consequently it contributes to the angular momentum for $\hbar$. On the other hand each quantum of action associated with p_r leaves it unchanged. In addition, the quantum number p relates to the radial motion as well as the motion which results from p_ω. It is necessary thus to distinguish the quantum numbers associated with these two types of speeds. It is this possibility for the rotation of having a constant or variable angular speed which is at the origin of the doublets.

In theory of Dirac it can seem that the sharing of the quantum number p does not exist. That comes from the fact that one put the accent on the two possible values of k, that is $k = \ell$ and $k = -(\ell + 1)$. These two values allow to find all the known quantum states, as a result there is no reason at first sight to consider that p can be the sum of two distinct numbers, each one being characteristic of variations of mass in two orthogonal directions. However, when the number k changes from $k = -(\ell+1)$ to $k = \ell$ the number p passes from $p = n - (\ell+1)$ to $p = n - \ell$. The number p thus varies of one unit, the total angular momentum remaining the same one. Indeed, one has $M = (\ell + \_)h$ [6 and 7], (let us recall that the numbers $j = \ell + \_$ and $j = \ell - \_$ characterises the maximum value in Bohr magneton of the observable and observed magnetic moment, according to the negative or positive values of k, the quantum number which in the model of Dirac takes the values $k = -(\ell+1)$ or $k = \ell$, [13, 6, and 7]). There is thus well a unit of quantification corresponding to the quantum number a in the model of Dirac. Thus for a same value of the principal quantum number n, the values $a = 0$ and $a = 1$ make it possible to find all the known quantum states. In the model of Dirac this point is known except that it is associated with both values of k without the introduction of the number a. This being, in the model of Sommerfeld or Dirac as well, nothing prohibits to consider values of a higher than one.

Let us now study if the integer values of a higher than one are possible. To this end it is useful to recall which are the possible values of quantum number r. When r varies, the eccentricity of the trajectory varies. In the hydrogenoid atoms we know that the energy levels $E_{n,\ell}$ for a same value of "n" but for different values of ℓ, are different. This property is interpreted by screening constant which affects the central potential, they are characteristic of the eccentricity of the trajectory. In addition, the states corresponding to various values of a with $r = 0$ are circular. Consequently, these interpretations show that the quantum number r takes well all the integer values positive or null obeying to the relation $r \leq n - 1$. As a results with the two values $a = 0$ and $a = 1$ all the known quantum states are interpreted. Consequently, the values $a > 1$ are either forbidden, or unstable. If such states exist they have characteristic energies which merge with those of the already known states. With such an eventuality, these states in electronic transitions, all being unstable, they must modify the statistical weights which one knows that they do not always correspond to the expected values.

5 THE DIFFERENT QUNTUM STATES

By studying rotation we saw that there is a quantum of action associated with rotation. As soon as the rotation exists with this quantum distributed on two orthogonal components, the other quanta can be distributed: either on the action associated with the variations of mass, or on the angular action by integers on each component of rotation. The existence of quantum of rotation allow the count of the various quantum states, to understand their magnetic contribution and to bring them closer of the count of model of Dirac where the two types of states are characterised by $k = -\ell - 1$ and $k = \ell$.

Let ℓ be the number of the quanta of action which are added to that of rotation and r that of the quanta of action associated with the variations of the radial speed. Let us consider the rotation either at constant mass with $k_1 = \ell + 1$ et $p_1 = r$, or at variable mass with $k_2 = \ell$ and $p_2 = r + 1$. In both cases the unit of quantification added to ℓ or to r contributes to the angular momentum. In addition as the rotation contributes for _$\hbar$, the total angular momentum M associated with ℓ is :

$$M = (\ell + \_)\hbar \tag{5.1}$$

As the number k is involved by his square in the expression of energy (3.11) it is also possible to consider negative values of k as Dirac does. To respect this use that we used in the study of magnetism [13] we characterise the two types of solutions with $k = -(\ell+1)$ and $k = \ell$. We have then for the angular momentum:

$$M = |k + \_|\hbar \tag{5.2}$$

Type I : *Rotation with constant mass*. The action of rotation ℓh can be distributed, by integer number of quanta h, on the two components of the rotation. Let m be the number of quanta associated with the equatorial plane. The possible values of m are $0 \leq m \leq \ell$. Let us add to m the contribution $\_$ of the own rotation. This leads to introduce the number $\eta = m + \_$. In addition any motion can as well take place in one direction as in the other. Consequently the possible values of η are :

$$\eta = \pm(m + \_) \tag{5.3}$$

With this definition of η the number m takes zero value twice. It is possible to avoid these two zero values by choosing for m all the values of the following interval :

$$-\ell \leq m \leq \ell + 1 \tag{5.4}$$

and by replacing the number η by the number u defined by the following relation:

$$u = -(m - \_) \tag{5.5}$$

This number is the magnetic quantum number of the theory of the Dirac [3] which gives the value of the projection of the total angular momentum on the perpendicular to the equatorial plane which we use in the calculation of the magnetic moment [13]. There are $2\ell + 2$ different quantum states for this subshell.

Type II : *Rotation with variable mass*. In this case there is one unit of action used for the variations speed and mass. These variations affect the angular motion and contribute consequently to the observable angular momentum but decrease by one unit the absolute value of the boundaries of variation of m. One has :

$$-(\ell - 1) \leq m \leq \ell \tag{5.6}$$

The value of u is still given by the relation (5.5) but the number of possible values is reduced by two units. There are consequently 2ℓ different quantum states for this subshell. That is for the whole shell $4\ell + 2$ different quantum states.

6 THE ACTION AND THE G FACTOR

It is interesting, in this study on the doublet, to recall that the g factor or the Landè factor which is introduced in the calculation of magnetic moment of a quantum state is a consequence of that the action of rotation is *h* and not _*h* as the half integer magnetic momentum can lead to believe it [6,7]. Let us consider an electron on a classical electronic orbit of which the angular momentum M has for projection $u\hbar$ in the direction of the magnetic fields H. It has a magnetic moment given by the relation :

$$\mu_e = u\mu_B \qquad \text{with} \qquad \mu_B = \hbar \frac{e}{2mc} \tag{6.1}$$

To establish this relation one has to use the classical expression giving the moment μ_e of a current in a single loop or circuit:

$$\mu_e = I\, S/c \tag{6.2}$$

where I is the current circulating in the loop (in electrostatic units), S is the area of the loop and c the velocity of the light. In terms of moving charge e and its period of revolution T, the current I is given by: I = e/T. On the other hand the area of a Kepler ellipse, valid in classical mechanics as well in special relativity, expressed on one period T_M with the mass *m* of the electron and its mechanical moment M, is given by the relation $2mS = T_M M$. For $M = \hbar$ we find the relation (6.1) but just when $T = T_M$. But the total angular momentum $M = |k + \_|\hbar$ does not correspond to the angular action $|k|\hbar$. As a result T and T_M are different. The period T_M is indeed fictitious and we have the relation:

$$2mS = T|k|\,\hbar = T_M|k + \_|\,\hbar \tag{6.3}$$

That is : $$T_M = g\,T \qquad \text{with} \qquad g = k/(k + \_) \tag{6.4}$$

For projection u the period T_M and T are the same one as for the total angular momentum M = |*k* + _|ħ. It results from it that the magnetic moment corresponding to projection *u* of the total angular moment M is given by the relation:

$$\mu_e = g u \mu_B \tag{6.5}$$

In particular for the states "s" $k = -1$ and g = 2. For these states the magnetic moment is the double of that obtained by the classical approach, but they are the only states.

7 THE ORIENTATION OF THE PLANES OF GRAVITATION

It is useful in the study of the crystal to know the orientation of the plane of gravitation G of the trajectory of a quantum state compared to the equatorial plane E. Let α be this angle. The radial action does not modify the orientation of the basic circular trajectory. It is thus enough to determine α for such states. Let us consider the *dl* displacement which brings the electron from B to C (figure 1). The displacement corresponding to the increase $d\varphi$ is $rd\varphi$. The component of the action parallel to the plane E is the product of the projections of the displacement and the projection of the momentum. One thus has:

$$\cos^2\alpha = \frac{p_\varphi r d\varphi}{pdl} \tag{7.1}$$

We saw that the projection of the angular action on the equatorial level E is *uh* while the total angular action is *kh*. It comes :

$$\cos^2\alpha = \left|\frac{u}{k}\right| = \left|\frac{m - 1/2}{k}\right| \tag{7.2}$$

The trajectory of a state *m* crosses the equatorial plane in an unspecified point of the circle homologous with that of radius PB of figure 1. When there are several states occupied on a same subshell the trajectories of the various states deviate the one from the others in such a way to minimise the interactions of repulsion between electrons. On the other hand, if the interactions electron-electron are neglected, the angle α keeps a constant value. In particular there is a minimum value. The exploitation of these properties

makes it possible to deduce certain properties characteristic of the considered atom. In particular the directions of space with a strong or small screening constant resulting of the electrons present on the atom. In other words the knowledge of these directions gives access to dissymmetries of the atom, which generate those of the crystal. Thus we proposed an interpretation of the preferential occupation of the octahedral site of spinels by chromium in $MnCr_2O_4$ [16].

8 CONCLUSION

In this study of the quantification, the motion is supposed the result of exchanges of matter between the electron and the proton. These exchanges take place in all the directions of space. From this fact the orbital rotation must be regarded as the result of a mechanical action which acts in two orthogonal directions. In other words the rotation can be regarded as the sum of two orthogonal rotations. As a results the magnetic field which corresponds to an action on only one direction of planes can modify only half of the action associated with the rotation. In addition the doublet have in fact for origin the possibility for the electron to be able to turn for same angular quantification with a constant or variable mass independently of radial speed and nothing indicate that the doublets correspond to state of spin-up and spin-down [13]. These results lead in examining in which extend the hypothesis of own rotation is confirmed.

The hypothesis of the spin was proposed in 1925 [11] to interpret the doublets before the discovery of the equations of wave of quantum mechanics 1926 [10] for that of Schrödinger and 1928 [17, 18] for that of Dirac. However the spin i.e. the own rotation of the electron are a property of volume and the study of the properties of a point in classical mechanics or special relativity could not reveal the properties of a volume. Moreover the introduction in quantum mechanics of the differential operators presumably giving again, while acting on the wave function, the properties of the centre of gravity of the electron, cannot more reveal the own rotation.

For lack of a clear comprehension of the quantum phenomena this remark into general is ignored. In fact the theory of Dirac have from the very start been impregnated of the model of Pauli where the wave function have two components : one being by hypothesis presumably attached to the one of the type of spin the other component being attached to the other type of spin. In theory of Dirac one can consider a similar step but the study of solutions show that it is the whole of four components of wave function which charac-

terises one quantum state. Thus the concept of spin escapes in fact from the theory of Dirac just like from the theory of Sommerfeld.

To discuss more completely the assumption of the spin, there remains the existence of the half integers angular momentum which belong according to this study to orbital rotation as well to the solutions of the equation of Dirac. It is with them that we calculated every magnetic moment by multiplying them by the factor $g=k(k+\_)^{-1}$ characteristic of the subshell. They contributed, before the equations of waves, to the hypothesis of the spin. These half integer numbers belonging to the model of Dirac, they consequently consolidated the assumption of the spin. However they rise not, through the studies propose to date, of a property of the own rotation, but of studies describing the properties of a point. As a result, we have not the right to present them as a proof of the existence of the spin. Thus it is possible that the orbital rotation of the electron is related to that of its own rotation but it is a question which remains open.

On the other hand this study shows that the action associated with the own rotation is the Planck's constant *h* and that it is this action which determines all the quantum numbers. This state of things clarifies the comprehension of the periodic table. Indeed the constant of action *h* does not depend on the medium, i.e. of the complexity of the atom, the molecule or of the solid, liquid or gas state of the matter in which moves the electron. Then we understand that the quantum numbers which characterise the quantum state remain whatever the situation of the atom. It is that which allows to understand the structure of the periodic table which synthesises so many chemical and physical properties of the matter. We also understand that the total angular momentum, reflection of the action associated with rotation, subsists whatever the complexity of the atom to which belongs the electron and the medium in which is the atom. It is this assumption which enabled us with George Lochak to propose a calculation of the magnetic moments of the atoms and whose coherence with the experiment remained mysterious [13, 19]. In the field of the prospects the understanding of the crystal state is enlighten by this approach. We already showed it [16] and hope to be able to again show it in the near future.

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