# Phenomenological description of interactions by energy-dependent metrics

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ABSTRACT. We review the foundations and the basic laws of "deformed special relativity" (DSR). DSR is a generalization of the special theory of relativity, based on a "deformation" of the Minkowski metric, with parameters dependent on the energy of the physical system considered. Such a deformed metric realizes, for any interaction, the "solidarity principle" between interactions and spacetime geometry (usually assumed for gravitation), according to which the peculiar features of every interaction determine - locally - its own spacetime structure.The DSR formalism permits to approach the problem of the breakdown of the local Lorentz symmetry in a quite general way. In particular, it allowed us to derive, for all four fundamental interactions, the explicit forms of the related deformed metrics as functions of the energy,which provide an effective dynamical description of the interactions (at least in the energy range considered). DSR admits also of an interpretation in terms of a Kaluza-Klein-like scheme, with energy as fifth dimension.

## 1 Introduction

The main aim of this paper is to review the formalism of *Deformed Special Relativity (DSR),* together with its main implications.

DSR was introduced quite recently[(1−7)] essentially to dealing in a phenomenological way with a possible breakdown of local Lorentz invariance (LLI) in some physical processes. They are: the anomalous behaviour of the lifetime of the (weakly decaying) $K_s^0$ meson[(8)]; the Bose-Einstein correlation in (strong) pion production[(9)]; the superluminal photon tunneling[(10)] . All such phenomena seemingly show a (local) breakdown of Lorentz invariance and, therefore, an inadequacy of the Minkowski metric in describing them, *at different energy scales and for* *the three interactions involved* (electromagnetic, weak and strong)[2−7]. On the contrary, they apparently admit of a consistent interpretation in terms of a *deformed Minkowski space-time, i.e. endowed with a metric depending on the energy E of the process considered*[2−7] of the type

$$\eta(E) = \quad diag(b_0^2(E), -b_1^2(E), -b_2^2(E), -b_3^2(E)). \tag{1}$$

The metric (1) is supposed to hold locally, i.e. in the space-time region where the process occurs. Moreover, it is assumed to describe, at least in an effective way, the nonpotential forces acting on the physical system considered. Such nonpotential forces may be of electromagnetic, weak, or strong nature. In other words, the energy-dependent metric (1) is supposed to play a *dynamical* role. *It therefore provides a geometric description of the interaction considered*, thus realizing, for all four interactions, the so-called "solidarity principle", between space-time and interaction (so that the peculiar features of every interaction determine - locally - its own space-time structure), that - following B.Finzi[11] - can be stated as follows: "Space-time is solid with interactions, so that their respective properties affect mutually". Moreover, it was shown that also the experimental results on the slowing down of clocks in a gravitational field[12] can be described in terms of a deformed energy-dependent metric[13].

"Deformed Special Relativity" (DSR) is just the generalization of Special Relativity based on the metric (1). We recall that - apart from the above-quoted processes, which however provide only an *indirect* evidence for a possible LLI breaking - a newly proposed electromagnetic test of LLI seems indeed to yield a first *direct* evidence of LLI violation[14].

Moreover, it has been recently shown that the deformed Minkowski space with energy-dependent metric admits a natural embedding in a five-dimensional space-time, with energy as extra dimension[15].

The content of the paper is as follows. In order to reformulate SR for generalized energy-dependent metrics of the type (1.1), we start from the axiomatic formulation of SR, and the critical reexamination of its very foundations, given at the beginning of '70's by Recami and Mignani[16] (sect.2). The structure and properties of the deformed Minkowski space are discussed in sect.3. Sect.4 illustrates the realization of the "solidarity principle" in terms of energy-dependent deformed metrics. The basic postulates of DSR, and the explicit form of the generalized Lorentz transformations, are given in sect.5. Sect.6 yields the basic kinematical laws of DSR. The propagation of a wave in the deformed Minkowski space-time is discussed in sect.7, together with its new features and implications. The explicit form of the energy-dependent deformed metrics for all four fundamental interactions (derived from the analysis of the experimental data concerning the four processes quoted above) is given in sect.8. Sect.9 contains the main results of the five-dimensional Kaluza-Klein -like scheme, which seems to be at the very basis of DSR. Sect.10 concludes the paper.

## 2 An axiomatic view to Special Relativity

As is well known, Special Relativity (SR) is essentially grounded on the properties of space-time, i.e. isotropy of space and homogeneity of space and time (as a consequence of the equivalence of inertial frames) and on the Galilei principle of relativity.

Let us summarize the two basic postulates of SR in its axiomatic formulation(16):

1 - *Space-time properties*: Space and time are homogeneous and space is isotropic.

2 - *Principle of Relativity*: All physical laws must be covariant when passing from an inertial reference frame $K$ to another frame $K'$, moving with constant velocity relative to $K$.

In the second postulate it is clearly understood that, for a correct formulation of SR, it is *necessary* to specify the total class, $C_T$, of the physical phenomena to which the relativity principle applies. The importance of such a specification is easily seen if one thinks that, from an axiomatic viewpoint, the only difference between Galilean and einsteinian relativities just consists in the choice of $C_T$ (i.e. the class of mechanical phenomena in the former case, and of mechanical and electromagnetic phenomena in the latter).

Depending on the explicit choice of $C_T$, one gets a priori *different* realizations of the theory of relativity (in its abstract sense), each one embedded in the previous. Of course, the principle of relativity, together with the specification of the total class of phenomena considered, necessarily implies, for consistency, the uniqueness of the transformation equations connecting inertial reference frames.

It is possible to show that, from the above two postulates, there follow-without any additional hypothesis - all the usual "principles" of

SR, i.e. the "principle or reciprocity", the linearity of transformations between inertial frames, and the invariance of light speed in vacuum[(16)].

Concerning this last point, it can be shown in general that postulates 1 and 2 above imply the existence of an invariant, real quantity, having the dimensions of the square of a speed[(16)], whose value must be experimentally determined in the framework of the total class $C_T$ of the physical phenomena[1].Such an invariant speed depends on the interactions involved in the physical phenomena considered. Therefore *there is*, a priori, *an invariant speed for every interaction*, namely, a maximal causal speed for every interaction. In the following, we shall denote by $u$ this invariant, maximal causal speed, without any reference to the interaction concerned.

All the formal machinery of SR in the Einsteinian sense (including Lorentz transformations and their implications, and the metric structure of space-time) is simply a consequence of the above two postulates and of the choice, for the total class of physical phenomena $C_T$, of the class of mechanical and electromagnetic phenomena.

The attempt at including the class of nuclear and subnuclear phenomena in the total class of phenomena for which special relativity holds true leads to considering a generalization of Minkowski metric, analogously to the generalization from the euclidean to the Minkowski metric in going from mechanics to electrodynamics.

However, in order to avoid misunderstandings, we have to stress that such an analogy with the extension of the euclidean metric must be understood not in the purely geometric meaning, but rather in the sense (already stressed by Penrose[(17)]) of euclidean geometry as a *physical* theory.

Indeed, as explained above, the metric deformation is endowed with a *dynamical* character and is not only a consequence, but also an effective description of (the interaction involved in) the class of phenomena considered (principle of solidarity).

The mathematical tool to achieve this generalization is *a deformation* (in the sense specified later on) of the Minkowski metric. This implies, among the others, new, generalized transformation laws, which admit, as a suitable limit, the Lorentz transformations (just like Lorentz transformations represent a covering of the Galilei-Newton transformations).

[1]The invariant speed is obviously $\infty$ for Galilei's relativity, and $c$ (light speed in vacuum) for Einstein's relativity.

## 3 Deformed Minkowski space-time

As is well known,and we already stressed above, the Minkowski metric

$$g = diag(1, -1, -1, -1) \tag{2}$$

is a generalization of the euclidean metric $\epsilon = diag(1, 1, 1)$. On the basis of the above discussion, we assume that the metric describes, in an effective way, the interaction, and that there exist interactions more general than the electromagnetic ones (which, as well known, are long-range and derivable from a potential). Therefore, such interactions can, in general, be also nonlocal and not derivable from a potential.

The simplest generalization of the space-time metric which accounts for such more general properties of interactions is provided by a *deformation*, $\eta$, of the Minkowski metric (2), defined as

$$\eta = diag(b_0^2, -b_1^2, -b_2^2, -b_3^2). \tag{3}$$

Of course, from a formal point of view metric (3) is not new at all. Deformed Minkowski metrics of the same type have already been proposed in the past[(18−21)] in various physical frameworks, starting from Finsler's generalization of Riemannian geometry[(18)] to Bogoslowsky's anisotropic space-time[(19)] to Santilli's isotopic Minkowski space[(20)]. A phenomenological deformation of the type (3) was also obtained by Nielsen and Picek[(21)] in the context of the electroweak theory. Moreover, although for quite different purposes, "quantum" deformed Minkowski spaces have been also considered in the context of quantum groups[(22)]. Leaving to later considerations the true specification of the exact meaning of the deformed metric (3) in our framework, let us right now stress two basic points.

Firstly, metric (3) is supposed to hold at a *local* (and not global) scale[2]. We shall therefore refer to metric (3) as a "topical" deformed

[2]Here, the term "local" is to be understood in the sense that the deformed metric describes the geometry of a 4-dimensional manifold "attached" to a point $x$ of the spacetime, in the same sense as e.g. a local Lorentz frame (a standard, flat Minkowski space, or, in mathematical terms, a tangent space) is attached to a point of (globally Riemannian) spacetime in General Relativity. Another example, which is in some respects more similar to our formalism, is provided by a fiber bundle structure of spacetime , in which a Riemannian space of constant curvature is attached to every point $x$ (see ref. [23]).

metric, because it is supposed to be valid not everywhere, but only in a suitable (local) space-time region (characteristic of both the system and the interaction considered).

Secondly, as already stressed above, we do regard (3) as playing a *dynamical role*. So, in order to comply with the solidarity principle, we assume that the parameters $b_\mu (\mu = 0, 1, 2, 3)$ are, in general, real and positive functions of the observables characterizing the system (in particular, of its total energy, as specified later). However, for the moment we shall confine ourselves to discuss the deformation of the Minkowski space from a formal point of view, disregarding the problem of the observables on which the coefficients $b_\mu$ actually depend.

Notice that the first point, i.e. the assumed local validity of (3), differentiates our approach from those based on Finsler's geometry or from the Bogoslowski's one (which, at least in their standard meaning, do consider deformed metrics at a *global* scale), and makes it similar, on some aspects, to the philosophy and methods of the isotopic generalizations of Minkowski spaces. However, it is well known that Lie-isotopic theories rely in an essential way, from the mathematical standpoint, on (and are strictly characterized by) the very existence of the so-called isotopic unit[20]. In the following, we shall make no use at all of such a formal device, so that our formalism is *not* an isotopic one. Moreover, from a physical point of view, the isotopic formalism is expected to apply only to strong interactions[20]. On the contrary, we assume that the (effective) representation of interactions through the deformed metric (3) does hold for *all* kinds of interactions (at least for their nonlocal component). In spite of such basic differences our formalism shows some common formal results - as we shall see in the following - with Santilli's isotopic relativity (like the mathematical expression of the generalized Lorentz transformations).

We can now define a generalized ("deformed") Minkowski space $\tilde{M}(x, \eta, R)$ with the same local coordinates $x$ of $M$ (the four-vectors of the usual Minkowski space), but with metric given by the metric tensor $\eta$ (3). The generalized interval in $\tilde{M}$ is therefore given by $(x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$, with $c$ being the usual light speed in vacuum)

$$ds^2 = b_0^2 c^2 dt^2 - (b_1^2 dx^2 + b_2^2 dy^2 + b_3^2 dz^2) = \eta_{\mu\nu} dx^\mu dx^\nu = dx * dx. \quad (4)$$

The last step in (4) defines the scalar product $*$ in the deformed Minkowski space $\tilde{M}$.

In order to derive the expression of the maximal causal speed $u$ in the deformed Minkowski space, let us consider (for simplicity' sake and without loss of generality) an isotropic 3-dimensional space, i.e.

$$b_1 = b_2 = b_3 = b, \tag{5}$$

so that the corresponding deformed metric reads

$$\eta = diag(b_0^2, -b^2, -b^2, -b^2). \tag{6}$$

We get, for null separation $ds^2 = 0$:

$$b^2(dx^2 + dy^2 + dz^2) = b_0^2 c^2 dt^2 \tag{7}$$

$$\frac{dx^2 + dy^2 + dz^2}{dt^2} = \left(\frac{b_0}{b}\right)^2 c^2. \tag{8}$$

From eq. (8) it is easily seen that the quantity $\left(\frac{b_0}{b}\right) c$ is the maximal causal speed in the generalized Minkowski space[3] :

$$u = \left(\frac{b_0}{b}\right) c \tag{9}$$

In eq. (9), the value of $u$ is parametrized in terms of $c$, and depends on the physical system (and its interactions). Such a result is in agreement with an analogous finding by Coleman and Glashow, who introduced a "maximum attainable velocity" for different particles, in their recent discussion of possible Lorentz-breaking effects[(24)].

Moreover, it is

$$u \underset{<}{\overset{\geq}{=}} c \quad according\ to\ \frac{b_0}{b} \underset{<}{\overset{\geq}{=}} 1. \tag{10}$$

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[3]Let us remark that $u$ depends explicity on the metric parameters $b_\mu$ , which are a priori different for every physical system. However, since the deformation of the metric represents, on average, the effects of the nonlocal interactions involved, it is expected that physical systems with the same kind of interactions (besides the electromagnetic ones) are described by metric parameters of the same order of magnitude (or, at least, this holds true for the ratio $b_0/b$ ). *In this sense it is possible to refer to u as a "speed of interaction"*, rather than "speed of the physical system" considered (of course, at the same energy scale).

In other words, there may be maximal causal speeds *superluminal*, depending on the interaction considered. As is easy to see, this a straightforward consequence of the non-Minkowskian nature of the metric. Indeed, the speed of light is constant and equal to $c$ only with respect to the standard Minkowski metric. A well known example is provided by light propagation in a gravitational field, whose speed - due to the very Riemannian structure of the metric - is in general different from $c$ and dependent on space-time coordinates.

The maximal causal speed $u$ can be interpreted, from a physical standpoint, as the speed of the quanta of the interaction which requires a representation in terms of a generalized Minkowski space[4]. Therefore, such quanta must be zero-mass particles, in analogy with photons in the usual SR[5].

We recall that the deformation of the metric, resulting in the interval (4), represents a geometrization of a suitable space-time region (corresponding to the physical system considered) that describes, in the average, the effect of nonlocal interactions on a test particle. It is clear that there exist infinitely many deformations of the Minkowski space (precisely, $\infty^4$), corresponding to the different possible choices of the parameters $b_\mu$, a priori different for each physical system.

Moreover, since the usual, "flat" Minkowski metric $g$ (2) is related in an essential way to the electromagnetic interaction, we shall always mean in the following - unless otherwise specified - that electromagnetic interactions imply the presence of a fully Minkowskian metric[6].

Let us explicitly stress that the theory of SR based on the topical metric (3) has nothing to do with General Relativity. Indeed, in spite of the formal similarity between the interval (4), with the $b_\mu$ functions of the coordinates, and the metric structure of a Riemann space, in this framework no mention at all is made of the equivalence principle between

[4]Such a physical interpretation of the maximal causal speed in our framework is to be compared with the isotopic case, where $u$ has a quite different meaning.[(20)].

[5] However, the carriers of a given interaction propagating with the maximal causal speed $u$ typical of that interaction are expected to be strictly massless only inside the space whose metric is determined by the interaction considered. A priori, nothing forbids that such "deformed photons" may acquire a nonvanishing mass in a deformed Minkowski space related to a different interaction (this, in our opinion, might be the case of the massive bosons $W$ and $Z$ which carry the weak interaction)..

[6]Actually, a deformed metric of the type (3) is required if one wants to account for possible nonlocal electromagnetic effects (like those occurring in the superluminal photon tunneling: see refs.[4,7]).

mass and inertia, and among non-inertial, accelerated frames. Moreover, General Relativity describes geometrization on a large-scale basis, whereas the special relativity with topical deformed metric describes *local* (small-scale) deformations of the metric structure (although the term "small scale" must be referred to the real dimensions of the physical system considered[7]).

But the basic difference is provided by the fact that *actually the deformed Minkowski space $\tilde{M}$ has zero curvature*, as it is easily seen remembering that, in a Riemann space, the scalar curvature is constructed from the derivatives, with respect to space-time coordinates, of the metric tensor. In others words, the space $\tilde{M}$ is *intrinsically flat* - at least in a mathematical sense. Namely, it would be possible, in principle, to find a change of coordinates, or a rescaling of the lengths, so as to recover the usual Minkowski space. However, such a possibility is only a mathematical, and not a physical one. This is related to the fact that the *energy of the process is fixed, and cannot be changed at will.* For that value of the energy, the metric coefficients do possess values different from unity, so that the corresponding space $\tilde{M}$, for the given energy value, is actually different from the Minkowski one. The usual space-time $M$ is recovered for a special value $E_0$ of the energy (characteristic of any interaction), such that indeed

$$\eta(E_0) = g = (1, -1, -1, -1). \tag{11}$$

Actually, as already quoted in the introduction, the deformed Minkowski space $\tilde{M}$ can be regarded as a subspace of a five-dimensional, genuine Riemannian manifold, with energy as fifth dimension (see sect.9)$^{(15)}$.

## 4 Description of interactions by energy-dependent metrics

We want now to go into the question of the dependence of the metric parameters $b_\mu$ on the observables of the system, and to examine closely their physical meaning.

To this aim, let us start from the idea, of immediate physical evidence, that exchanging energy between particles amounts to measure operationally their space-time separation. Of course such a process depends on the interaction involved in the energy exchange; moreover, each exchange occurs at the maximal causal speed characteristic of the given

[7] For instance, in the gravitational case, "local" may well mean "in the neighborhood of Earth". See ref.[13].

interaction. It is therefore natural to assume that the measurement of distances, performed by the energy exchange according to a given interaction, realizes the "solidarity principle" between space-time and interactions at the microscopic scale. This allows us to identify the energy $E$ of a physical process as the observable on which the coefficients $b_\mu$ depend. Such an energy is to be understood as the energy measured by the detectors via their electromagnetic interaction in the usual Minkowski space, and must be therefore regarded as a merely phenomenological variable.

Thus, the distance measurement is accomplished by means of the metric tensor, given explicitly by

$$\eta(E) = \mathrm{diag}(b_0^2(E), -b_1^2(E), -b_2^2(E), -b_3^2(E)). \tag{12}$$

An interaction, even if not derivable from a potential, can be therefore phenomenologically described by metric (12).

We are therefore led to put forward a revision of the concept of "geometrization of an interaction": each interaction produces its own metric, formally expressed by the metric tensor (12), but realized via different choices of the set of parameters $b_\mu(E)$. In other words, the $b_\mu(E)$'s are peculiar to every given interaction. The statement that (12) provides us with a metric description of an interaction must be just understood in such a sense.

We stress since now that, in our formalism, $E$ is to be considered as a dynamical variable (on the same footing as the space-time coordinates), because it specifies the dynamical behavior of the process under consideration, and, via the metric coefficients, it provides us with a dynamical map - in the energy range of interest - of the interaction ruling the given process. Let's recall that the use of momentum components as dynamical variables on the same foot of the space-time ones can be traced back to Ingraham[25]. Moreover, Dirac[26], Hoyle and Narlikar[27] and Canuto et al.[28] treated mass as a dynamical variable in the context of scale-invariant theories of gravity. Such a point of view has been advocated also in the framework of modern Kaluza-Klein theories by the so-called "Space-Time-Mass" (STM) theory , in which the fifth dimension is the rest mass, proposed by Wesson[29] and studied in detail by a number of authors[30]. Moreover, the concept of energy as a dynamical variable was recently revived in a quite different framework[31].

Let us notice that metric (12) plays, for nonpotential interactions, a role analogous to that of the Hamiltonian $H$ for a potential interaction.

In particular, the metric tensor $\eta$ as well is not an input of the theory, but must be built up from the experimental knowledge of the physical data of the system concerned (in analogy with the specification of the Hamiltonian of a potential system). However, there are some differences between $\eta$ and $H$ worth to be stressed. Indeed, as is well known, $H$ represents the total energy $E_{tot}$ of the system irrespective of the value of $E_{tot}$ and the choice of the variables. On the contrary, $\eta(E)$ describes the variation in the measurements of space and time, in the physical system considered, as $E_{tot}$ changes; therefore, $\eta$ does depend on the numerical value of $H$, but not on its functional form. The explicit expression of $\eta$ depends only on the interaction involved.

We want moreover to recall that the use of an energy-dependent space-time metric can be traced back to Einstein himself, who generalized the Minkowski interval as follows

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right) c^2 dt^2 - (dx^2 + dy^2 + dz^2)$$

(where $\phi$ is the Newtonian gravitational potential), in order to account for the modified rate of a clock in presence of a gravitational field[(12)].

It is worth stressing that, due to the dependence on energy of the metric parameters $b_\mu$, the maximal causal speed of any interaction, too, is a function of the energy, according to the relation

$$u(E) = \frac{b_0(E)}{b(E)} c. \tag{13}$$

However, it remains invariant - for fixed energy values - under generalized Lorentz transformations from a given reference frame to another. Clearly, in eq.(13), the light speed in vacuum, $c$, does merely play the role of a phenomenological parameter on which the value of $u$ depends.

We do realize that one may be puzzled about the dependence of the metric on the energy, which is not an invariant under usual Lorentz transformations, but transforms like the time-component of a four vector[8]. Let us therefore briefly discuss the phenomenological aspects of such a dependence.

[8]Actually, energy is to be regarded, in our formalism, from two different points of view. We have, on one side, the energy as measured in full Minkowskian conditions, which, as such, behaves as a genuine four-vector *under usual Lorentz transformations (*in the sense that it changes in the usual way if we go, say, from the laboratory frame to another frame in uniform motion with respect to it). Once we fixed the frame,

We start noting that, in particle collisions, the energy in the laboratory frame, $E = E_L$, can be related to the invariant quantity $\sqrt{s}$ (with $s$ being, as is well known, the total energy of the colliding particles in their centre-of-mass system) by means of the relation, valid at sufficiently high energies

$$s \approx 2M\ E_L \quad , \tag{14}$$

where $M$ is the mass of the target at rest.[9]

Therefore, in particle collisions, the metric parameters can be indeed considered as dependent on the invariant quantity $\sqrt{s}$.[10] This interpretation is supported by the case of colliding beam reactions with different energies: indeed, it would be impossible, otherwise, to define what energy must be used as parameter. Such a point of view can be adopted e.g. in the analysis of the so called "ramping run" of UA1 in order to extract the parameters of the hadronic metric (as functions of the energy) from the experimental data [(3)].

In other cases, the phenomenological energy parameter may not so easy to identify, neither it can be directly related to an invariant quantity. This is the case, for instance, of the lifetime of unstable particles [(2)].

To end this section, let us notice that the deformation of the Minkowski metric (12) is expected to apply to the description not only of extended particles, but also of quantum pointlike particles, as far as

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we get *a measured value* of the energy for a given process (for instance, the energy of the Bose-Einstein correlation phenomenon in pion production, as measured at CERN by the UA1 collaboration). This is the value which enters, *as a parameter,* in the expression (12) of the *DSR* metric. Such an energy, therefore, *is no longer to be considered as a four vector in the deformed Minkowski space,* but it is just a quantity whose value determines the deformed geometry of the process considered (or, otherwhise speaking, which selects the deformed spacetime we have to use to describe the phenomenon). Let us stress that such a problem is overcome in what seems the natural framework of *DSR*, i.e. a five-dimensional Kaluza-Klein-like scheme[15]. In this context, the energy is regarded as a *bona fide* fifth dimension, on the same footing of the spacetime coordinates. In such a framework, one has to look for the isometries of the new 5-dimensional space, in order to account for the transformation property of the energy. Such a topic will be faced in our future work.

[9] Eq.(14) is analogous to the relation between the laboratory momentum $p_L$ and the invariant flux $\phi\ :\ \phi = p_L\ M$.

[10] Actually, $\sqrt{s}$ is a scalar under usual Lorentz transformations, and, in general, an usual relativistic invariant is no longer unchanged for transformations preserving the deformed interval (4). However, let us recall that the energy we are choosing as phenomenological parameter is that measured electromagnetically above the threshold of $5\mu eV$, and therefore in presence of a Minkowski metric.

their energy is such that one cannot neglect their associated cloud of virtual quanta.[11]

The problem of a metric description of a given interaction is thus formally reduced to the determination of the coefficients $b_\mu(E)$ from the data on some physical system, whose dynamical behaviour is ruled by the interaction considered.

To this aim, it is necessary to get the generalized Lorentz transformations.

## 5 Generalized principle of Relativity and Lorentz transformations

In order to develop the relativity theory in a deformed Minkowski space-time, we have to suitably generalize and clarify the basic concepts which are at the very foundation of SR.

Let us first of all define a "topical inertial frame":

i) a *topical "inertial" frame* is a reference frame in which space-time is homogeneous, but space is not necessarily isotropic.

Then, we can state a *"generalized principle of relativity"* (or *"principle of metric invariance"*) as follows:

ii) all physical measurements within every topical "inertial" frame must be carried out via the *same* metric.

We shall call *"Deformed Special Relativity"* (DSR) the generalization of SR based on the above two postulates, and whose space-time structure is given by the deformed Minkowski space $\tilde{M}$ introduced in sect.3.

It follows from the above points i), ii) that the transformation equations connecting topical "inertial" frames, i.e. the generalized Lorentz transformations, are those which leave invariant the metric when passing from a topical "inertial"frame $K$, to another frame $K'$, moving with constant velocity with respect to $K$. Then, physical laws are to be covariant with respect to such generalized transformations.

In other words, the generalized Lorentz transformations are the isometries of the deformed Minkowski space $\tilde{M}$. Their explicit form can be derived by the same procedure followed in order to find the Lorentz transformations in the usual Minkowski space. Assuming, for simplicity,

[11] L. Chiatti: private communication.

an isotropic three-space (see eq.(6)), we get, for a boost, say, along the $x$-axis:(1,6,19)

$$\begin{cases} x' = \tilde{\gamma}(x - vt); \\ y' = y; \\ z' = z; \\ t' = \tilde{\gamma}\left(t - \tilde{\beta}^2 \dfrac{x}{v}\right), \end{cases} \tag{15}$$

where $v$ is the relative speed of the reference frames, and(1,6)

$$\tilde{\beta} = \frac{v}{u} \; ; \tag{16}$$

$$\tilde{\gamma} = (1 - \tilde{\beta}^2)^{-1/2} \; . \tag{17}$$

Transformations (5.1) do formally coincide with the isotopic Lorentz transformations introduced by Santilli(19). However, in our context their physical meaning is *different*, as it is easily seen e.g. by the identification of the maximal causal speed $u$ with the speed characteristic of the quanta of a *given* interaction (see sect. 3). In particular, the parametrization (5.2) of the deformed velocity parameter $\tilde{\beta}$ in terms of $u$ (first introduced by us in ref.[1]) immediately shows that is always $\tilde{\beta} < 1$, so that $\tilde{\gamma}$ *never* takes imaginary values (contrarily to the isotopic case: see ref.[19]). Moreover, no reference at all is made, in our framework, to the existence of an underlying "medium".

Apparently eqs.(15) are asymmetrical in the behaviour of $x'$ and $t'$, unlike the usual Lorentz transformations, which are fully symmetric when putting $x^0 = ct$. However, such asymmetry is only formal. It can be removed by introducing, in analogy with the electromagnetic case, a time coordinate defined in terms of the maximal causal speed $u$ in the generalized Minkowski space considered:

$$x^0 = ut = \left(\frac{b_0}{b} c\right) t \tag{18}$$

and changing the metric tensor $\eta$ into

$$\eta' = diag(b^2, -b^2, -b^2, -b^2) = b^2 g \; . \tag{19}$$

Then, the generalized Lorentz transformations in $\tilde{M}'$ take the symmetrical form(1)

$$\begin{cases} x^{0\prime} = \tilde{\gamma}(x^0 - \tilde{\beta}x^1); \\ x^{1\prime} = \tilde{\gamma}(x^1 - \tilde{\beta}x^0); \\ x^{2\prime} = x^2; \\ x^{3\prime} = x^3. \end{cases} \tag{20}$$

It is easily seen that the deformed Minkowski spaces $\tilde{M}$ and $\tilde{M}'$, with metrics (6) and (19) respectively, are isometric, because they have the same interval (4). They are therefore fully equivalent in every respect, and it is therefore possible to use indifferently either transformations (15) or (20). The main advantage of the latter ones is that, due to relation (19), the formulae holding for $\tilde{M}'$ are immediately got from those of the standard special relativity by simply replacing everywhere $c$ by $u$.

It must be carefully noted that, like the metric, also the generalized Lorentz transformations depend on the energy. This means that one gets *different transformation laws* for *different* values of $E$, but still with the same functional dependence on the energy. As to the invariance of the deformed interval (4), it is always ensured (provided the process considered does always occur via the same interaction) by the fact (already stressed in sect. 4) that energy is to be regarded as a fixed parameter - whose value is determined by experimental measurements, performed via electromagnetic interactions in the usual Minkowski space -, which selects the particular deformed spacetime ruling the phenomenon under consideration[12].

Differentiating eqs.(15), we get therefore

$$dx' = \tilde{\gamma}(dx - \tilde{\beta}udt) + [d\tilde{\gamma}(x - \tilde{\beta}ut) - \tilde{\gamma}utd\tilde{\beta}] \quad ; \tag{21}$$

$$udt' = \tilde{\gamma}(udt - \tilde{\beta}dx) + [d\tilde{\gamma}(ut - \tilde{\beta}x) - \tilde{\gamma}xd\tilde{\beta}] \quad , \tag{22}$$

where, by the above arguments, $dE = 0$ and therefore $d\tilde{\gamma} = d\tilde{\beta} = 0$. Squaring (21),(22) and subtracting, we find

$$dx'^2 - u^2dt'^2 = \tilde{\gamma}^2[(dx - \tilde{\beta}dt)^2 - (udt - \tilde{\beta}dx)^2] = dx^2 - u^2dt^2 \tag{23}$$

[12] If one works in a quantum framework, the energy $E$ can be considered fixed also because, from such a point of view, energy can be transferred only by *finite* amounts. On the other side, in a classical Kaluza-Klein-like context, with energy as fifth dimension, selecting a given deformed 4-dimensional spacetime means to work on a special slice of the five-dimensional space at $E = const$ . See ref. [15].

where in the last step use has been made of eq.(17). Exploiting the explicit expression of $u$, eq.(9), one has finally

$$ds'^2 = ds^2, \tag{24}$$

i.e. the generalized Lorentz transformations (15) are actually the isometries of the deformed Minkowski space $\tilde{M}$, in spite of their dependence on the energy.

As already stressed in sect.3 (see eq.(10)), it is possible to have $u \geq c$ and therefore $c \leq v \leq u$, i.e. superluminal motions are allowed. Let us remark that the possibility of tachyonic speeds is accomplished, within this framework, without any recourse neither to imaginary quantities nor to singularities in the transformation laws (unlike the standard case(16)), because it is always $v \leq u$ (even if $v \geq c$), so that the relativistic factor $\tilde{\gamma}$ (eq. (17)) takes only real values, as already stressed above.

## 6 Relativistic kinematics in a deformed Minkowski space

From the knowledge of the generalized Lorentz transformations it is easy to derive the main kinematical and dynamical laws valid in DSR(1,6,19).

We just confine ourselves to list them:

- *Velocity composition law*:

$$V_{tot} = \frac{v_1 + v_2}{1 + \dfrac{v_1 v_2}{u^2}}, \tag{25}$$

which obviously for, say, $v_1 = u$ yields $V_{tot} = u$;[13]

[13] If we give up the condition of spatial isotropy, the composition law for motion, say, along the $x_k$-axis, becomes

$$V = \frac{v_1 + v_2}{1 + \dfrac{v_1 v_2}{u_k^2}}; \; u_k = \frac{cb_0}{b_k}$$

and, therefore, the speed that has an invariant character is

$$u_k = \frac{cb_0}{b_k}.$$

It follows that, in a given Minkowski space with deformed metric, there exist infinitely many different, maximal causal speeds, corresponding to the different possible directions of motion (although, of course, only three of them are independent). Clearly, this result is a strict consequence of the spatial anisotropy of the space-time region considered. Let us notice that there is indeed a phenomenon-the Bose-Einstein correlation-which can be fully described in the framework of such a Minkowski space, but with the consequence of a local loss of space isotropy(3).

- *Time dilation:*

$$\Delta t = \tilde{\gamma} \Delta t_0; \tag{26}$$

- *Length contraction:*

$$\Delta L = \tilde{\gamma}^{-1} \Delta L_0; \tag{27}$$

- *Four-velocity:*

$$V^\mu = \frac{d}{dt_0} x^\mu, \tag{28}$$

whose explicit form (with $dt_0$ derived from eq.(26)) reads

$$V^0 = \tilde{\gamma} u; \tag{29}$$

$$V^k = \tilde{\gamma} u^k. \tag{30}$$

Therefore, the generalized expression of the momentum fourvector is

$$p^\mu = m_0 V^\mu = (m_0 \tilde{\gamma} u, m_0 \tilde{\gamma} v^k). \tag{31}$$

Lastly, let us consider a plane wave propagating with speed $u$ (e.g. in the $xy$ plane, at angles $\theta, \theta'$ in frames $K, K'$) with dispersion relation $u = \lambda\nu = \lambda'\nu'$, where $\nu, \nu'$ are the wave frequencies in $K, K'$. Applying the generalized Lorentz transformations, it is easy to get the following laws:

- *Doppler effect:*

$$\nu = \hat{\gamma} \nu' (1 + \hat{\beta} \cos\theta'); \tag{32}$$

- *Aberration law:*

$$tg\theta = \frac{\sin\theta'}{\hat{\gamma}(\hat{\beta} + \cos\theta')}. \tag{33}$$

We want now to provide a comparison between the main kinematical laws in the usual Minkowski space and in the deformed one (in the hypothesis of spatial isotropy), because their different behaviours may help us to understand the peculiar features of leptonic and hadronic interactions with respect to the electromagnetic one. Such laws are summed up in Table I.

**Table I**

| *Minkowski space* | *Deformed Minkowski space* |
|---|---|
| $V_{tot} = \dfrac{v_1 + v_2}{1 + \dfrac{v_1 v_2}{c^2}}$ | $V_{tot} = \dfrac{v_1 + v_2}{1 + \left(\dfrac{b}{b_0}\right)^2 \dfrac{v_1 v_2}{c^2}}$ |
| $\Delta t = \dfrac{\Delta t_0}{\left(1 - \dfrac{v^2}{c^2}\right)^{1/2}}$ | $\Delta t = \dfrac{\Delta t_0}{\left[1 - \left(\dfrac{b}{b_0}\right)^2 \dfrac{v^2}{c^2}\right]^{1/2}}$ |
| $\Delta L = \Delta L_0 \left(1 - \dfrac{v^2}{c^2}\right)^{1/2}$ | $\Delta L = \Delta L_0 \left[1 - \left(\dfrac{b}{b_0}\right)^2 \dfrac{v^2}{c^2}\right]^{1/2}$ |

We have expressed $u$ in terms of $c$, in order to emphasize the dependence of the topical laws on the parameter ratio $b/b_0$ and exhibit their scale invariance. In the limiting case $v = c$, we explicitly get

$$v_1 = c \qquad V_{tot} = \frac{c + v_2}{1 + \left(\dfrac{b}{b_0}\right)^2 \dfrac{v_2}{c}}; \tag{34}$$

$$v = c \qquad \Delta t = \frac{\Delta t_0}{\left[1 - \left(\dfrac{b}{b_0}\right)^2\right]^{1/2}}; \tag{35}$$

$$v = c \qquad \Delta L = \Delta L_0 \left[1 - \left(\frac{b}{b_0}\right)^2\right]^{1/2}. \tag{36}$$

We recall that, in this framework, $c$ has lost its meaning of maximal causal speed, by preserving the mere role of maximal causal speed for electromagnetic phenomena in the usual Minkowski space.

To the purpose of an experimental verification, it is worth to express the topical kinematical laws of time dilation and length contraction for a particle of rest mass $m_0$ in terms of the *usual* energy $E$. Clearly, for $E \gg m_0 c^2$, $E$ can be considered the total energy of the particle, measured (as already stressed) by electromagnetic methods in the usual Minkowski space. We report such laws in Table II (in comparison with the standard, Einsteinian ones)[1,6] :

**Table II**

| *Minkowski space* | *Deformed Minkowski space* |
|---|---|
| $\Delta t = \Delta t_0 \frac{E}{m_0}$ | $\Delta t = \frac{\Delta t_0}{\left[1 - \left(\frac{b}{b_o}\right)^2 + \left(\frac{b}{b_o}\right)^2 \left(\frac{m_0}{E}\right)^2\right]^{1/2}}$ |
| $\Delta L = \Delta L_0 \frac{m_0}{E}$ | $\Delta L = \Delta L_0 \left[1 - \left(\frac{b}{b_o}\right)^2 + \left(\frac{b}{b_o}\right)^2 \left(\frac{m_0}{E}\right)^2\right]^{1/2}$ |

It is easily seen that, in the case of the time-dilation law, the main difference is the loss of linearity in the dependence on the energy of the topical law, as compared to the Lorentzian one. Such a different behaviour is therefore a clear signature of the presence of nonlocal effects in the interaction considered. A first, although preliminary, evidence is provided by the lifetime of the meson $K_s^0$ in the range $3 - 400 GeV$[2].

## 7 Wave propagation in a deformed space-time

We want now to approach the problem of wave propagation in a deformed Minkowski space-time[4,6,7]. To this end, let us introduce the generalized d'Alembert operator $\tilde{\Box}$, defined by means of the scalar product $*$ in $\tilde{M}$ (see eq.(4)):

$$\tilde{\Box} \equiv \partial * \partial = \eta_{\mu\nu}\partial^\mu \partial^\nu = \frac{b_0^2}{c^2}\partial_t^2 - (b_1^2\partial_x^2 + b_2^2\partial_y^2 + b_3^2\partial_z^2) \qquad (37)$$

Therefore, the generalized Helmholtz-D'Alembert wave equation is given by

$$\tilde{\Box} f = 0 \qquad (38)$$

with $f$ being any component of the field associated to the wave considered. For instance, the field of such a wave propagating in the Minkowski space $\tilde{M}$ can be written as

$$\mathbf{f}(x) = \mathbf{A}(\mathbf{x}) e^{ik*x} \qquad (39)$$

where $k$ is the wavevector and $e^{ik*x}$ is the generalized phase.

Assuming a topical metric spatially isotropic (see eq. (6)), in the corresponding deformed space-time the generalized phase takes the "minkowskian-like" form $e^{i\tilde{k}\cdot x}$ (where the dot denotes the usual scalar product in the Minkowski space), with

$$\tilde{k}^{\mu} = \left( \frac{2\pi\nu}{c}, \tilde{k_x}, \tilde{k_y}, \tilde{k_z} \right) \tag{40}$$

and $\nu$ is the frequency measured in the ordinary space-time. Then, eq. (39) becomes

$$\mathbf{f}(x) = \mathbf{A}(\mathbf{x}) e^{i\tilde{k}\cdot x}. \tag{41}$$

As an application of the previous results leading to interesting physical consequences, we want now to discuss the case of an electromagnetic (e.m.) field propagating in a conducting waveguide, with section of width $a$, in the principal transversal electric mode (commonly denoted $TE_{10}$). If the guide axis coincides with the $z$-axis, we can put $E_x = E_z = 0$, and the only nonvanishing component $E_y$ can be written as

$$E_y(x, z, t) = g(x) e^{\,i(\tilde{k}_z z - \omega t)}. \tag{42}$$

Therefore the generalized Helmholtz equation (38) for $E_y$ becomes

$$b^2 \partial_x^2 E_y + b^2 \partial_z^2 E_y - \frac{b_0^2}{c^2} \partial_t^2 E_y = 0 \tag{43}$$

or

$$b^2 \partial_x^2 g - b^2 \tilde{k}_z^2 g + \frac{b_0^2}{c^2} (2\pi\nu)^2 g = 0. \tag{44}$$

This last equation for $g(x)$ has solution

$$g(x) = C_1 \cos(\tilde{k}_x x) + C_2 \sin(\tilde{k}_x x) \tag{45}$$

with $C_1, C_2$ constants. Imposing the boundary conditions on $E_y$:

$$E_y = 0, x = 0, a \tag{46}$$

it is obviously

$$g(x) = 0, x = 0, a \tag{47}$$

so that $C_1 = 0, C_2 \neq 0$ and

$$\tilde{k}_x = \frac{\pi}{a} m, \quad m = 1, 2, ... \tag{48}$$

Replacing (45) and (48) in eq. (44), since $m = 1$ in the $TE_{10}$ mode, we get

$$-b^2 \left(\frac{\pi}{a}\right)^2 - b^2 \tilde{k}_z^2 + b_0^2 \left(\frac{2\pi}{c}\right)^2 \nu^2 = 0. \tag{49}$$

Such a relation gives us the wavevector component $\tilde{k}_z$ along the propagation direction in a deformed Minkowski space with metric (6). Putting

$$\frac{\pi}{a} = \frac{2\pi}{c} \nu_c, \tag{50}$$

where $\nu_c$ is the cutoff frequency of the waveguide, one finds from (49) the explicit expression of $\tilde{k}_z^2$:

$$\tilde{k}_z^2 = \left(\frac{2\pi}{c}\right)^2 \nu^2 \left[\left(\frac{b_0}{b}\right)^2 - \left(\frac{\nu_c}{\nu}\right)^2\right]. \tag{51}$$

In the case of a Minkowskian metric ($b_0^2 = b^2 = 1$), eq.(51) is nothing but the usual relation for guided e.m. waves. The condition of actual propagation is obviously given by $\nu > \nu_c$ (high-pass filter), whereas for $\nu < \nu_c$ there is no propagation in a strict sense, but only an evanescent mode with $\tilde{k}_z^2 < 0$ (imaginary wavevector).

On the contrary, in the case of a deformed space-time, the condition of actual propagation depends in an essential way on the ratio $b_0^2/b^2$. Indeed, even for $\nu < \nu_c$ it is possible to have $\tilde{k}_z^2 > 0$, provided relation

$$\left(\frac{b_0}{b}\right)^2 > \left(\frac{\nu_c}{\nu}\right)^2 \tag{52}$$

holds.

This defines the validity conditions of (38) and then of (51). Moreover, since $(\nu_c/\nu) > 1$, it follows $(b_0/b) > 1$, namely, by eq. (10), $u > c$. Relation (52) can be therefore regarded as the condition of propagation for a wave with *real* wavevector, and *superluminal speed*, in the deformed Minkowski space, in correspondence to frequency values ($\nu < \nu_c$) for which, in the usual Minkowski space, the wavevector is imaginary, and the related e.m. wave is therefore in an evanescent mode. Otherwise speaking, for suitable values of the ratio $b_0/b$, and in suitable space-time regions, an evanescent e.m. wave in the usual space can be interpreted as an actual wave propagating, with superluminal speed, in the deformed Minkowski space.

Such a (classical) interpretation of an imaginary wavevector can be straightforwardly extended to the quantum case (which amounts therefore to describing virtual photons as tachyonic "photons" with speed $u > c$). Let us notice that, in this framework, the wave speed is already fixed by the topical metric (6), and *not* by the Helmholtz equation (38), which yields only the propagation conditions in a space with generalized metric $\eta$ and maximal causal speed $u$ (which, as stressed at the end of section 3, is the speed of the "photons" associated to the interaction described by the given metric).

In conclusion, let us notice that the choice of the example used to discuss the generalized Helmholtz-D'Alembert equation (38) is due to two reasons. First, this allowed us to provide a concrete case of metric description of electromagnetic interaction in non-Minkowskian conditions[4,6,7]. Moreover, the condition $\nu < \nu_c$ in a waveguide corresponds, in optics, to light transmission by a partially reflecting mirror, and, in quantum mechanics, to the crossing of a barrier potential by tunnel effect. The latter analogy suggests a possible (although highly speculative) interpretation of quantum tunneling as a wave propagation in a deformed space-time, whose metric is determined by the interaction generating the barrier, with coefficients $b_\mu$ evaluated at that energy value corresponding on average to the barrier height. Of course, such a metric would have to be restricted to the space region occupied by the barrier.

Let us also stress that, by Fourier-expanding the evanescent wave inside the barrier as

$$e^{-\chi z} = \sum_{n=-\infty}^{\infty} c_n e^{in2\pi z/L}, \tag{53}$$

it is possible to describe the deformation of the space-time inside the barrier region in terms of an *effective metric tensor*

$$\bar{\eta}_{\mu\nu}(c_n) = \frac{\sum_n |c_n|^2 \eta_{\mu\nu}^{(n)}}{\sum_n |c_n|^2}, \tag{54}$$

where $\eta^{(n)}_{\mu\nu}$ is the deformed metric "seen" by the n-th Fourier component of the evanescent wave. Definition (54) is analogous to the well-known Cauchy-stress tensor for a deformable medium. We refer the reader to ref.[4].

## 8 Energy-dependent phenomenological metrics for the four interactions

As far as the phenomenology is concerned, we recall that a local breakdown of Lorentz invariance may be envisaged for all four fundamental interactions (electromagnetic, weak, strong and gravitational) whereby *one gets evidence for a departure of the space-time metric from the Minkowskian one* (at least in the energy range examined). The experimental data analyzed were those of the four physical processes mentioned in the introduction, i.e. the lifetime of the (weakly decaying) $K^0_s$ meson(21); the Bose-Einstein correlation in (strong) pion production(22); the superluminal photon tunneling(23); the comparison of clock rates in the gravitational field of Earth. A detailed derivation and discussion of the energy-dependent phenomenological metrics for all the four interactions can be found in refs. [1-6]. Here, we confine ourselves to recall their following basic features[14]:

1) Both the electromagnetic and the weak metric show the same functional behavior, namely(4,6)

$$\eta(E) = diag(1, -b^2(E), -b^2(E), -b^2(E)); \tag{55}$$

$$b^2(E) = \begin{cases} (E/E_0)^{1/3}, & 0 < E \le E_0 \\ 1, & E_0 \le E \end{cases} \tag{56}$$

with the only difference between them being the threshold energy $E_0$, i.e. the energy value at which the metric parameters are constant, i.e. the metric becomes Minkowskian; the fits to the experimental data yield

$$E_{0el} = 5.0 \pm 0.2 \mu eV \; ; \; E_{0w} = 80.4 \pm 0.2 GeV; \tag{57}$$

2) for the strong interaction, the metric reads:

[14] In all the expressions of the metrics reported below, we assume $E \neq 0$ . Indeed, formally, some metrics become degenerate for $E = 0$. Moreover, actually it does not makes sense, from a physical point of view, to consider a vanishing energy in our framework, because no physical process at all does take place at zero energy.

$$\eta(E) = diag(b_0^2(E), -b_1^2(E), -b_2^2(E), -b_3^2(E)); \tag{58}$$

$$b_0^2(E) = b_3^2(E) = \begin{cases} 1, & 0 < E \leq E_{0s} \\ (E/E_{0s})^2, & E_{0s} \leq E \end{cases} \tag{59}$$

with

$$E_{0s} = 367.5 \pm 0.4 GeV. \tag{60}$$

Let us stress that, in this case, contrarily to the electromagnetic and the weak ones, *a deformation of the time coordinate occurs*; moreover, *the three-space is anisotropic,* with two spatial parameters constant (but different in value) and the third one variable with energy in an "*over-Minkowskian*" *way*(3,6).

3) in the gravitational case, the form of the metric is(13):

$$\eta(E) = diag(b_0^2(E), -b_1^2(E), -b_2^2(E), -b_3^2(E)); \tag{61}$$

$$b_0^2(E) = b_3^2(E) = \begin{cases} 1, & 0 < E \leq E_{0grav} \\ (1 + E/E_{0grav})^2, & E_{0grav} \leq E \end{cases} \tag{62}$$

with

$$E_{0grav} = 20.2 \pm 0.1 \mu eV. \tag{63}$$

Intriguingly enough, this is approximately of the same order of magnitude of the thermal energy corresponding to the $2.7^o K$ cosmic background radiation in the Universe. Analogously to the strong case, the gravitational metric (8.6), (8.7) is time-deformed and over-Minkowskian. The values of the threshold energies for the electromagnetic, weak, gravitational and strong interactions, given by eqs. (57), (60), (63), can be ordered as follows:

$$E_{0el} < E_{0grav} < E_{0w} < E_{0s} \tag{64}$$

i.e. we have an increasing arrangement of $E_0$ from the electromagnetic to the strong interaction. Moreover

$$\frac{E_{0grav}}{E_{oel}} = 4.49 \pm 0.02; \quad \frac{E_{0s}}{E_{0w}} = 4.57 \pm 0.01, \tag{65}$$

namely

$$\frac{E_{0grav}}{E_{oel}} \simeq \frac{E_{0s}}{E_{0w}} \tag{66}$$

an intriguing result indeed.

## 9 Five-dimensional relativity with energy as fifth dimension

In conclusion, we want to briefly discuss the five-dimensional formalism, seemingly underlying DSR[15].

It is easily seen, from the examination of the phenomenological metrics considered in the previous section, that, in the formalism of the deformed Minkowski space, energy does play a *dual* role. Indeed, on one side, $E$ is to be considered as a true dynamical variable (as already stressed in sect.4). On the other hand, *a fixed value* of the energy determines the space-time structure of the interaction region for the given process *at that given energy.* In this respect, therefore, $E$ is to be regarded as a *geometrical quantity*, intimately connected to the very geometrical structure of the physical world itself. The simplest way of taking into account such a double role of $E$ *is to assume that energy does in fact represent an extra dimension* -besides the space and the times ones - *namely, to embed the deformed Minkowski space-time* $\tilde{M}$ *in a larger, five-dimensional space* $\Re$[15].

Let us specify the metric structure of the five-dimensional space $\Re$. We assume that the generalized interval in $\Re$ is given by[15]

$$\begin{aligned} ds^2_{(5)} &\equiv b_0^2(E)c^2dt^2 - b_1^2(E)dx^2 - b_2^2(E)dy^2 - b_3^2(E)dz^2 + f(E)\ell_0^2dE^2 = \\ &= \eta_{\mu\nu}(E)dx^\mu dx^\nu + f(E)(dx^4)^2 \equiv g_{AB}(E)dx^A dx^B, \end{aligned} \tag{67}$$

where A,B = 0,1,2,3,4, $x^4 \equiv \ell_0 E$, with $\ell_0$ being a constant with suitable dimensions (for instance, if $\ell_0$ is a length the fifth coefficient $f(E)$ has dimension (for [energy]$^{-2}$) while the functions $b_\mu^2(E)$ are positive. The five-dimensional metric tensor $g^{(5)}$ reads therefore

$$g^{(5)} = diag\,(b_0^2, -b_1^2, -b_2^2, -b_3^2, f) \tag{68}$$

*and is a function of the energy*: $g^{(5)} = g^{(5)}(E)$.

In analogy with the space-time metric coefficients $b_\mu$, one assumes that also the fifth coefficient depends only on the energy: $f = f(E)$. Moreover, *a priori* $E$ can be considered either as a timelike or a spacelike coordinate in $\Re$; in other words, we may have either f $> 0$ or f $< 0$. Actually, in the standard Kaluza-Klein scheme, the fifth-dimension must necessarily be spacelike, because the number of time-like dimension cannot exceed one, if one wants to avoid causal anomalies. But - and this is just another point worth stressing - *this five-dimensional scheme is not a "true" Kaluza-Klein one*, due to the fact that the four-dimensional space-time is endowed with the deformed metric (3). It is therefore an open issue whether or not, in such a framework, more timelike dimensions do give rise to causal anomalies.

The vacuum Einstein equations in the space $\Re$ are

$$R_{AB} - \frac{1}{2} g^{(5)}_{AB} R = \Lambda g^{(5)}_{AB} \tag{69}$$

where $R_{AB}$ and $R = R^A_A$ are the five-dimensional Ricci tensor and scalar curvature, respectively, and $\Lambda$ is the "cosmological" constant, which may, in principle, depend on both the energy $E$ and the space-time coordinate $x$: $\Lambda = \Lambda(x, E)$.

The important point to be stressed is that *the formalism of the five-dimensional, deformed Kaluza-Klein scheme, with energy as fifth dimension, permits to recover, as solutions of the vacuum Einstein equations* (i.e. with $\Lambda = 0$), *all the phenomenological energy-dependent metrics of the electromagnetic, weak, strong and gravitational type*(15).

Work is in progress on such a formalism(15).

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