Annales de la Fondation Louis de Broglie, Volume 25, no 1, 2000

Additional Equations Derived from the Ryder Postulates in the $(1/2, 0) \oplus (0, 1/2)$ Representation of the Lorentz Group

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ABSTRACT. Developing recently proposed constructions for description of particles in the $(1/2, 0) \oplus (0, 1/2)$ representation space, we derive the second-order equations. The similar ones have been proposed in the sixties and the seventies in order to understand the nature of various mass and spin states in the representations of the $O(4, 2)$ group. We give some additional insights into this problem. The used procedure can be generalized for *arbitrary* number of lepton families.

A correct equation for adequate description of neutrinos was sought for a long time [1, 2, 3, 4, 5]. This problem is, in general, connected with the problem of taking the massless limit of relativistic equations. For instance, it has been known for a long time that *"one cannot simply set the mass equal to zero in a manifestly covariant massive-particle equation, in order to obtain the corresponding massless case"*, e. g., ref. [5a].

Secondly, in the seventies the second-order equation in the 4-dimensional representation of the $O(4, 2)$ group was proposed by Barut *et al.* in order to solve the problem of the mass splitting of leptons [6, 7, 8] and by Fushchich *et al.*, for describing various spin states in this representation [9, 10]. The equations (they proposed) may depend on two parameters. Recently we derived the Barut-Wilson equation on the basis of the first principles [11].[1]

[1] Briefly, the scheme for derivation of the equation

$$[i\gamma^\mu \partial_\mu + \alpha_2 \partial^\mu \partial_\mu - \kappa]\, \phi(x^\mu) = 0 \tag{1}$$

is the following. First, apply the generalized Ryder-Burgard relation (see below, Eq. (10)) and the standard scheme for the derivation of relativistic wave equations [12, Thirdly, we found the possibility of generalizations of the $(1,0)\oplus(0,1)$ equations (namely, the Maxwell's equations and the Weinberg-Tucker-Hammer equations[2]) also on the basis of including two independent constants [13], cf. also [14]. This induced us to look inside the problem on the basis of the first principles; my research was started in [15].

In this paper, we first apply the Ahluwalia reformulation [12, 16, 17] of the Majorana-McLennan-Case construct for neutrino [18, 19] with the purpose of the derivation of relevant equations, we recalled above.

The following definitions and postulates are used:

- The operators of the discrete symmetries are defined as follows: a) the space inversion operator:

$$S^s_{[1/2]} = \begin{pmatrix} 0 & \mathbb{1} \\ \mathbb{1} & 0 \end{pmatrix} , \tag{6}$$

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footnote # 1]. Then form the Dirac 4-spinors; the left- and right parts of them are connected as follows:

$$\phi_L^{\uparrow}(p^\mu) = -\Theta_{[1/2]}[\phi_R^{\downarrow}(p^\mu)]^* \quad , \quad \phi_L^{\downarrow}(p^\mu) = +\Theta_{[1/2]}[\phi_R^{\uparrow}(p^\mu)]^* \; , \tag{2a}$$

$$\phi_R^{\uparrow}(p^\mu) = -\Theta_{[1/2]}[\phi_L^{\downarrow}(p^\mu)]^* \quad , \quad \phi_R^{\downarrow}(p^\mu) = +\Theta_{[1/2]}[\phi_L^{\uparrow}(p^\mu)]^* \; , \tag{2b}$$

in order to obtain

$$\left[a\,\frac{i\gamma^\mu\partial_\mu}{m} + b\,\mathcal{C}\mathcal{K} - \mathbb{1}\right]\Psi(x^\mu) = 0 \; , \tag{3}$$

in the coordinate space. Transfer to the Majorana representation with the unitary matrix

$$U = \frac{1}{2}\begin{pmatrix} \mathbb{1} - i\Theta_{[1/2]} & \mathbb{1} + i\Theta_{[1/2]} \\ -\mathbb{1} - i\Theta_{[1/2]} & \mathbb{1} - i\Theta_{[1/2]} \end{pmatrix} \quad , \quad U^\dagger = \frac{1}{2}\begin{pmatrix} \mathbb{1} - i\Theta_{[1/2]} & -\mathbb{1} - i\Theta_{[1/2]} \\ \mathbb{1} + i\Theta_{[1/2]} & \mathbb{1} - i\Theta_{[1/2]} \end{pmatrix} . \tag{4}$$

Finally, one obtains the set

$$\left[a\frac{i\gamma^\mu\partial_\mu}{m} - \mathbb{1}\right]\phi - b\,\chi = 0 \; , \tag{5a}$$

$$\left[a\frac{i\gamma^\mu\partial_\mu}{m} - \mathbb{1}\right]\chi - b\,\phi = 0 \tag{5b}$$

for $\phi(x^\mu) = \Psi_1 + \Psi_2$ or $\chi(x^\mu) = \Psi_1 - \Psi_2$ (where $\Psi^{MR}(x^\mu) = \Psi_1 + i\Psi_2$). With the identification $a/2m \to \alpha_2$ and $m(1-b^2)/2a \to \kappa$ the above set leads to the second-order equation of the Barut type.

[2] In general, the latter does *not* completely reduce to the former after taking the massless limit in the accustomed way.

is the 4×4 anti-diagonal matrix; b) the charge conjugation operator:

$$S^c_{[1/2]} = \begin{pmatrix} 0 & i\Theta_{[1/2]} \\ -i\Theta_{[1/2]} & 0 \end{pmatrix} \mathcal{K} \,, \tag{7}$$

with $\mathcal{K}$ being the operation of complex conjugation; and $\left(\Theta_{[j]}\right)_{h,\,h'} = (-1)^{j+h}\delta_{h',\,-h}$ being the Wigner operator.

- The left- ($\phi_{_L}$ and $\zeta\Theta_{[j]}\phi^*_{_R}$) and the right- ($\phi_{_R}$ and $\zeta'\Theta_{[j]}\phi^*_{_L}$) spinors are transformed to the frame with the momentum p^μ (from the zero-momentum frame) as follows:

$$\phi_{_R}(p^\mu) = \Lambda_{_R}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\,\phi_{_R}(\overset{\circ}{p}{}^\mu) \;=\; \exp(+\mathbf{J}\cdot\boldsymbol{\varphi})\,\phi_{_R}(\overset{\circ}{p}{}^\mu)\;, \tag{8a}$$
$$\phi_{_L}(p^\mu) = \Lambda_{_L}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\,\phi_{_L}(\overset{\circ}{p}{}^\mu) \;=\; \exp(-\,\mathbf{J}\cdot\boldsymbol{\varphi})\,\phi_{_L}(\overset{\circ}{p}{}^\mu)\;. \tag{8b}$$

$\Lambda_{_{R,L}}$ are the matrices for the Lorentz boosts; $\mathbf{J}$ are the spin matrices for spin j, e. g., ref.[20]; $\boldsymbol{\varphi}$ are parameters of the given boost. If we restrict ourselves by the case of bradyons they are defined, *e. g.*, refs. [22, 14], by means of:

$$\cosh(\varphi) = \gamma = \frac{1}{\sqrt{1-v^2}} = \frac{E}{m}, \quad \sinh(\varphi) = v\gamma = \frac{|\mathbf{p}|}{m}, \quad \hat{\boldsymbol{\varphi}} = \mathbf{n} = \frac{\mathbf{p}}{|\mathbf{p}|}\;. \tag{9}$$

- The Ryder-Burgard relation between spinors in the zero-momentum frame[3] is established

$$\phi^h_{_L}(\overset{\circ}{p}{}^\mu) = a(-1)^{\frac{1}{2}-h}e^{i(\vartheta_1+\vartheta_2)}\Theta_{[1/2]}[\phi^{-h}_{_L}(\overset{\circ}{p}{}^\mu)]^* + be^{2i\vartheta_h}\Xi^{-1}_{[1/2]}[\phi^h_{_L}(\overset{\circ}{p}{}^\mu)]^*\;, \tag{10}$$

with the *real* constant a and b being arbitrary at this stage. h is the quantum number corresponding to the helicity,

$$\Xi_{[1/2]} = \begin{pmatrix} e^{i\phi} & 0 \\ 0 & e^{-i\phi} \end{pmatrix}\;, \tag{11}$$

[3]This name was introduced by D. V. Ahluwalia when considering the $(1,0)\oplus(0,1)$ representation [14]. If one uses $\phi_{_R}(\overset{\circ}{p}{}^\mu) = \pm\phi_{_L}(\overset{\circ}{p}{}^\mu)$, cf. also [21, 22], after application of the Wigner rules for the boosts of the 3-component objects to the momentum p^μ, one immediately arrives at the "Bargmann-Wightman-Wigner type quantum field theory", ref. [23] (cf. also the old papers [24, 25, 26] and the recent papers [27, 28]), in this representation. The reader can still reveal some terminological obscurities in [14].

ϕ is here the azimuthal angle related to $\mathbf{p} \to \mathbf{0}$; in general, see the cited papers for the notation.[4]

- One can form either Dirac 4-spinors:

$$u_h(p^\mu) = \begin{pmatrix} \phi_R(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} \quad , \quad v_h(p^\mu) = \gamma^5 u_h(p^\mu) \ , \tag{13}$$

or the second-type spinors [12], see also [26, 28]:

$$\lambda(p^\mu) = \begin{pmatrix} (\zeta_\lambda \Theta_{[j]})\phi_L^*(p^\mu) \\ \phi_L(p^\mu) \end{pmatrix} \quad , \quad \rho(p^\mu) = \begin{pmatrix} \phi_R(p^\mu) \\ (\zeta_\rho \Theta_{[j]})^* \phi_R^*(p^\mu) \end{pmatrix} \ , \tag{14}$$

or even more general forms of 4-spinors depending on the phase factors between their left- and right- parts and helicity sub-spaces that they belong to. For the second-type spinors, the author of ref. [12] proposed several forms of the field operators, e. g.,

$$\nu^{DL}(x^\mu) = \sum_\eta \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{1}{2E_p} \left[\lambda_\eta^S(p^\mu) c_\eta(p^\mu) \exp(-ip\cdot x) + \right.$$
$$\left. + \lambda_\eta^A(p^\mu) d_\eta^\dagger(p^\mu) \exp(+ip\cdot x) \right] \ . \tag{15}$$

On the basis of these definitions on using the standard rules, e. g. [12, footnote # 1] one can derive:

- In the case $\vartheta_1 = 0$, $\vartheta_2 = \pi$ the following equations are obtained

[4] In general, one can connect also $\phi_L^\uparrow$ and $\phi_L^\downarrow$. with using the Ω matrix (see formulas (22a,b) in ref. [12]):

$$\phi_L^\uparrow(\overset{\circ}{p}^\mu) = \Omega \phi_L^\downarrow(\overset{\circ}{p}^\mu) \ , \quad \Omega = \begin{pmatrix} cotan(\theta/2) & 0 \\ 0 & -tan(\theta/2) \end{pmatrix} = \frac{|\mathbf{p}|}{\sqrt{\mathbf{p}^2 - p_3^2}} (\sigma_3 + \frac{p_3}{|\mathbf{p}|}) \ . \tag{12}$$

We did not yet find any explicitly covariant form of the resulting equation.

for $\phi_L(p^\mu)$ and $\chi_R = \zeta_\lambda \Theta_{[1/2]} \phi_L^*(p^\mu)$:[5]

$$\begin{aligned}
\phi_L^h(p^\mu) &= \Lambda_L(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\phi_L^h(\overset{\circ}{p}{}^\mu) \\
&= \frac{a}{\zeta_\lambda}(-1)^{\frac{1}{2}+h}\Lambda_L(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\Lambda_R^{-1}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\chi_R^h(p^\mu) + \\
&+ \frac{b}{\zeta_\lambda}\Lambda_L(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\Xi_{[1/2]}^{-1}\Theta_{[1/2]}^{-1}\Lambda_R^{-1}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\chi_R^{-h}(p^\mu)\,, \qquad (16a) \\
\chi_R^{-h}(p^\mu) &= \Lambda_R(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\chi_R^{-h}(\overset{\circ}{p}{}^\mu) \\
&= a\zeta_\lambda(-1)^{\frac{1}{2}-h}\Lambda_R(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\Lambda_L^{-1}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\phi_L^{-h}(p^\mu) + \\
&+ b\zeta_\lambda\Lambda_R(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\Theta_{[1/2]}\Xi_{[1/2]}\Lambda_L^{-1}(p^\mu \leftarrow \overset{\circ}{p}{}^\mu)\phi_L^h(p^\mu)\;. \qquad (16b)
\end{aligned}$$

Hence, the equations for the 4-spinors $\lambda_\eta^{S,A}(p^\mu)$ take the forms:[6]

$$ia\frac{\widehat{p}}{m}\lambda_\uparrow^S(p^\mu) - (b\mathcal{C}\mathcal{K} - \mathbb{1})\lambda_\downarrow^S(p^\mu) = 0\;, \qquad (17a)$$

$$ia\frac{\widehat{p}}{m}\lambda_\downarrow^S(p^\mu) + (b\mathcal{C}\mathcal{K} - \mathbb{1})\lambda_\uparrow^S(p^\mu) = 0\;, \qquad (17b)$$

$$ia\frac{\widehat{p}}{m}\lambda_\uparrow^A(p^\mu) - (b\mathcal{C}\mathcal{K} + \mathbb{1})\lambda_\downarrow^A(p^\mu) = 0\;, \qquad (17c)$$

$$ia\frac{\widehat{p}}{m}\lambda_\downarrow^A(p^\mu) + (b\mathcal{C}\mathcal{K} + \mathbb{1})\lambda_\uparrow^A(p^\mu) = 0\;, \qquad (17d)$$

$a = \pm(b-1)$ if we want to have $p_0^2 - \mathbf{p}^2 = m^2$ for massive particles.

- We can write several forms of equations in the coordinate representation depending on the relations between creation/annihilation operators. For instance, provided that we imply $d_\uparrow(p^\mu) = +ic_\downarrow(p^\mu)$ and $d_\downarrow(p^\mu) = -ic_\uparrow(p^\mu)$; the $\mathcal{K}$ operator acts on $q-$ numbers as hermitian conjugation, the first generalized equation in the coordinate space reads

$$\left[ia\frac{\gamma^\mu\partial_\mu}{m} - (b - \mathbb{1})\gamma^5\mathcal{C}\mathcal{K}\right]\Psi(x^\mu) = 0\;. \qquad (18)$$

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[5]The phase factors ζ are defined by various constraints imposed on the 4-spinors (or corresponding operators), e. g., the condition of the self/anti-self charge conjugacy gives $\zeta_\lambda^{S,A} = \pm i$. But, one should still note that phase factors also depend on the phase factor in the definition of the charge conjugation operator (7). The "mass term" of resulting dynamical equations may also be different.

[6]η is the chiral helicity quantum number introduced in ref. [12].

Transferring into the Majorana representation one obtains two real equations:[7]

$$ia\frac{\gamma^\mu\partial_\mu}{m}\Psi_1(x^\mu) - i(b-\mathbb{1})\gamma^5\Psi_2(x^\mu) = 0 \ , \qquad (19a)$$

$$ia\frac{\gamma^\mu\partial_\mu}{m}\Psi_2(x^\mu) - i(b-\mathbb{1})\gamma^5\Psi_1(x^\mu) = 0 \ . \qquad (19b)$$

for real and imaginary parts of the field function $\Psi^{^{MR}}(x^\mu) = \Psi_1(x^\mu) + i\Psi_2(x^\mu)$. In the case of $a = 1 - b$ and considering the field function $\phi = \Psi_1 + \Psi_2$ we come to the Sokolik equation for the spinors of the *second kind* [26, Eq.(8)] and ref. [28, Eqs.(14,18)]. Next, we come to the second-order equation in the coordinate representation for massive particles

$$\left[a^2\frac{\partial_\mu\partial^\mu}{m^2} + (b-1)^2\right]\begin{cases}\Psi_1(x^\mu)\\ \Psi_2(x^\mu)\end{cases} = 0 \ . \qquad (20)$$

Of course, it may be reduced to the Klein-Gordon equation. In general, there may exist mass splitting between various $CP-$ conjugate states. We shall return to this question in other papers.

- One can find the relation between creation/annihilation operators for another equation (β_1, $\beta_2 \in \Re e$)

$$\left[ia\frac{\gamma^\mu\partial_\mu}{m} - e^{i\alpha_1}\beta_1\gamma^5\mathcal{C}\mathcal{K} + e^{i\alpha_2}\beta_2\right]\Psi(x^\mu) = 0 \ , \qquad (21)$$

which would be consistent with the equations (17a-17d).[8] Here they are:

$$(b-1)c_\uparrow = ie^{i\alpha_1}\beta_1 d_\downarrow - ie^{i\alpha_2}\beta_2 c_\downarrow \ , \qquad (22a)$$

$$(b-1)c_\downarrow = -ie^{i\alpha_1}\beta_1 d_\uparrow + ie^{i\alpha_2}\beta_2 c_\uparrow \ , \qquad (22b)$$

$$(b-1)d_\uparrow^\dagger = -ie^{i\alpha_1}\beta_1 c_\downarrow^\dagger - ie^{i\alpha_2}\beta_2 d_\downarrow^\dagger \ , \qquad (22c)$$

$$(b-1)d_\downarrow^\dagger = ie^{i\alpha_1}\beta_1 c_\uparrow^\dagger + ie^{i\alpha_2}\beta_2 d_\uparrow^\dagger \ . \qquad (22d)$$

The condition of the compatibility ensures that $\alpha_2 = 0, \pi$ and $\beta_1^2 + \beta_2^2 = (b-1)^2$. We assumed that two annihilation operators

[7] It seems that this procedure (the transfer to the Majorana representation) can be carried out for any spin, cf. [29].

[8] As one can expect from this consideration the equation (21) may be reminiscent of the works of the 60s, refs. [3, 4, 5, 30].

are linear independent. If $\beta_1 = 0$, we recover the Dirac equation but with additional constraints put on the creation/annihilation operators, $c_\uparrow = \mp i c_\downarrow$ and $d_\uparrow = \pm i d_\downarrow$. The phase phactor α_1 remains unfixed at this stage.

In the Majorana representation the resulting set of the real equations are

$$\left[ia\frac{\gamma^\mu\partial_\mu}{m} + i\beta_1 \sin\alpha_1\gamma^5 + \beta_2\right]\Psi_1 - i\beta_1\cos\alpha_1\gamma^5\Psi_2 = 0\ , \tag{23a}$$

$$\left[ia\frac{\gamma^\mu\partial_\mu}{m} - i\beta_1 \sin\alpha_1\gamma^5 + \beta_2\right]\Psi_2 - i\beta_1\cos\alpha_1\gamma^5\Psi_1 = 0\ . \tag{23b}$$

For instance in the $\alpha_1 = \frac{\pi}{2}$ we obtain

$$\left[ia\frac{\gamma^\mu\partial_\mu}{m} + i\beta_1\gamma^5 + \beta_2\right]\Psi_1 = 0\ , \tag{24a}$$

$$\left[ia\frac{\gamma^\mu\partial_\mu}{m} - i\beta_1\gamma^5 + \beta_2\right]\Psi_2 = 0\ . \tag{24b}$$

But, in any case one can recover the Klein-Gordon equation for both real and imaginary parts of the field function, Eq. (20). It is not yet clear, if the constructs discussed recently in ref. [30] are permitted.

- But, we are able to consider other constraints on the creation/annihilation operators, introduce various types of fields operators (as in [13]) and/or generalize even more the Ryder-Burgard relation (see footnote # 4 of the present paper, for instance). In the general case, we suggest to start from

$$(a\frac{\widehat{p}}{m} - 1\!\!1)u_h(p^\mu) + ib(-1)^{\frac{1}{2}-h}\gamma^5\mathcal{C}u^*_{-h}(p^\mu) = 0\ ; \tag{25}$$

i. e., the equation (11) of [11]. But, as opposed to the cited paper, we write the coordinate-space equation in the form:

$$\left[a\,\frac{i\gamma^\mu\partial_\mu}{m} + b_1\,\mathcal{C}\mathcal{K} - 1\!\!1\right]\Psi(x^\mu) + b_2\gamma^5\mathcal{C}\mathcal{K}\widetilde{\Psi}(x^\mu) = 0\ , \tag{26}$$

thus introducing the third parameter. Then we can perform the same procedure as in ref. [11]. Implying $\Psi^{^{MR}} = \Psi_1 + i\Psi_2$ and

$\widetilde{\Psi}^{MR} = \Psi_3 + i\Psi_4$, one obtains real equations in the Majorana representation

$$(a\frac{i\gamma^\mu\partial_\mu}{m} - \mathbb{1})\phi - b_1\chi + ib_2\gamma^5\widetilde{\phi} = 0\,, \tag{27a}$$

$$(a\frac{i\gamma^\mu\partial_\mu}{m} - \mathbb{1})\chi - b_1\phi - ib_2\gamma^5\widetilde{\chi} = 0\,, \tag{27b}$$

for $\phi = \Psi_1 + \Psi_2$, $\chi = \Psi_1 - \Psi_2$ and $\widetilde{\phi} = \Psi_3 + \Psi_4$, $\widetilde{\chi} = \Psi_3 - \Psi_4$. After algebraic transformations we have:

$$(a\frac{i\gamma^\mu\partial_\mu}{m} - b_1 - \mathbb{1})\left[2ia\frac{\gamma^\nu\partial_\nu}{m} + a^2\frac{\partial^\nu\partial_\nu}{m^2} + b_1^2 - \mathbb{1}\right]\Psi_1 - \\ -ib_2\gamma^5\left[2ia\frac{\gamma^\mu\partial_\nu}{m} - a^2\frac{\partial^\mu\partial_\mu}{m^2} - b_1^2 + \mathbb{1}\right]\Psi_4 = 0\,; \tag{28a}$$

$$(a\frac{i\gamma^\mu\partial_\mu}{m} + b_1 - \mathbb{1})\left[2ia\frac{\gamma^\nu\partial_\nu}{m} + a^2\frac{\partial^\nu\partial_\nu}{m^2} + b_1^2 - \mathbb{1}\right]\Psi_2 - \\ -ib_2\gamma^5\left[2ia\frac{\gamma^\mu\partial_\mu}{m} - a^2\frac{\partial^\mu\partial_\mu}{m^2} - b_1^2 + \mathbb{1}\right]\Psi_3 = 0\,, \tag{28b}$$

the third-order equations. However, the field operator $\widetilde{\Psi}$ may be linear dependent on the states included in the Ψ. So, relations may exist between $\Psi_{3,4}$ and $\Psi_{1,2}$. If we apply the most simple (and accustomed) constraints $\Psi_1 = -i\gamma^5\Psi_4$ and $\Psi_2 = i\gamma^5\Psi_3$ one should recover the Dirac-Barut-like equation with *three mass eigenvalues*:

$$\left[i\gamma^\mu\partial_\mu - m\frac{1 \pm b_1 \pm b_2}{a}\right]\cdot\left[i\gamma^\nu\partial_\nu + \frac{a}{2m}\partial^\nu\partial_\nu + m\frac{b_1^2 - 1}{2a}\right]\Psi_{1,2} = 0\,. \tag{29}$$

Thus, we can conclude that as in several previous works we observe that the physical results depend on the stage where one applies the relevant constraints. Furthermore, we apparently note that the similar results can be obtained by consecutive applications of the generalized Ryder-Burgard relations.

As indicated by Barut himself, several ways for introdcution of interaction with 4-vector potential exist in second-order equations. Only considering the correct one (and, probably, taking into account γ^5 axial currents introduced in [17]), we shall be able to answer the question of why the α_2 parameter of the Barut works is fixed by means of the use of the *classical* value of anomalous magentic moments; and on what physical basis we have to fix other parameters we introduced above.

In conclusion, we presented a very natural way of deriving the massive/massless equations in the $(1/2,0)\oplus(0,1/2)$ representation space, which leads to those given by other researchers in the past. It is known that present-day neutrino physics has come across serious difficulties. No experiment and observation are in agreement with theoretical predictions of the standard model. Furthermore, Barut's way of solving the hierarchy problem was almost forgotten, in spite of its simplicity and beauty. In fact, an idea of imposing certain relations between convective and rotational motions of a fermion is much more physical than all other modern-fashioned models. It may be very fruitful, because as we have shown previously and here, the $(j,0)\oplus(0,j)$ representation spaces have a very rich internal structure. I hope the question of whether proposed equations have some relevance to the description of the physical world, will be solved in the near future.

Acknowledgments. The work was motivated by old [6-8,25,26] and recent [12,13,27], by very useful frank discussions and phone conversations with Prof. A. F. Pashkov during last 15 years, by Prof. A. Raspini who kindly sent me his papers, and by the critical report of anonymous referee of the "Foundation of Physics" which, nevertheless, was of crucial importance in initiating my present research. I also acknowledge the discussion with Profs. A. E. Chubykalo, Y. S. Kim, R. M. Santilli and Yu. F. Smirnov. After completing the preliminary version I had fruitful discussions with Dr. G. Quznetsov. Many thanks to all them.

Zacatecas University, México, is thanked for awarding a professorship. This work has been partly supported by the Mexican Sistema Nacional de Investigadores and the Programa de Apoyo a la Carrera Docente.

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(Manuscrit reçu le 26 octobre 1999)