# Josephson junctions as E.P.R. pairs?

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ABSTRACT. We apply the principle of indistinguishability to macroscopic quantum objects in the form of superconducting bodies. A quantum object being defined as that which is Lorentz and CPT invariant. In case of two close but separate superconducting bodies, we thereby obtain the Josephson equation $j\tilde{}\ \sin\left(\chi_2 - \chi_1 - \frac{2e}{\hbar}\int_1^2 A_1 da^i\right)$ from a different perspective. Generalising to once connected junctions we find the localisation in time of macroscopic SC phenomena is lost. The formalism is discussed.

## 1. Introduction

It is known that matter is time extended and this is formalised in the Lorentz and CPT invariance of the quantum theory. It has been argued [1] that this -time extension- shows up macroscopically in a superconducting body and is the principal cause of the whole macroscopic superconducting (SC) electrodynamics.

## 2. Case of 2 close but separate superconducting bodies

With this in mind, let us examine the case of two separate superconducting bodies which are brought into close proximity, of the order of a coherence length, so as to allow supercurrent to pass from one to the other.

We wish to calculate the amplitude for the propagation of supercurrent from SC body 1 to SC body 2, i.e. the amplitude for the occurrence $J\,(1 \rightarrow 2)$.

We note that the amplitude for this process is dependent on the relative macroscopic phases (phase angle) of each superconducting body.

We also note that time reversal symmetry is present. More precisely, the quantum field of material points is Lorentz and CPT invariant [1].

We cannot define a definite frame and therefore must use a generalised path $\Gamma_{ab}$ for two points $a$ and $b$ within a superconducting body, whereby $\Gamma_{ab}$ is valid whatever the character of the spacetime path, including curved spacetime paths. Usually we can define a definite spacetime path according to the inertial frame, but we must relax this condition within a SC body. A statement may be made about the phase. It must satisfy the equation

$$\chi_{;uv} - \chi_{;vu} = 0 \tag{1}$$

which is valid in any frame [1].

Following the notation of Hoyle and Narlikar [2] but relaxing the specification of an inertial frame, one might think that the amplitude for the occurrence $J\,(1 \rightarrow 2)$ is of the form[1]

$$P\left(\Gamma_{21}\right) = \prod_i P\left(\Gamma^{\pm}_{i,i+1}\right) \tag{2}$$

where $\Gamma^+$ is a generalised path going forwards in time and $\Gamma^-$ is a generalised path going backwards in time.

We must now include the coupling of the electromagnetic and quantum fields. Due to the time symmetric nature of the occurrences in a superconducting body, we must use both advanced and retarded waves in the description of electromagnetic phenomena. To do this, we write the potential as

$$A_i = \begin{cases} \frac{1}{2}A_i^{adv} & t_2 < t_1 \\ \frac{1}{2}A_i^{ret} & t_2 > t_1 \end{cases} \tag{3}$$

where $A_i^{ret}$ influences a forward time process (from initial to final state) and $A_i^{adv}$ influences a backwards in time process (from final to initial state). The amplitudes for the paths $\Gamma^+_{21}$ or $\Gamma^-_{12}$ in the presence of the potential $A_i$ are therefore of the form

$$P^A\left(\Gamma^+_{21}\right) = P\left(\Gamma^+_{21}\right)\exp\left[-i\frac{2e}{\hbar}\int_{\Gamma^+_{21}}\left(\frac{1}{2}\right)A_i^{ret}da^i\right] \tag{4}$$

[1]Here we encompass general properties of a quantum object (forwards and backwards in time occurrences), (product of amplitudes) following the lead by London [3]; that a superconductor may be thought of as a macroscopic quantum object with phase $\chi$ and wavefunction $\psi = |\psi|\exp\{(ie/\hbar c)\,\chi\}$ where $\chi$ is a scalar function of position.

$$P^A\left(\Gamma_{12}^-\right) = P\left(\Gamma_{12}^-\right)\exp\left[-i\frac{2e}{\hbar}\int_{\Gamma_{12}^-}\left(\frac{1}{2}\right)A_i^{adv}da^i\right]$$

If we put $P\left(\Gamma_{21}^+\right)$ equal to $\left(\exp\left[i\left(\chi_2-\chi_1\right)\right]\right)^{1/2}$, the amplitude for a forward time propagation of supercurrent and $P\left(\Gamma_{12}^-\right)$ equal to $\left(\exp\left[-i\left(\chi_1-\chi_2\right)\right]\right)^{1/2}$, the amplitude for a backward time propagation, then the total amplitude for the process $J\left(1\rightarrow 2\right)$ is[2]

$$P^A\left(\Gamma_{21}^+\right)P^A\left(\Gamma_{12}^-\right) \tag{5}$$

or

$$\exp\left[i\left(\chi_2-\chi_1-\frac{2e}{\hbar}\int_{\Gamma_{21}^+,\Gamma_{12}^-}A_i da^i\right)\right] \tag{6}$$

but this is not the whole picture as we shall see shortly.

Consider the standard equation for supercurrent propagation [4,5]:

$$j\left(r\right) = \int\int K\left(r,s,s'\right)\Delta\left(s\right)\Delta^*\left(s'\right)ds ds' \tag{7}$$

where $\Delta\left(s\right)$ and $\Delta^*\left(s'\right)$ may be thought of as wavefunctions and $K\left(r,s,s'\right)$ contains the influence of the electromagnetic field on the supercurrent propagation and is given by;

$$T\sum_{\omega}G_{-\omega}\left(s,s'\right)\times\left[\frac{ie}{m}\left\{\tilde{G}_\omega\left(r,s\right)\nabla_r\tilde{G}_\omega\left(s',r\right)-\tilde{G}_\omega\left(s',r\right)\nabla_r\tilde{G}_\omega\left(r,s\right)\right\}\right.$$

$$\left.-\frac{2e^2}{mc}A\left(r\right)\tilde{G}_\omega\left(r,s\right)\tilde{G}_\omega\left(s',r\right)\right] \tag{8}$$

where $\tilde{G}$ and $G$ are statistical Green's functions for the superconducting and normal states respectively.$T$ is the temperature. Equation (7) is valid within superconducting bodies (case 1) and across two close but separate superconducting bodies (case 2). For the latter case, it reduces to $j(r) = iK\left(\Delta_1\Delta_2^*-\Delta_2\Delta_1^*\right)$ , $K$ real with $\Delta_1$and $\Delta_2$ taken to be constant either side of the barrier. Putting $\chi_2-\chi_1 = \arg\Delta_2-\arg\Delta_1$ and $j_1=\frac{1}{2}K\left|\Delta_1\right|\left|\Delta_2\right|$, $j=j_1\sin\left(\chi_2-\chi_1\right)$ is obtained [5].

[2]We are not concerned here with an ab initio calculation of the form for the amplitude. The choice is such that it is consistent with the standard result.

Josephson has given the Feynman diagrams for the propagation of supercurrent (Cooper pairs) between two close but separate superconducting bodies, shown below.

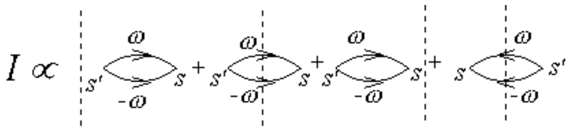


**Figure 1.**

The lines represent single particle Greens functions. A dot at a vertex represents a factor $\Delta^*$ or $\Delta$ according to whether two directed lines emerge from or go to the vertex. The sum of the frequencies labelling the vertex must be zero. The dashed line represents the barrier. In addition to the 4 principal Feynman diagrams shown above there are other corresponding to the propagation or more than one Cooper pair. There is, for instance, a set of 4 diagrams corresponding to the propagation of 2 Cooper pairs (4 electrons) across, and on each side of, the divide. The supercurrent propagation across the divide is proportional to the sum of these diagrams [5].

We note that the supercurrent propagation $J\,(1 \rightarrow 2)$ is in general proportional to processes occurring both across the barrier and on either side of the barrier. This point is important and we will return to it later.

There is a correspondence between equ. (5) and the standard equation for supercurrent propagation (7):

(i)both relations take into account forward and backward in time occurrences

(ii)both relations contain the coupling of the electromagnetic and quantum fields

(iii)in both cases only the complex part is taken to represent supercurrent propagation.

As yet, we have not included the basic concept of indistinguishability of identical particles in quantum mechanics. We may assume that each Cooper pair is indistinguishable from any other, further that each volume of SC charge carriers is indistinguishable from any other, further that we must interchange each and all Cooper pairs, labelled with phase angle $\chi_1$ on one side of the SC junction with each and all Cooper pairs, labelled with phase angle $\chi_2$ on the other side. That is, we are effectively performing a macroscopic interchange. We do this to facilitate the principle of indistinguishability [6].

The amplitude for the process $J\,(1 \rightarrow 2)$ is therefore of the form

$$P^A\left(\Gamma_{21}^{+}\right) P^A\left(\Gamma_{12}^{-}\right) + P^A\left(\Gamma_{11}^{+}\right) P^A\left(\Gamma_{22}^{-}\right) \tag{9}$$

with the path $\Gamma_{11}^{+}$ arriving back onto itself as the close but separate SC bodies are interchanged during the propagation process. It is formally similar to the indistinguishability principle as it is applied to an electron and a positron (time reversed electron) except that we subtract amplitudes for Fermion objects. In Feynman propagator notation, the amplitude for the electron-positron process is given by $K_{+}^{(A)}\,(3,1)\,K_{+}^{(A)}\,(4,2) - K_{+}^{(A)}\,(4,1)\,K_{+}^{(A)}\,(3,2)$, where the + index indicates the -Feynman propagator- which takes account of quantum objects travelling backwards as well as forwards in time. This is achieved by having[3]

$$\begin{aligned} K_{+}\,(2,1) &= \sum_{POS.\ E_n} \phi_n\,(2)\,\widetilde{\phi}_n\,(1) \times \exp\left(-iE_n\,(t_2 - t_1)\right) \qquad \text{for} \qquad t_2 > t_1 \\ &= -\sum_{NEG.\ E_n} \phi_n\,(2)\,\widetilde{\phi}_n\,(1) \times \exp\left(-iE_n\,(t_2 - t_1)\right) \qquad \text{for} \qquad t_2 < t_1 \end{aligned} \tag{10}$$

the sum being taken over positive energy states $(t_2 > t_1)$ or negative energy states $(t_2 < t_1)$ so that between times $t_2$ and $t_1$ the time direction for the occurrence is not assumed to be necessarily positive [7]. These processes are compared below.

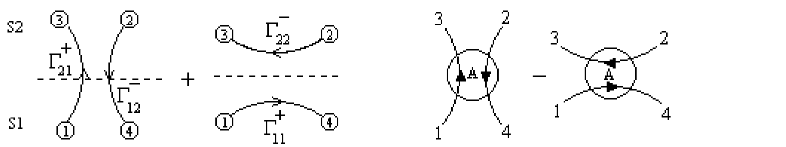


Diagram for the propagation of supercurrent from superconducting body 1 (SC1) to superconducting body 2 (SC2) in the presence of time reversal symmetry. Alternatively, interchange of SC1 and SC2 for the occurrence J(1→2) after the superconducting bodies are brought into close proximity.

Feynman diagram for an electron- positron pair found at 1,4 is found at 3,2. Or interchange of an electron, -- time reversed electron, after interaction at one time in a region A.

**Figure 2.**

[3] $\phi_n\,(1)$ and $\phi_n\,(2)$ are energy eigenfunctions at points 1 and 2.

We should note, though, that in the above we have actually applied the Indistinguishability principle to macroscopic quantum objects, possessing a phase field $\chi$ which satisfies equation (1). From figure 2 we see that supercurrent propagation between two close but separate superconducting bodies depends on processes occurring both across the divide and on each side of the divide as was also apparent from figure 1. But now we have given a physical meaning to this, namely as a consequence of superconducting body 1, superconducting body 2 interchange.

Equation (9) is therefore

$$\begin{aligned}\exp &\left[i\left(\chi_2-\chi_1-\tfrac{2e}{\hbar}\int_{\Gamma^+_{21},\Gamma^-_{12}}A_i da^i\right)\right]\\ &+\exp\left[i\left(\tfrac{1}{2}\right)\left(\chi_1'-\chi_1\right)-i\left(\tfrac{1}{2}\right)\left(\chi_2-\chi_2'\right)-\tfrac{2e}{\hbar}i\int_{\Gamma^+_{11},\Gamma^-_{22}}A_i da^i\right]\end{aligned} \tag{11}$$

By following arguments[4] similar to those used by Josephson [8], we find that the right hand term of equation (11) is negligable and so

$$j=\sin\left(\chi_2-\chi_1-\frac{2e}{\hbar}\int_{\Gamma^+_{21},\Gamma^-_{12}}A_i da^i\right) \tag{12}$$

by taking the imaginary part, $j$ being real. The standard formula [9],

$$j=\sin\left(\chi_2-\chi_1-\frac{2e}{\hbar}\int_1^2 A_i da^i\right) \tag{13}$$

is thereby recovered, the linkage being taken around a closed loop lying across superconducting bodies 1 and 2, *but this time as a consequence of applying the indistinguishability principle to two close but separate SC bodies.*

In the above description, we have assumed the equality of advanced and retarded waves within the quantum field of material points which constitutes the superconducting body.

We could now proceed to generalise directly to the case of more than 2 SC bodies which have been brought into close proximity. Before we do

[4]These arguments show that for a rectangular path which contains the barrier whose short sides are parallel to the normal to the barrier, then the flux enclosed by this path is given by the contribution to the alteration of the phase by the paths $\Gamma^+_{21}$ and $\Gamma^-_{12}$, the contribution from the paths $\Gamma^+_{11}$ and $\Gamma^-_{22}$ being negligible. The flux enclosed is therefore $\Delta\chi\left(\Gamma^+_{21}\right)-\Delta\chi\left(\Gamma^-_{12}\right)=\left(\frac{2e}{\hbar}\right)\Phi$.

this, for the purpose of illustration, let us consider 4 quantum objects; an electron and a positron (time reversed electron), an electron and a positron (time reversed electron). The time directions for interchange of each are then commensurate with the time directions for the propagation of supercurrent for 4 SC bodies, brought into close proximity, each having respective phases $\chi_1$, $\chi_2$, $\chi_3$ and $\chi_4$.

## 3. Illustration: Occurrences for an electron-positron electron-positron exchange

In this section we consider the existence of indistinguishability for 4 quantum objects having once interacted in a region A. That is, an electron positron pair found at 1,6 is found at 5,2 while an electron positron pair found at 3,2 is found at 7,4. This is easily done using the Feynman propagator method. The amplitude for the above occurrence is given by the antisymetric sum $\sum\limits_{perm(5,6,7,8)} \pm K_{+}^{A}\left(perm;1,2,3,4\right)$ with a + or − sign being taken according to whether the permutation is even or odd. There are 10 terms in all. These quantities are listed below,

$$\begin{array}{c}
K_{+}\left(5,1\right)K_{+}\left(6,2\right)K_{+}\left(7,3\right)K_{+}\left(8,4\right)\\
K_{+}\left(6,1\right)K_{+}\left(5,2\right)K_{+}\left(7,3\right)K_{+}\left(8,4\right)\\
K_{+}\left(7,1\right)K_{+}\left(6,2\right)K_{+}\left(5,3\right)K_{+}\left(8,4\right)\\
K_{+}\left(8,1\right)K_{+}\left(6,2\right)K_{+}\left(7,3\right)K_{+}\left(5,4\right)\\
K_{+}\left(5,1\right)K_{+}\left(7,2\right)K_{+}\left(6,3\right)K_{+}\left(8,4\right)\\
K_{+}\left(5,1\right)K_{+}\left(8,2\right)K_{+}\left(7,3\right)K_{+}\left(6,4\right)\\
K_{+}\left(5,1\right)K_{+}\left(6,2\right)K_{+}\left(8,3\right)K_{+}\left(7,4\right)\\
K_{+}\left(6,1\right)K_{+}\left(5,2\right)K_{+}\left(8,3\right)K_{+}\left(7,4\right)\\
K_{+}\left(7,1\right)K_{+}\left(8,2\right)K_{+}\left(5,3\right)K_{+}\left(6,4\right)\\
K_{+}\left(8,1\right)K_{+}\left(7,2\right)K_{+}\left(6,3\right)K_{+}\left(5,4\right)
\end{array} \tag{14}$$

and shown in fig.3 by Feynman diagrams (i) to (x)

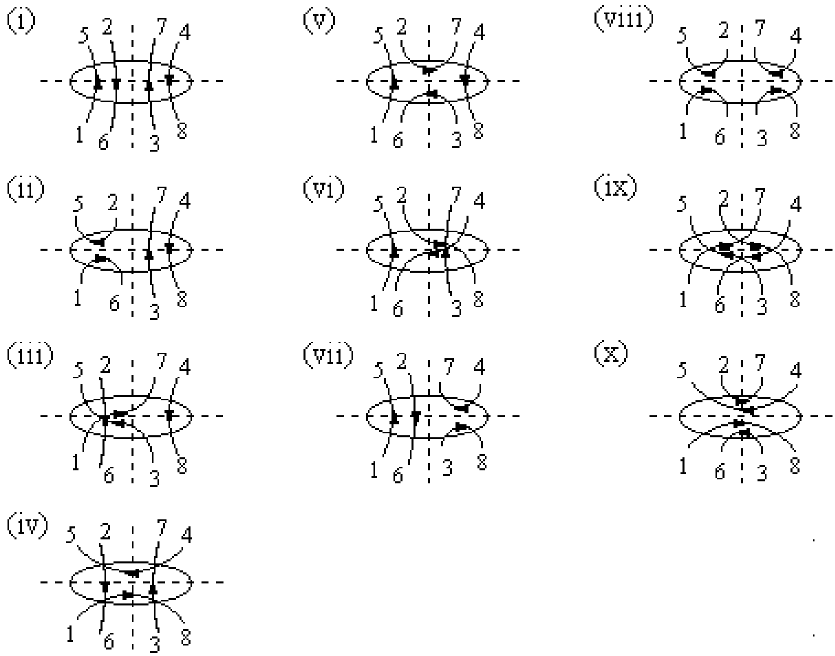


**Figure 3.**

## 4. Occurrences for interchange of 4 superconducting bodies

The analogous situation in a superconducting body is the occurrence $J\,(1 \rightarrow 2)$ and $J\,(2 \rightarrow 1)$ (time reversed process) with application of the indistinguishability property basic to quantum objects. In this case, the inertial frame is not fixed and we must use a generalised path. The important point is that the amplitude for the occurrence $J\,(1 \rightarrow 2)$ and $J\,(2 \rightarrow 1)$ are dependent only on the relative phase angles of each superconducting body. All possible paths for these occurrences are shown below, the notation for $\Gamma$ being consistent with that used in equation

(9). i.e. we put $K_+(5,1) \equiv \Gamma^+_{12}$, $K_+(6,2) \equiv \Gamma^-_{21}$, $K_+(7,3) \equiv \Gamma^+_{34}$...etc.

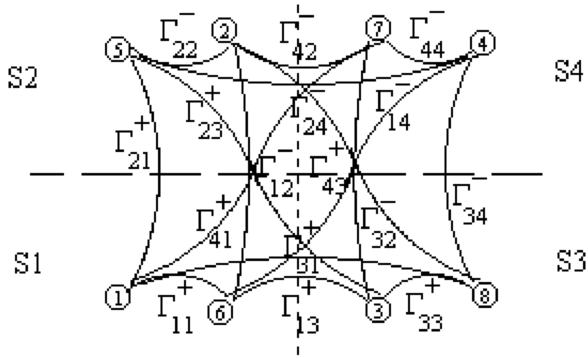


**Figure 4.**

The occurrences which may happen for interchange of each of the 4 SC bodies are listed below using the generalised paths Γ.

$$\begin{gathered}
P\left(\Gamma^+_{21}\right) P\left(\Gamma^-_{12}\right) P\left(\Gamma^+_{43}\right) P\left(\Gamma^-_{34}\right) \\
P\left(\Gamma^+_{11}\right) P\left(\Gamma^-_{22}\right) P\left(\Gamma^+_{43}\right) P\left(\Gamma^-_{34}\right) \\
P\left(\Gamma^+_{23}\right) P\left(\Gamma^+_{41}\right) P\left(\Gamma^-_{12}\right) P\left(\Gamma^-_{34}\right) \\
P\left(\Gamma^+_{31}\right) P\left(\Gamma^-_{12}\right) P\left(\Gamma^+_{43}\right) P\left(\Gamma^-_{24}\right) \\
P\left(\Gamma^+_{21}\right) P\left(\Gamma^-_{42}\right) P\left(\Gamma^+_{13}\right) P\left(\Gamma^-_{34}\right) \\
P\left(\Gamma^+_{21}\right) P\left(\Gamma^-_{32}\right) P\left(\Gamma^+_{43}\right) P\left(\Gamma^-_{14}\right) \\
P\left(\Gamma^+_{21}\right) P\left(\Gamma^-_{12}\right) P\left(\Gamma^+_{33}\right) P\left(\Gamma^-_{44}\right) \\
P\left(\Gamma^+_{11}\right) P\left(\Gamma^-_{22}\right) P\left(\Gamma^+_{33}\right) P\left(\Gamma^-_{44}\right) \\
P\left(\Gamma^+_{41}\right) P\left(\Gamma^-_{32}\right) P\left(\Gamma^+_{23}\right) P\left(\Gamma^-_{14}\right) \\
P\left(\Gamma^+_{31}\right) P\left(\Gamma^-_{42}\right) P\left(\Gamma^+_{13}\right) P\left(\Gamma^-_{24}\right)
\end{gathered} \tag{15}$$

Here $\Gamma^+_{ij}$ denotes propagation from quantum object $i$ to $j$ in the positive time sense and $\Gamma^-_{ji}$ propagation from quantum objects $j$ to $i$ in the negative time sense. We therefore have $\Gamma^+_{21} \equiv \left(e^{i(\chi_2-\chi_1)}\right)^{1/2}$,

$\Gamma_{12}^{-} \equiv \left(e^{-i(\chi_1-\chi_2)}\right)^{1/2}$, etc. Following this nomenclature, with $A_i$ set to zero, (15) becomes

$$\begin{array}{ll}
 & \text{(taking the imaginary part)} \\
\left(e^{i(\chi_2-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_1-\chi_2)}\right)^{\frac{1}{2}} \left(e^{i(\chi_4-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_3-\chi_4)}\right)^{\frac{1}{2}} & \sin\left(\chi_2-\chi_1+\chi_4-\chi_3\right) \\
\left(e^{i(\chi_1-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_2-\chi_2)}\right)^{\frac{1}{2}} \left(e^{i(\chi_4-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_3-\chi_4)}\right)^{\frac{1}{2}} & \sin\left(\chi_4-\chi_3\right) \\
\left(e^{i(\chi_2-\chi_3)}\right)^{\frac{1}{2}} \left(e^{i(\chi_4-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_1-\chi_2)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_3-\chi_4)}\right)^{\frac{1}{2}} & \sin\left(\chi_2-\chi_3+\chi_4-\chi_1\right) \\
\left(e^{i(\chi_3-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_1-\chi_2)}\right)^{\frac{1}{2}} \left(e^{i(\chi_4-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_2-\chi_4)}\right)^{\frac{1}{2}} & \text{-(real)} \\
\left(e^{i(\chi_2-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_4-\chi_2)}\right)^{\frac{1}{2}} \left(e^{i(\chi_1-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_3-\chi_4)}\right)^{\frac{1}{2}} & \sin\left(\chi_2-\chi_3\right) \\
\left(e^{i(\chi_2-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_3-\chi_2)}\right)^{\frac{1}{2}} \left(e^{i(\chi_4-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_1-\chi_4)}\right)^{\frac{1}{2}} & \sin\left(\chi_2-\chi_1+\chi_4-\chi_3\right) \\
\left(e^{i(\chi_2-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_1-\chi_2)}\right)^{\frac{1}{2}} \left(e^{i(\chi_3-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_4-\chi_4)}\right)^{\frac{1}{2}} & \sin\left(\chi_2-\chi_1\right) \\
\left(e^{i(\chi_1-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_2-\chi_3)}\right)^{\frac{1}{2}} \left(e^{i(\chi_3-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_4-\chi_4)}\right)^{\frac{1}{2}} & \text{-(real)} \\
\left(e^{i(\chi_4-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_3-\chi_2)}\right)^{\frac{1}{2}} \left(e^{i(\chi_2-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_1-\chi_4)}\right)^{\frac{1}{2}} & \sin\left(\chi_2-\chi_3+\chi_4-\chi_1\right) \\
\left(e^{i(\chi_3-\chi_1)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_4-\chi_2)}\right)^{\frac{1}{2}} \left(e^{i(\chi_1-\chi_3)}\right)^{\frac{1}{2}} \left(e^{-i(\chi_2-\chi_4)}\right)^{\frac{1}{2}} & \text{-(real)}
\end{array}$$

Therefore, the amplitude for the occurrence $J\,(1 \to 2)$ while all other processes occur is

$$\begin{aligned}
J\,(1 \to 2) \quad \approx \quad & \sin\left(\chi_2-\chi_1+\chi_4-\chi_3\right)+ \\
& \sin\left(\chi_4-\chi_3\right)+ \\
& \sin\left(\chi_2-\chi_3+\chi_4-\chi_1\right)+ \\
& \sin\left(\chi_2-\chi_3\right)+ \\
& \sin\left(\chi_2-\chi_1+\chi_4-\chi_3\right)+ \\
& \sin\left(\chi_2-\chi_1\right)+ \\
& \sin\left(\chi_2-\chi_3+\chi_4-\chi_1\right) \qquad \text{with 7 terms in all.}
\end{aligned} \tag{16}$$

We notice immediately that the supercurrent propagation at one junction depends on the relative phase angles at each of the 4 quantum objects. Furthermore, the above relation should hold for an arbitrary time t after one junction (S1,S2) has been separated in space from the other junction (S3,S4). That is after the 4 quantum objects have interacted. This parallels the case of the positron-electron system; the wavefunctions remain nonfactorizable at any time t after their interaction in region A.

A variation of the phase difference $\chi_4 - \chi_3$ (by application of a vector potential $A_i$, for instance) on one of a pair of once connected junctions, should influence, instantaneously and over arbitrary distance, the maximum voltage- free supercurrent propagation at the other junction, now spatially separate from the former.

Consider a Josephson junction S1,S2 below it's transition temperature which is subsequently broken up into two junctions S1,S2 and S3,S4 as illustrated in figure 5a,b such that there exists no material connections between the junctions. The maximum voltage free supercurrent $J\,(1 \rightarrow 2)$ tunneling across S1,S2 is then measured (by a standard 4 terminal method) versus the field strength $B$ applied parallel to the plane of junction S3,S4. According to equation (16) one should find that the applied field across S3,S4 alters the phase difference $(\chi_4 - \chi_3)$ by an amount $-\frac{2e}{\hbar}\int_3^4 A_i da^i$ thereby altering the maximum voltage free supercurrent across S1,S2, the junctions being at arbitrary separation.

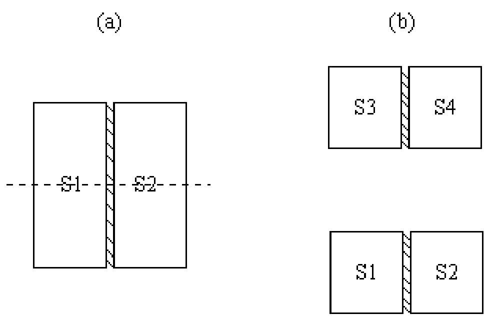


**Figure 5.**

The case described here is similar to that of an EPR correlation tied in the barrier except that the occurrence $J\,(1 \rightarrow 2)$ is proportional to the sum of the amplitudes for the respective processes and *not* the square of the modulus of the sum as is usually the case in quantum processes. Normally the act of squaring the modulus of the amplitude wipes out initial or final phase relations and we move from an internal reversibility to the de facto irreversibility of classical phenomena (see reference [10]).

In our example, the internal reversibility together with the respective phase relations remain intact and we have

$$\begin{aligned} \text{Occurrence} \quad J\,(1 \rightarrow 2) \;\approx\; & \text{sum of the amplitudes for the} \\ & \text{respective processes} \\ \approx\; & \text{respective phase relations} \end{aligned}$$

Retrocausation together with forward causation is implied. The formalism is relativistically covariant and has time reversal symmetry [1]. Strict adherence to this formalism implies the instantaneous transmission of information by the apparatus described herein.

However, in criticism to the above, the application of the magnetic field (parallel to the plane of one of the separated junctions) may very well have the effect of destroying the indistinguishability previously set up, in which case the superluminal telegraph will not function. We are therefore left with an extremely badly coupled EPR system. One in which the " passion at a distance" of Shimony does not show up. This is very probable but not absolutely certain because the effect of the magnetic field is bound up with questions about measurements on macroscopic quantum systems, and this is recondite, and so should properly be settled by experiment (like the one described above).

Lastly, as examples of experimental results which may have a bearing on the above we note that recent experiments on stacked Josephson junctions find no description at present through inductive or quasiparticle coupling [11]. These experiments may find description through a coupling of a non local quantum mechanical nature. These results exhibit current locking or phase locking. The locking of critical currents to a single value has also been observed in two closely spaced indium microbridges biased in series [12]. Locking of phases (and associated current locking) is to be expected from equation (16).

## 5. Discussion

The macroscopic phase $\chi$ owes its existence to a novel manifestation of macroscopic matter. One in which off diagonal matrix elements are present. It occurs below the Einstein- Bose transition. C. N. Yang [13] has illustrated this concept with aid of reduced density matrices defined by $Tr\rho = 1$ , $\langle j|\,\rho_1\,|i\rangle = Tra_j\rho a_i^+$, $\langle kl|\,\rho_2\,|ij\rangle = Tra_k a_l \rho a_j^+ a_i^+$ . . . In superconductors, as is well known, we may have many particle elements behaving as though they have exactly the same momentum.

e.g. persistent currents. In thermal equilibrium, $\rho$ commutes with the total momentum. Thus in the momentum representation, $\rho_1$ is diagonal:

$$\langle p'|\,\rho_1\,|p\rangle = \delta_{pp'} n_p \tag{17}$$

where $n_p$ is the average occupation number (from Bose statistics) of single particle state $p$. In the coordinate representation, however,

$$\langle x'|\,\rho_1\,|x\rangle = 1/\Omega \sum n_p \exp ip\,(x' - x) \tag{18}$$

and $\langle x'|\,\rho_1\,|x\rangle \rightarrow (N\alpha/\Omega) \exp ip\,(x' - x)$ as $|x' - x| \rightarrow \infty$ where $\alpha$ is the fraction of particles in the state. And we have off diagonal matrix elements (ODME). These ODME are retained (have persistence) in the presence of macroscopic measurements (J,V). The macroscopic wavefunction must therefore possess some rigidity.

Costa de Beauregard [14] has clarified the importance of off diagonal matrix elements (the phase) in connection with the cause-effect symmetry apparent in the EPR and time reversed (spin 0 photon pair anticascade or ‘ echelon absorption’ ) EPR paradox at the microlevel.

It follows logically that a superconducting body exhibits time symmetry or more precisely CPT and Lorentz invariance. Also, we must use time symmetric electrodynamics, that is, we must have equality of advanced and retarded waves in the description.

It is interesting that the idea of the difficulty of localising macroscopic phenomena in time is not new. It was deduced, from the principles of quantum mechanics, by Einstein, Tolman and Podolsky in 1931 in connection with the opening and closing of a shutter [15].

## Notes

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## References

[1] Stephens, G. (1996) Found. Phys. Lett. 9, 145

[2] Hoyle, F. and Narlikar, J. V. (1995) Rev. Mod. Phys. 67, 113; see also Feynman, R. P. and Vernon, F. L. jr (1963) Ann. Phys. 24, 118

[3] London, F Superfluids (Dover)

[4] Gor’ kov, L. P. (1959) Sov. Phys. JETP 36, 1364; Josephson, B. D. (1964) Ph.D. thesis, University of Cambridge, England.

[5] Josephson, B. D. (1964) Ph.D. thesis, University of Cambridge, England
[6] Pauli, W. (1940) Phys. Rev. 58, 716
[7] Feynman, R. P. (1949) Phys. Rev. 76, 749
[8] Josephson, B. D. (1969) Chp. 9, pp438-439 in Superconductivity, vol.1, ed. R. D. Parks: Physics Lett. (1962) 1, 251
[9] Mercereau, J. E. (1969) Chp. 8 in Superconductivity, vol.1, ed. R. D. Parks
[10] Costa de Beauregard, O. (1993) ' Maximum Entropy and Bayesian Methods' p9-14, A. Mohammad & G. Demoments (eds.); (1996) Foundations of Physics 26, 391
[11] Nevirkovets, I. P., Ustinov, A. V., Goldobin, E. (1996) preprint.
[12] Neumann, L. G., Dai, Y. D., Kao, Y. H. (1982) J. Low Temp. Phys. 49, 457
[13] Yang, C. N. (1962) Rev. Mod. Phys. 34, 694
[14] Costa de Beauregard, O. (1983) Proc. Int. Symp. Foundations of Quantum Mechanics, Tokyo, p233
[15] Einstein, A., Tolman, R. C., Podolsky, B. (1931) Phys. Rev. 37, 780

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