THE SHIFTING OF ELECTRONIC INTERFERENCE FRINGES IN A FIELDLESS MAGNETIC POTENTIAL

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Abstract: It is shown that, if an interference or a diffraction phenomenon is created in a magnetic fieldless potential (for instance owing to a Tonomura torus) and without the possibility that electronic trajectories surround any induction vector, a shifting of fringes will appear and indeed even a change of the interfringe. For this to happen an angle is needed between the potential and the incident beam. With rapid electrons the shifting is of the first order in $\frac{eA}{mv}$ (quotient of the two terms of de Broglie's wave-vector). The case of slow electrons is also examined. Finally we compare the local method (applied here) to the global method based on circular integrals and we show that the latter confuses the phase and electron trajectories, which prevents it from predicting the phenomena that are the subject of the present paper. A new variant of the Aharonov-Bohm experiment is suggested, in which the closed curves defined by pairs of trajectories coming from Young slits cannot surround any induction vector, so that the integral of **A** above such curves is always equal to zero. According to the local method the effect must subsist, according to the global method, it disappears.

1. Introduction. The present paper is a development of a recent suggestion of Olivier Costa de Beauregard and myself, to try electronic interference or diffraction experiments in the vicinity of the center of a Tonomura magnetic torus, in order to verify an old de Broglie's formula already given in his Thesis, and to which he later came back several times, but that was never submitted to the experiment. This formula is the one which gives the wavelength of an electron in a magnetic potential **A**:

$$h\mathbf{k} = m\mathbf{v} + e\mathbf{A} \;\rightarrow\; \lambda = \frac{h}{p}\,; \quad p = |m\mathbf{v} + e\mathbf{A}| \tag{1}$$

($\mathbf{k} = \left(\frac{\nu}{\lambda}\right)\mathbf{n}$ = wave vector, $\mathbf{p}$ = Lagrange momentum, ν = frequency, V = phase velocity, λ = wavelength)

Such experiments are interesting for several reasons:

1) To verify the formula (1) which is *exclusively* based on the identity between the principle of Fermat and that of least action: the equations of quantum mechanics only repeat this result on which they are based.

2) The formula (1) *is gauge dependent* and so are the phenomena themselves.

3) If the fringe shifting is experimentally confirmed, it will be due to the potential alone because no induction line can play a role. The force generated by a fieldless potential is only exerted through the wavelength and it may be considered as an *inertial force*, just as may be considered, in general, the forces due to the de Broglie wave which are at the origin of diffraction and interference phenomena. De Broglie drew attention, starting from his first notes[1], on the fact that the diffraction of material particles violates the Galileo principle of inertia, because particles are deviated from their inertial motion in the absence of external forces and without colliding with any obstacle (unless the wave exerts a pressure on matter). The existence of the wave forces a more general expression of the inertia principle, the old expression of which remains approximately true when the diffraction effects are negligible.

4) Finally, such experiments would make the exact relations between waves and particules more evident than usual experiments can do in the interference phenomena. Let us recall some principles:

a) *Everything is governed by the wave*, to which the Fresnel arguments may be applied. Particles (including photons) don't interfere "between themselves" but only *one by one*. All the interferences may be realized with isolated quanta and they often cannot be realized in any other way. Phase coherence is only due to the wave. Even bosons are linked only by the wave; electrons, which are fermions, are phase independent (supraconductivity is another thing). It does not make sense to speak of "coherent electrons, each passing by a slit". We can speak only of one particle at the same time. We don't know from which slit the electron comes and if we try to know it, the interference is destroyed.

b) The calculation of phases is based on *wavelength*, defined as $\lambda = \frac{h}{p}$ where p is *the length of the Lagrange momentum*. In the ordinary case $p = |m\mathbf{v}|$, but here we have: $p = |m\mathbf{v} + e\mathbf{A}|$ and λ is the only length that must be used. It makes no sense to speak about a phase difference due to $m\mathbf{v}$, to which would be "added" a difference due to $e\mathbf{A}$: such a formulation mixes two different directions.

c) The phase differences are defined along the lines tangent to the *normal to the wave*: the wave vector **k** defined by the formula (1).

d) The wave propagates with a phase velocity V. Just as the frequency ν, this velocity is defined in relativity[2]. Therefore, ν and V are wrong in the Schrödinger equation and correct in the Klein-Gordon and Dirac equations. They are not (until now) directly mesured but they can be deduced, owing to a relativistic reasoning, from the sole knowledge of the wavelength λ[2].

e) The velocity of the electrons is not the phase velocity V (which besides is greater than the velocity of light), but the *group velocity*. This result of de Broglie is intuitive because the group velocity is the one of a wave packet, thus of energy and finally of the particle. Usual experiments in electron interference or diffraction are based on the wavelength only: wave or particle velocities play no role because, in the absence of potentials, they have the *same direction*.

f) On the contrary, in the experiments we are dealing with, *the distinction between phase and group velocities plays an essential role* because they are no more parallel:

- The phase goes, as precedingly, at the phase velocity along the *normal* of the wave, while the particle goes (with the group velocity) along the *ray* which is no more parallel to the normal.
- Therefore, the interference is controled by the wave through phase differences along the normal of the wave, which defines *non observable fringes* of phase coherence, while the *observable fringes* are different (despite the fact that they are indirectly defined by phase differences): they are those on which the electrons fall and they fall in a direction which is defined by the ray and *not* by the normal to the wave.

2. The problem of the calculation of the effect. First of all we must formulate more precisely the question: "What is observable?"

A vector potential creates an *anisotropy of space,* which is locally transformed in a *uniaxis crystal* [2] the *optical axis* of which is defined in the neighbourhood of each point of space by the magnetic potential. In analogy with Fresnel's ideas on crystal optics, this point will be taken as a focus of a revolution ellipsoid of velocities, the equation of which is (**A** defines the z axis):

$$\frac{x^2 + y^2}{(mv)^2 - (eA)^2} + \frac{(z + eA)^2}{(mv)^2} = 1 \qquad (2)$$

The section by a plane containing the axis (Fig. 1) is an ellipse with parameters:

(3) $a = mv,\; b = \sqrt{(mv)^2 - (eA)^2},\; c = \sqrt{a^2 - b^2} = (eA)^2 \quad \left(mv > eA \rightarrow (mv)^2 - (eA)^2 > 0\right)$

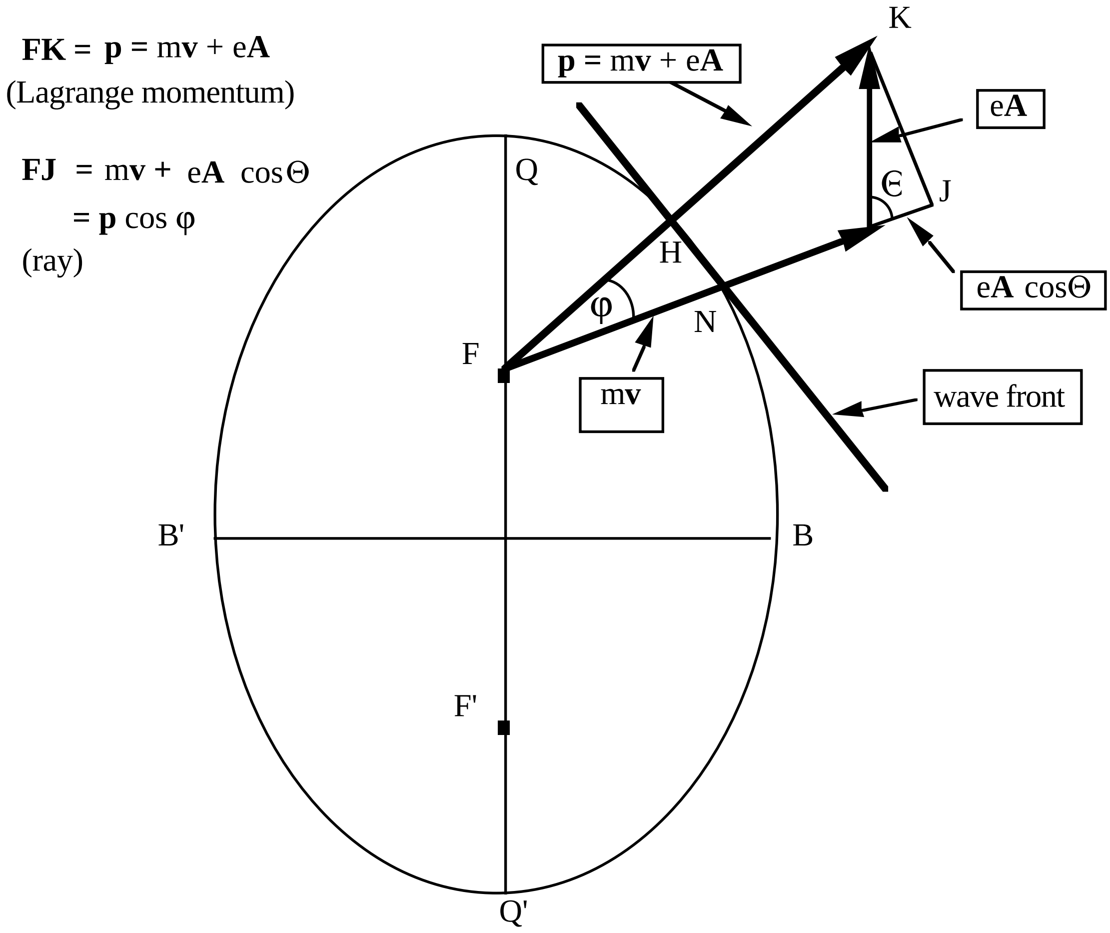

Fig . 1

The *normal wave velocity* V (*phase velocity*) is defined along the wave vector **k**, and thus along the Lagrange momentum **p**, because the phase is carried by the *wave front*. The theory of interferences is based on the wavelength λ defined in the same direction FK.

It must be once more emphasized that the direction FK in which phase coherence is defined is not the one of an observable fringe. The electrons travel with the group velocity, along the ray FJ, which is the direction of the observable fringe.

Contrary to the phase velocity, the group velocity does not depend on the potential. Consider the **superposition of two waves** in a potential **A**, with slightly different velocities (and masses):

$$\Psi = \sin 2\pi \left\{ \nu t - \left(\frac{m\mathbf{v} + e\mathbf{A}}{h} \right) . \mathbf{r} \right\} + \sin 2\pi \left\{ (\nu + \delta\nu) t - \left[\frac{m\mathbf{v} + \delta(m\mathbf{v}) + e\mathbf{A}}{h} \right] . \mathbf{r} \right\} \tag{4}$$

$$\rightarrow \Psi = 2 \cos \pi\, \delta\nu \left\{ t - \frac{\partial\left(\frac{m\mathbf{v}}{h}\right)}{\partial\nu} . \mathbf{r} \right\} \sin 2\pi \left\{ \nu t - \left(\frac{m\mathbf{v} + e\mathbf{A}}{h} \right) . \mathbf{r} \right\}$$

The potential **A** appears only in the second factor: in the **phase of the wave.** The **amplitude slowly oscillates** with a frequency $\delta\nu$ and **propagates** with the velocity $\frac{\partial\nu}{\partial\left(\frac{m\mathbf{v}}{h}\right)}$ equal to the **group velocity of the free particle**: the **velocity v.**

It may be surprising that the group velocity, which is the velocity of energy, does not depend on the potential[3]. But the same happens in crystal optics: whereas all the phenomena are described in terms of electrical induction and are a consequence of the electrical polarization, the propagation of energy is given by the Poynting vector which is expressed in terms of fields and not of inductions. But we shall see that the propagation of energy is indirectly influenced by the potential through the phase propagation (just as in crystal optics it is influenced by the polarization of the medium).

The above calculation holds *before* the interference. But it is also valid *after* the interference because the energy of the electrons is conserved, and thus the velocity. But the conservation of velocity is *only in absolute value*. Its *direction* may be modified and this will be the observable effect.

3. Calculation of the shifting of Young fringes. We shall consider three cases: the incident beam is parallel to **A**, orthogonal to **A**, or under any angle.

1st case: The incident beam is parallel to A.

The incident beam, the axis of the torus and the potential A are on Fz in the plane of the Fig. 2. The torus is orthogonal to Fz. The midpoint of Young slits is F, in the center of torus. The slits are separated by a distance **a** on a straight line orthogonal to **A** in the plane of the figure. The circle of radius mv and center F defines all the vectors m**v** of diffracted electrons.

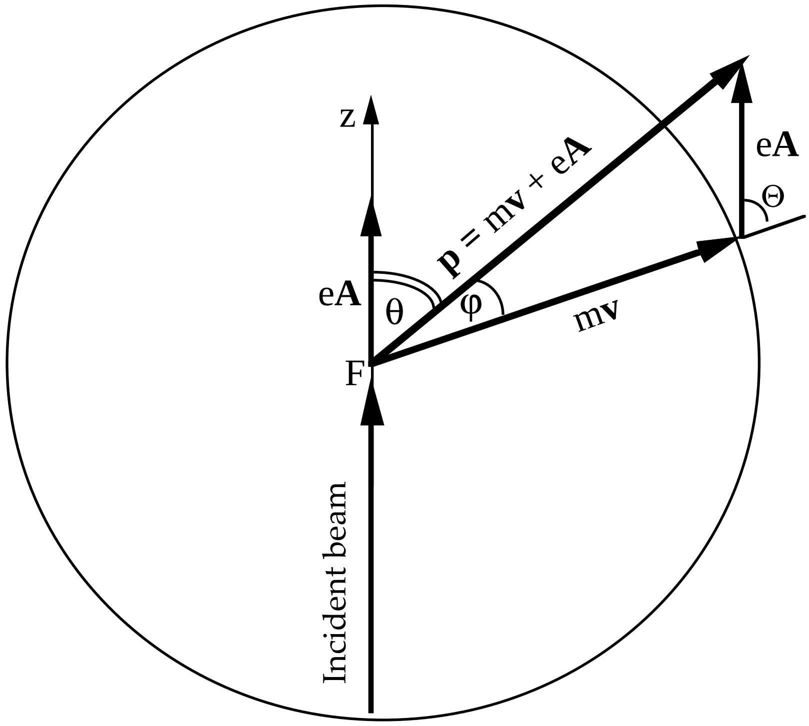

Fig. 2

For symmetry reasons, the central brilliant fringe is along Fz: is is true just as well for the point of zero phase difference as for the fringe on which the electron fall because here, m**v** and **A** have the same direction.

Let us find the following fringe. On Fig. 2 we have: θ = (**p**, Fz) = angle of the fringe corresponding to a phase difference λ, φ = (m**v**, **p**) and Θ = (mv, Fz): the angle with respect to Fz, under which the electrons fall:

(5) $$\Theta = \theta + \varphi$$

(6) $$\theta \cong \sin\theta = \frac{\lambda}{a} = \frac{h}{a|\mathbf{p}|} = \frac{h}{a|m\mathbf{v} + e\mathbf{A}|}$$ (a = distance between the Young slits)

(7) $$|\mathbf{p}| = p = \left[(mv)^2 + (eA)^2 + 2mv\, eA \cos\Theta\right]^{1/2}$$

We shall make use of the following definitions (note the expression of ε):

$$\varepsilon = \frac{eA}{mv}\ ;\ \delta = \frac{h}{mva}\ \text{(interfringe without potential)}$$

(8)
$$\gamma = \left[1 + 2\varepsilon \cos \Theta + \varepsilon^2\right]^{1/2} = \left(1 + \varepsilon \cos \Theta\right)\left[1 + \left(\frac{\varepsilon \sin \Theta}{1 + \varepsilon \cos \Theta}\right)^2\right]^{1/2}$$

Remark: The wavelength is $\lambda=\frac{V}{\nu}=\frac{h}{p}$. The phase velocity is $V=h\nu p^{-1}$. In the presence of a potential **A**, p is given by (7): $p = mv\,\gamma$. In the vacuum, $p_0=mv$ and the phase velocity is $V_0=h\nu p_0^{-1}$ with the same frequency $\nu=E/h$, that does not depend on **A.** Thus we have $V_0/V=p/p_0=\gamma$: γ is the quotient of the phase velocities in the vacuum, and in the presence of the potential **A**: **γ is the index of refraction of de Broglie's wave, due to the potential** .

Now, we have $p=mv\gamma \rightarrow \theta=\delta\gamma^{-1}$, from which:

(9)
$$\theta = \delta\left(1 + \varepsilon \cos \Theta\right)^{-1}\left[1 + \left(\frac{\varepsilon \sin \Theta}{1 + \varepsilon \cos \Theta}\right)^2\right]^{-1/2}$$

On the other hand, we see on Fig. 2 that:

(10)
$$p \cos \varphi = mv + eA \cos \Theta$$

and, taking (8) into account:

(11)
$$\cos \varphi = \left[1 + \left(\frac{\varepsilon \sin \Theta}{1 + \varepsilon \cos \Theta}\right)^2\right]^{-1/2}$$

Expanding (9) and (11) with respect to ε, we find at the first order:

(12)
$$\theta = \delta\,(1 - \varepsilon),\ \ \varphi = \varepsilon\Theta\ \rightarrow \Theta = \theta + \varphi = \delta\,(1 - \varepsilon) + \varepsilon\Theta\ \rightarrow \boxed{\Theta = \delta}$$

It is the same result as in the absence of potential. It is confirmed at higher orders in ε and is easily verified in the particular case $e\mathbf{A} = m\mathbf{v}$, where we cannot expand with respect to $\varepsilon = \frac{eA}{mv}$ but the result is then trivial because the triangle of Fig. 2 is isosceles, which entails:

(13) $$\varphi = \theta \rightarrow \Theta = 2\theta$$

From (7) we have:

(14) $$p^2 = 2\,(mv)^2\,(1 + \cos\Theta) = 4\,(mv)^2 \cos^2\frac{\Theta}{2} \rightarrow p = 2\,mv\cos\theta$$

and owing to (13) and (14):

(15) $$\sin\theta = \frac{\lambda}{a} = \frac{h}{ap} = \frac{h}{a\left(2\,m\,v\cos\theta\right)} \rightarrow 2\sin\theta\cos\theta = \sin 2\theta = \sin\Theta = \frac{h}{a\,m\,v} = \delta$$

Finally we find $\Theta \cong \delta$ again.

Are these results astonishing? No. Because the anisotropy of space due to the potential cannot appear if the incident beam is parallel to the optical axis: there is too much symmetry. As Pierre Curie said: *"It is dissymmetry that creates the phenomenon"*. We need an angle between the beam and the potential.

2nd case: the incident beam is orthogonal to the potential: parallel to the plane of the torus and close to it.

The line passing through the centers of the Young slits is orthogonal to the incident beam and parallel to the magnetic potential. It could be a good idea to join two parallel tori as indicated on the Fig. 3.

a) Central fringe. See Fig. 4. There is, like previousely, a wave vector $\mathbf{k}$ (and a Lagrange momentum $\mathbf{p} = m\mathbf{v} + e\mathbf{A}$) parallel to the incident beam. It corresponds, by symmetry, to a zero phase difference between the waves coming from the Young slits: it is the *central fringe* in the sense of phase equality.

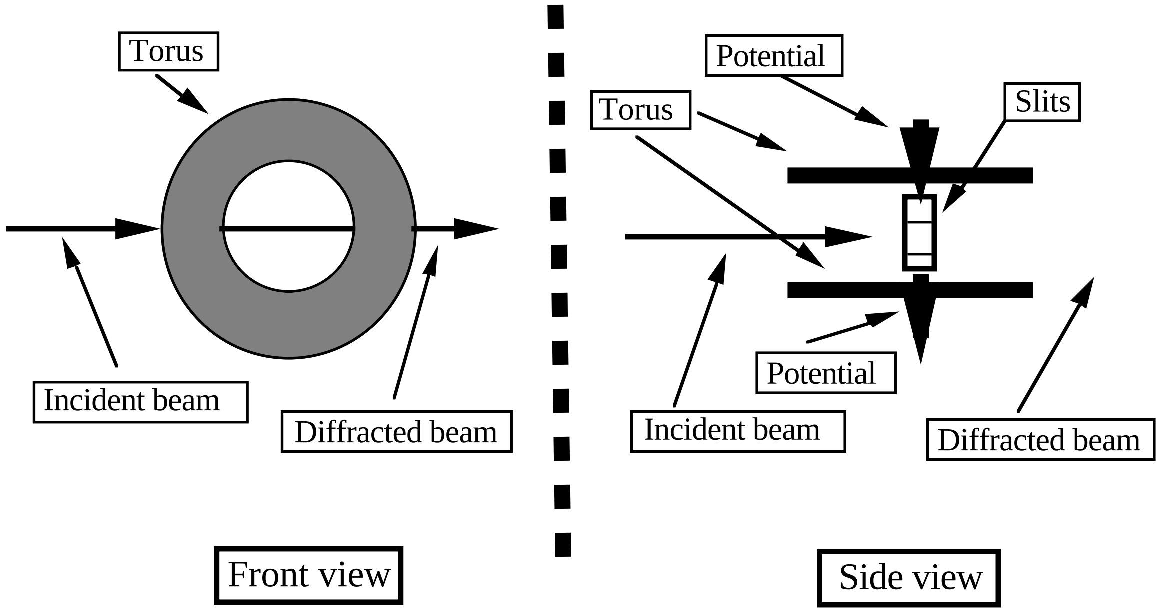
Torus
Incident beam
Diffracted beam
Front view
Potential
Torus
Slits
Potential
Incident beam
Diffracted beam
Side view

Fig. 3

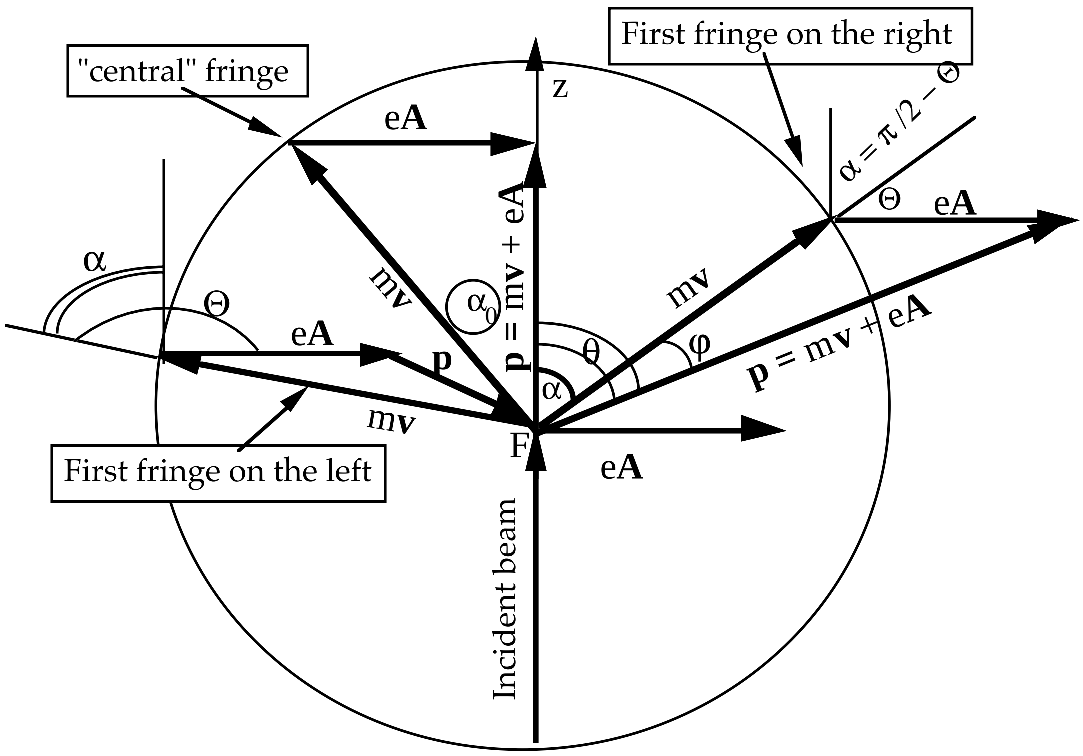
"central" fringe
First fringe on the right
z
eA
α = π /2 − Θ
Θ
eA
α
Θ
eA
mv
α0
p = mv + eA
mv
p
θ
φ
α
p = mv + eA
mv
F
eA
First fringe on the left
Incident beam

Fig. 4

But the electrons will not fall on this fringe: they follow the linear momentum m**v** corresponding to the group velocity **v** and, contrary to the first case, this velocity is no more parallel to **p** even for the central fringe.

The vector m**v** is given by the intersection of the circle of radius mv (centered on F) and the straight line parallel to Fz at a distance e**A** (Fig. 4).

This is the observable "central" fringe which is shifted with respect to the incident beam, by an angle:

(16) $$\boxed{\alpha_0 = \varepsilon = \frac{e\,A}{m\,v}}$$

As we shall see, **at the first order, the whole interference pattern will be shifted** in the direction opposite to e**A**.

b) First fringe on the right of Fz: We keep the same angles θ, φ with the same definitions, but Θ = (e**A**, m**v**) is not any more small. The small angle is the angle α between m**v** and the beam:

(17) $$\alpha = (m\mathbf{v},\ Fz) = \frac{\pi}{2} - \Theta$$

We shall have, instead of (9) and (11):

(18) $$\theta = \delta\,(1 + \varepsilon \sin\alpha)^{-1}\left[1 + \left(\frac{\varepsilon\cos\alpha}{1 + \varepsilon\sin\alpha}\right)^2\right]^{-1/2};\ \ \cos\varphi = \left[1 + \left(\frac{\varepsilon\cos\alpha}{1 + \varepsilon\sin\alpha}\right)^2\right]^{-1/2}$$

and we find at the first order in ε and δ:

(19) $$\theta = \delta;\ \ \varphi = \varepsilon$$

With the new orientation of e**A**, we have instead of (5):

(20) $$\alpha = \theta - \varphi.$$

So that **the first observable fringe, on the right of the central fringe is**:

(21) $$\boxed{\alpha = \delta - \varepsilon}$$

At the first order the interfringe δ is not modified by the potential A but the shifting $\alpha_0 = \frac{\mathbf{eA}}{\mathbf{mv}}$ of the interference pattern is confirmed.

c) First fringe on the left of Fz. The calculation is the same as for the right fringe, but it is evident on Fig. 4, without any calculation, that there is a dissymmetry because the momentum **p** and therefore the wavelength λ are different on the right and on the left: a consequence of the orientation of e**A**.
Nevertheless, there is a certain symmetry of the position of the fringes.
We have here the relation:

(22) $$\alpha = \theta + \varphi$$

and the **first observable fringe on the left** is:

(23) $$\boxed{\alpha = \delta + \varepsilon}$$

in full analogy with the formula (21) for the right fringe, in spite of the change of sign before ε, because the angles are not algebraic, only their absolute value is well-defined: they are angular distances with respect to Fz. The term δ is the classical interfringe, as previously, and the term $+\varepsilon$ is a shift on the *left* just as was $-\varepsilon$ in formula (21) for the right fringe.

d) Other fringes. They are given by the substitution of $n\lambda$ to λ in (6), and $n\delta$ to δ in (8). Thus we find, for the right and left fringes:

(24) $$\boxed{\alpha = n\delta - \varepsilon} \; ; \; \boxed{\alpha = n\delta + \varepsilon}$$

At the first order, the classical interfringe δ is not changed by the potential A, but we find a shift $\alpha_0 = \frac{\mathbf{eA}}{\mathbf{mv}} = \varepsilon$ of the whole interference pattern.

e) Higher order effects. We must solve more carefully the equations (18) for θ and φ, which give respectively for the right and left fringes:

(25) $$\alpha_r = (n\delta - \varepsilon)\left[1 - \varepsilon(n\delta - \varepsilon)\right] ; \ \alpha_l = (n\delta + \varepsilon)\left[1 - \varepsilon(n\delta + \varepsilon)\right]$$

The interfringe and the shift are both modified by a second order effect.

3rd case: Other angles between A and the beam.

We give the figure and the result only for the central and the right fringe.

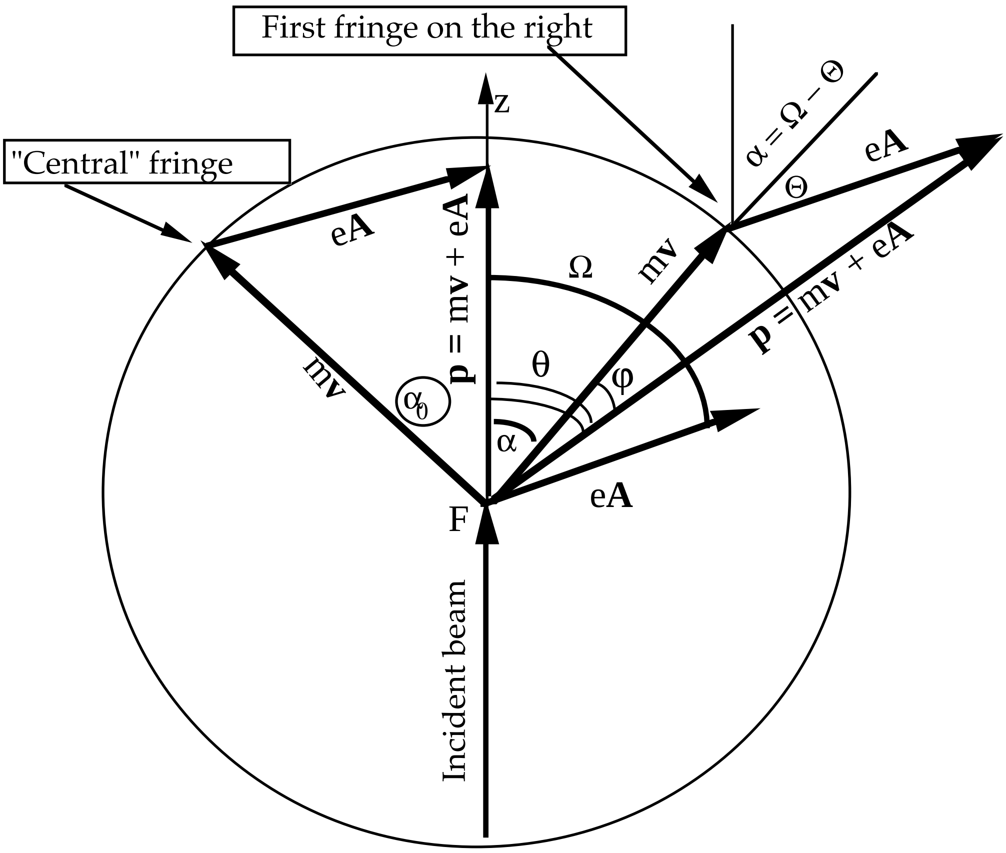

Fig. 5

The angles α, θ and φ keep their definition, but we must introduce the angle Ω between the potential and the beam (Fig. 5):

(26) $$\Theta = \Omega - \alpha \ ; \ \Omega = (e\mathbf{A}, \ Fz)$$

It is realistic to suppose that $\Omega >> \alpha$ because α is a small diffraction angle while Ω is the tilt of the torus, which makes sense only if the beam is able to come through the torus.
We shall use the equations (9) and (11) again, introducing (26) and expanding $\cos(\Omega - \alpha)$ and $\sin(\Omega - \alpha)$, which gives:

(27) $$\theta = \delta\left(1 - \varepsilon \cos \Omega\right) ; \varphi = \varepsilon \sin \Omega$$

We have:

(28) $$\alpha = \theta - \varphi$$

And the central fringe is thus:

(29) $$\boxed{\alpha_0 = \varepsilon \sin \Omega}$$

The first fringe on the right is:

(30) $$\boxed{\alpha = \delta\left(1 - \varepsilon \cos \Omega\right) - \varepsilon \sin \Omega}$$

Actually the term $\varepsilon \cos \Omega$ is a correction of the second order. This is why, if $\Omega = 0$ (beam parallel to e**A**) we find $\alpha_0 = 0$ as in our 1st case, but for the first fringe we find $\alpha = \delta(1 - \varepsilon)$ instead of $\alpha = \delta$.
If $\Omega = \frac{\pi}{2}$ (beam orthogonal to e**A**) we find $\alpha_0 = \varepsilon$, $\alpha = \delta - \varepsilon$, as in the 2nd case.
Formula (30) shows that, despite the fact that the parallelism between the incident beam and the magnetic potential annihilates the effect, one must not be worried about a small angle between these two vectors.
For instance, if we have: $\Omega = (e\mathbf{A}, Fz) = 80^0$, i. e. an angle of 20^0 between the beam and the plane of the torus, formula (30) gives:

(31) $$\alpha = \delta\,(1 - 0{,}17\,\varepsilon) - 0{,}\ 98\ \varepsilon$$

The difference with the result of formula (21) is very small.

4. General method and calculation of the fringes in other cases.

The method is given by formulae (7), (9), (10), abandoning ε which is no more supposed to be small:

$$\text{(32)} \qquad \sin\theta = \delta\left[(mv)^2 + (eA)^2 + 2mv\ eA\cos\Theta\right]^{-1/2}$$
$$\cos\varphi = \left(mv + eA\cos\Theta\right)\left[(mv)^2 + (eA)^2 + 2mv\ eA\cos\Theta\right]^{-1/2}$$

1st example: slow electrons in the particuliar case e**A** = m**v**.
We have seen it with formulae (13), (14), (15) in the 1st case (Fz parallel to e**A**). Let us consider now the more interesting 2nd case (Fz ⊥ e**A**). We give only an outline. Let us go back to Fig. 4 with $eA = mv$. Obviously there cannot be a phase equality in the center, otherwise the isoceles triangle would be right-angled, with the hypotenuse equal to a side of the right angle. All the same, electrons cannot fall on the center, because we would have θ = π/4, which is impossible because θ depends on δ. On the other hand, we see on the figure that:

$$\text{(33)} \qquad \varphi = \frac{\Theta}{2} = \frac{\pi}{2} - \theta\ ;\ \ \alpha = \theta - \varphi$$

and we find from (32):

$$\text{(34)} \qquad \sin\theta = \frac{\delta}{2\cos\frac{\Theta}{2}} = \frac{\delta}{2\cos\varphi} = \frac{\delta}{2\sin\theta} \ \rightarrow\ \sin\theta = \sqrt{\frac{\delta}{2}}$$

So that the fringes are, on the right and on the left:

$$\text{(35)} \qquad \alpha_n = \frac{\pi}{2} - \sqrt{2n\delta}$$

actually, we can see only the right fringes... if at all.

2-nd example: slow electrons in a more general case.
a) We can, at first, take a perturbation of the preceding case:

(36) $$mv = eA + \eta$$

The calculation is trivial.
b) We can suppose that eA >> mv, which is quite possible. Going back to (18), the calculations are easy, in principle, but they require some care because the definition of ε is reversed, the relations between angles must be reexamined and it must be kept in mind that **v remains the group velocity**.

3-rd example: other interference and diffraction phenomena.
We can apply the same procedure:
a) Interferences on a grating or a crystal. The results will be, *mutatis mutandis*, the same as for the Young slits.
b) The diffraction through an aperture, a slit or on the side of a screen.
the classical theory, in optics, gives a series of values: $\delta_1, \delta_2, \ldots \delta_n \ldots$ for the successive angles of diffraction (for instance the Fresnel fringes on a straight edge). If we introduce these values in (32), making precise in each case the relations of type (5) or (20), as we have done until now, we shall get a series of phase coherence angles $\theta_1, \theta_2, \ldots \theta_n \ldots$ and then a series of angles $\Theta_1, \Theta_2, \ldots \Theta_n \ldots$ or $\alpha_1, \alpha_2, \ldots \alpha_n \ldots$ under which the electrons will be observed.

5. Some theoretical remarks.

We have made use, for the calculation of interferences, of the Fresnel formula (6) that corresponds to Fig. 6. On the basis of this formula, de Broglie argued that interferences are gauge dependent. He pointed out that the simple experience of Young slits with an electron beam, even without any potential, implicitely defines a gauge.
Actually, the phase difference between paths coming from both slits is:

(37) $$\frac{\Delta l}{\lambda} = \frac{a\,\theta}{\lambda} = \frac{a\,\theta\, mv}{h}$$

and if we add $e\nabla\chi$ to the electron linear momentum, this difference becomes:

(38) $$\frac{\Delta l}{\lambda} = \frac{a\,\theta}{\lambda} = \frac{a\,\theta\left(mv + e\nabla\chi\right)}{h}$$

so that (37) fixes a gauge value, even when the potential equals zero.

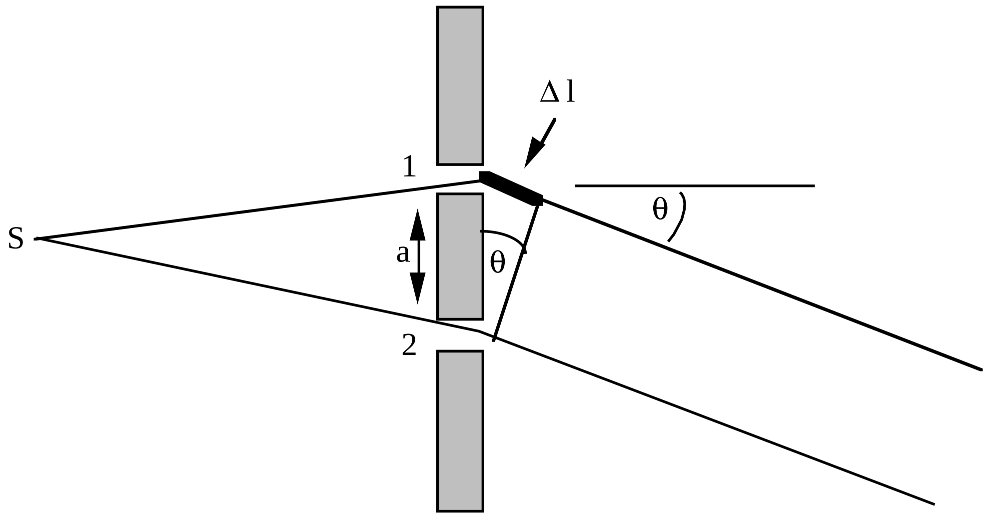

Fig. 6

Feynman developed a different argument about the Aharonov-Bohm[4] effet. Before recalling this problem, let us make some preliminary remarks.

First of all, Feynman never says that the magnetic potential must be added to the Lagrange momentum, giving a new wave vector $\mathbf{p} = m\mathbf{v} + e\mathbf{A}$ that controls the interferences owing to the law (1), and resulting in a change in the orientation of group velocity. In short, he omits the essential change:

(39) $$h\mathbf{k} = m\mathbf{v} \rightarrow h\mathbf{k} = m\mathbf{v} + e\mathbf{A}$$

He considers separately the two terms of $\mathbf{k}$ (actually, he is only interested by $e\mathbf{A}$) and he makes no distinction between phase velocity and group velocity, or between the normal to the wave and the ray. He overlooks the fact that electrons do not fall on the points of equal phase and that the latter define only indirectly the observable fringes. The angles that appear on our figures escape him.

It is not so easy to realize it in his book because, in the Aharonov-Bohm effect, $m\mathbf{v}$ is approximately parallel to $e\mathbf{A}$ in the active part of the potential, so that one can hardly see that *integrals are taken, in fact, along the electron trajectories and not on the lines of phase*. It is not very important in the particular case he is dealing with, but the result would be wrong in a more general case like ours: looking at the Fig. 4, it is obvious that, if the phase difference was taken in the direction m**v** instead of **p** = m**v** + e**A**, forgetting that the wavelength is the one defined by (1), the result would be wrong.

Feynman distinguishes the phase in the absence of the solenoid (Fig. 7), denoted as Φ (**B** = 0), and the additional phase introduced by the potential **A**. And he writes the total phase difference between both "paths", but it is difficult to understand if he speakes of phase lines or electron trajectories:

(40) $$\delta\Phi = \left[\Phi_1(\mathbf{B}=0) - \Phi_2(\mathbf{B}=0)\right] + \left[\int_{(1)} \frac{e\mathbf{A}}{h}.\mathbf{ds} - \int_{(2)} \frac{e\mathbf{A}}{h}.\mathbf{ds}\right]$$

At first glance, one could think that this formula gives (37), because if we remember that $\Phi(\mathbf{B}=0) = \int \frac{m\mathbf{v}}{h}.\mathbf{ds}$, (40) may be written as:

(41) $$\delta\Phi = \int_{(l_1 - l_2)} \frac{m\mathbf{v} + e\mathbf{A}}{h}.\mathbf{ds} \cong (l_1 - l_2)\frac{|m\mathbf{v} + e\mathbf{A}|}{h} = \frac{\Delta l}{\lambda}$$

which is (37) indeed. But in reality, I have introduced two hypotheses:

a) I suppose that the wavelength is given by (1), contrary to Feynman's discussion.

b) When I write the mean value of the integral $(l_1 - l_2)|m\mathbf{v} + e\mathbf{A}|$, I assume implicitely that the integral is taken along a curve tangent to **k** (to the normal of the wave), what Feynman never does. In his case, the cosine of the angle between **k** and **ds** must be introduced.

In any case, the gauge problem appears again in (41), because if ∇χ is added to the potential, the phase difference is modified (except for the central fringe):

(42) $$\delta'\Phi = \cong (l_1 - l_2)\frac{e\nabla\chi}{h}$$

But Feynman pays attention, in (40), only to the second bracket which he writes in the form of a circular integral:

(43) $$\int_{(1)} \frac{e\mathbf{A}}{h}.\mathbf{ds} - \int_{(2)} \frac{e\mathbf{A}}{h}.\mathbf{ds} = \int_{(1-2)} \frac{e\mathbf{A}}{h}.\mathbf{ds}$$

Here the gauge invariance appears and also the induction vector because:

(44) $$\int \mathbf{A}.\mathbf{ds} = \iint \text{rot}\, \mathbf{A}\, \mathbf{d\sigma} = \iint \mathbf{B}\ \mathbf{d\sigma}$$

Therefore, the Aharonov-Bohm effect seams to be due to the induction flux through the closed loop of the paths followed by the electrons, but in doing so, the orthogonal lines to the surfaces of equal phase are confused with electron trajectories. And it should be noted that, in the circular integral,*the localisation of the interference fringes is lost.*

To summarize, we do get a gauge invariant integral but it does not describe the phenomenon because: a) the integration path is not the one that corresponds to the calculation of phase differences; b) the wavelength never appears; c) the localisation of observable fringes - the first question which must be answered - is forgotten.

In the Aharonov-Bohm effect, all that is more or less rubbed out because phase lines and electronic trajectories are not very different, but it is no more the case for the phenomena described in the present paper.

It must be added that, according to Feynman, in a zero induction flux, the added phase is zero. Therefore, the effects described above cannot exist because the potential is supposed to be uniform, i.e. a gradient field. We have indeed:

(45) $$\mathbf{A} = \nabla\ \text{U}\ ;\ \ \text{U} = \nabla\,(\mathbf{A}.\mathbf{x})$$

And it is so for any fieldless vector potential in a connected domain D, because rot $\mathbf{A}$ = 0 and $\int_l \mathbf{A}.\mathbf{ds} = 0$ along any closed loop and there is a scalar potential U such that $\mathbf{A} = \nabla\text{U}$. It is true for the potential of a magnetic torus because the potential of a magnetic dipole $\mathbf{M}$ is equal to:

(46) $$\mathbf{A} = \frac{\mathbf{M}\times\mathbf{r}}{\text{r}^3}$$

and a magnetic torus is nothing but a closed chain of such dipoles. The total potential is thus (Φ = magnetic flux trapped in the torus):

(47) $$\mathbf{A}=\Phi\int_l \frac{\mathbf{ds}\times\mathbf{r}}{r^3}$$

and we have:

(48) $$\frac{\mathbf{r}}{r^3} = -\nabla\left(\frac{1}{r}\right) \rightarrow \mathbf{A}=\Phi\int_l \nabla\left(\frac{1}{r}\right)\times\mathbf{ds}$$

so that the potential **A** of a torus is a gradient field[5]. According to Feynman, all the effects described here would be annihilated for this reason, but I claim that they do exist.

This paper may be considered as a test for local and global theories of interferences. If the predicted effects are observed, the local theories are right, otherwise, the global are, as their predictions are concerned.

Now, according to the local theory, the Aharonov-Bohm effect (Fig. 7) is due to the simple fact that $\frac{e\,\mathbf{A}}{h}$ **is added to the wave vector, on the phase line passing through one slit and substracted from the other phase line**, without putting forward any induction line surrounded by an integration path.

One can thus suggest the following variant of the Aharonov-Bohm experiment: if the solenoid between the slits is substituted with two parallel solenoids situated on both sides (Fig. 8), according to the local theory the effect subsists, according to the global theory the effect disappears.

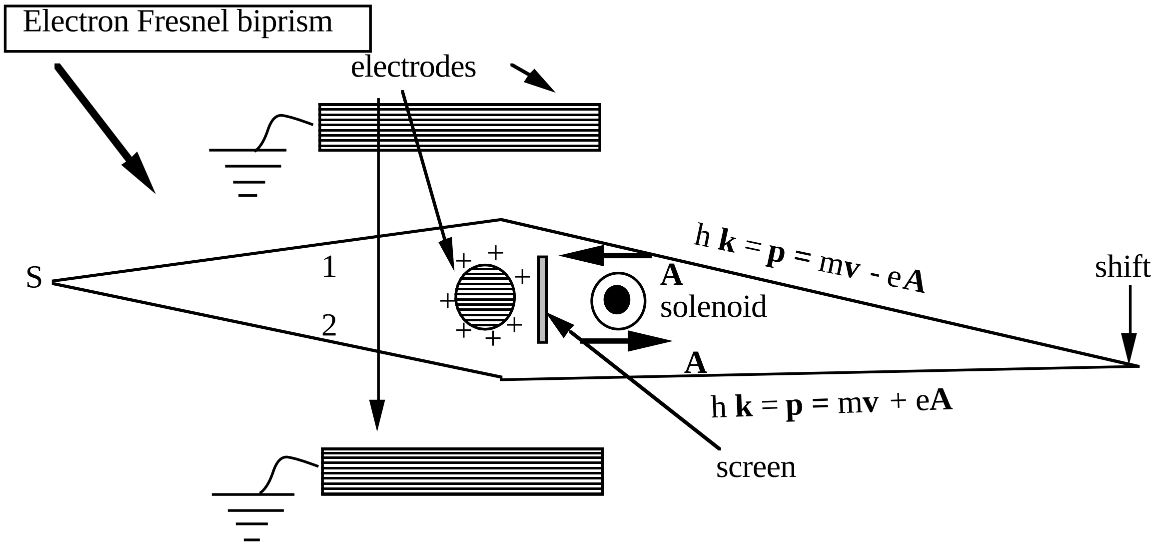

Fig. 7

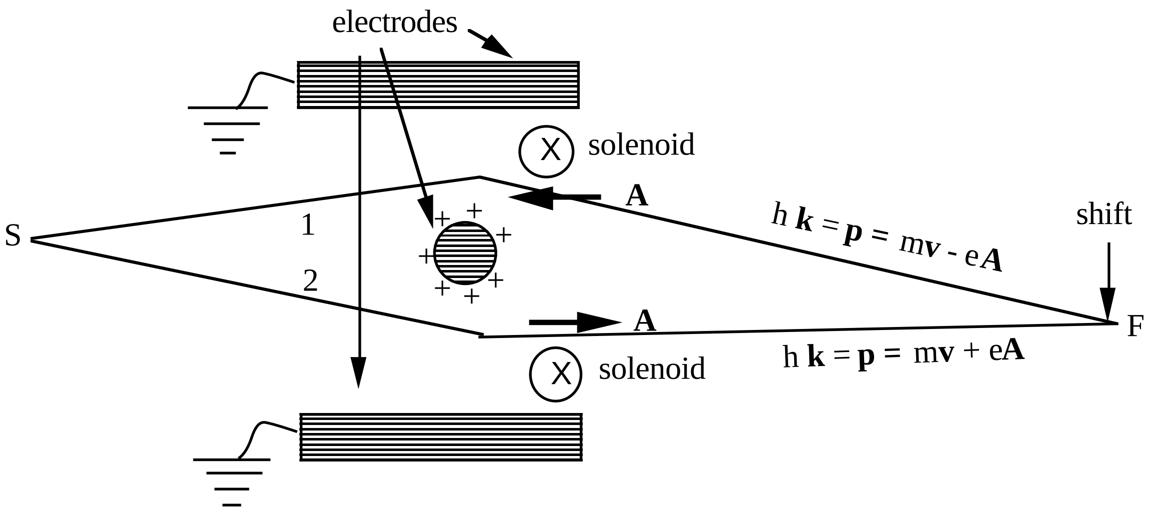

Fig. 8

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[1] Louis de Broglie, *Comptes rendus de l'Académie des Sciences*, **177**, 1923, p. 548.

[2] Louis de Broglie,

-*Thèse de 1924,* Annales de Physique, 10^{e} série, **III**, 1925 réimpression: Annales de la Fondation Louis de Broglie, **17**, 1992, p. 1.
- *Ondes et mouvements*, Gauthier-Villars, Paris, 1926 (réimpression: Jacques Gabay, Paris, 1988)
- *Optique électronique et corpusculaire*, Hermann, Paris, 1950

[3] In his *Thesis* , de Broglie proved this result as a consequence of Hamilton equations.

[4] R. Feynman, *The Feynman Lectures on physics*, vol. 2, Ch. 15, § 5.

[5] The vector **r** goes from a running point on the torus to the point where **A** is defined. The gradient is taken with respect to this last point. The point on the torus is a parameter.