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"La 'théorie' de Bell, est-elle la plus grande méprise de l'histoire de la physique?"

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ABSTRACT. The fundamental error in Bell's Theorem is presented in its raw, most easily recognised form. In addition, an error in the Bell-Kochen-Specker theorem is exposed; and, a realistic local model for EPR correlations is presented.

RÉSUMÉ: *L'erreur fondamentale dans le théorème de Bell est présentée de façon plus élémentaire et reconnaissable. En plus, une erreur dans le théorème de Bell, Kochen et Specker est exposée, et une formulation réaliste et causale des corrélations EPR est proposée*

1 Introduction

The above title is taken directly from an article by D. Canals-Frau appearing in this journal in 1991.[1] This question is stylistically and morally legitimate because the answer is: *Oui*—a startlingly significant result! Nevertheless, that article is part of a critical literature[2]-[8], to which this writer has also contributed[9]-[11], which, while unrefuted, still has had next to no impact in the greater physics community. There may be several reasons for this but certainly one of the most potent is that a majority of eminent physicists have committed themselves publicly to the validity of Bell's analysis. Some even have praised it as extraordinarily beautiful or the harbinger of subtle, deep truth accessible only to those of particular intellectual alacrity. To now accede that in fact it was just an error, will reveal such pompous pronouncements as symptomatic of the unscientific enthusiasm for the preternatural and excessive appetite for approbation characterising the *Zeitgeist* of the profession.

At the same time, it must be acknowledged that the arguments in the critical literature on Bell's 'Theorem' have focused on what can be called derivative errors. Fundamental errors leave a froth of secondary, consequential errors, misunderstandings and obscurities in their wake. Although many of these sequential errors unambiguously and rigorously establish the existence of a lacuna in the logic of the proposition at hand, frequently pointing them out fails to convince, especially those of a conformist yen. Thus, it should be of use to render a fundamental error in its native, raw form, that is to say, in that form in which the offending hypothesis is made explicitly in a minimal context. It is one point of this article to reformulate Bell's analysis so as to expose its error in its fundamental form.

2 The fundamental error in Bell's analysis

Bell's Theorem is the current stroke in a conceptual volley that began with de Broglie's proposal for interpreting undulatory behaviour of matter. His 'double solution' envisioned the ontological essence of a particle to be a material kernel embedded somehow in a wave which guides the particle as a pilot.[12] If this imagery is accepted, then the coordinates of the kernel can be considered 'hidden variables' which a theory underlying quantum mechanics (QM) should be able to specify. Taking some liberty with historical details, it can be said that the famous Einstein, Podolsky and Rosen paper gave an argument in favour of the existence of such hidden variables.[13] Responding (at least conceptually if not factually) to this concept, John von Neumann presented a "no-go" theorem proving that the existence of hidden variables (at least those having nice properties) is inconsistent with the structure of QM.[14] In turn, de Broglie and then Bell found a lacuna in von Neumann's argument, namely, an irrelevant supposition. (Actually, it was found first by one unheralded Greta Hermann.[15]) Finally, Bell formulated a new theorem which he believed should allow an experimental test to settle the issue. His 'theorem' provides certain inequalities which, per Bell, must be respected by coincidence probabilities if they are extractable from a local, objective underlying theory. All tests have failed, indicating, ostensibly, that such an underlying theory is impossible.[16]-[19] An incidental result of Bell's analysis is essentially what appears to be proof that the reason an extension (with nice properties) is not possible, is that QM is intrinsically and ineluctably nonlocal.

Bell's Theorem is explicitly formulated so as to be testable by the Einstein-Podolsky-Rosen (EPR) experiment as modified by Bohm (EPRB).[20] This experiment considers the coincidences to be obtained when measuring the symmetric twin emissions of certain objects (usually photons or electrons with spin are considered, but the actual objects are immaterial) when they have the following property: regardless of how $\hat{x}$ is chosen, but perpendicular to the line of flight, if one of the twins is passed by a polariser in the $\hat{x}$ direction, the other will be blocked in this direction. Alternately, if one has spin 'up' in this direction, the other will have spin 'down.' Only when they are measured with respect to non-parallel directions, is this perfect anticorrelation broken. Indisputably, however, these twin objects are created with *some* correlation.

The question now is: what is the correlation when the measuring axes differ by an angle ϕ? With a simple calculation QM provides the answer: $\cos^2(k\phi)/2$ ($k = 1$ for photons, $1/2$ for electrons) which has been verified by experiment, while not beyond all argument ("accidentals," geometry and the like complicate reality [21]), but to general satisfaction.

Now, if an objective local extension of QM as envisioned by de Broglie, Einstein and others is to be found, it must duplicate this result. Bell argued that such an extended theory also must be consistent with probability theory, and on this basis asserted that its coincidence correlation be of the form:

$$P(a,b) = \int P(a,\lambda)P(b,\lambda)P(\lambda)d\lambda, \tag{1}$$

where it was taken that some heretofore 'hidden' variables, λ, specify the probabilities of the outcomes at the measuring stations A and B according to an objective and local extended theory whose average, given the density $P(\lambda)$, goes to QM. Locality is enforced, Bell supposed, by demanding that the probabilities of outcomes at station A, namely $P(a,\lambda)$, not depend on settings b at station B, and vise versa. In other words, the coincidence probability must simplify to the form of the integrand of Eq. (1). On the basis of Eq. (1), Bell, and subsequently others with modifications to accommodate comparison with experiments, derived the celebrated inequalities that the relevant coincidence probabilities derived from an underlying theory must satisfy.[20]

These considerations, however plausible they may sound, are not entirely in accord with basic probability theory. The general form of a coincidence correlation, or coincidence coefficient is:

$$Cor(A, B) = \frac{< |AB| > - < A >< B >}{\sqrt{< A^2 >< B^2 >}}. \tag{2}$$

when $< A >$ or $< B >$ equals zero and $< A^2 >=< B^2 >= 1$, then $Cor(A, B)$ reduces to the expression used by Bell. In this case the derivation of Bell inequalities goes through just as he and others presented it. However, if these conditions are not met, then the resulting inequalities take on another form; e.,g., the four-setting version becomes:

$$|P(A,\, B) - P(A,\, B')| + |P(A', B') + P(A', B)| \leq 2 + \frac{2 < A >< B >}{\sqrt{< A^2 >< B^2 >}}. \tag{3}$$

Does this matter? The sequences generated by EPR experiments are considered to be dichotomic and comprised of ±1's for which the averages are zero. In reality, however, photoelectrons are generated in proportion to the square of an electric field, a positive entity which constitutes the random variable having physical significance. Thus, for example, if the evoking electric field is taken to be proportional to $\cos^2(x)$, then $< A >= 1/2$ and $< A^2 >= 3/8$, so that the rhs of Eq. (3) becomes 10/3. (This simple model does not faithfully represent the EPR situation, however. For that see below.)

At the same time, note that according to Malus' Law, the relative intensity of different polarisation states is proportional to $\cos^2(\delta)$, where δ is the angle between the two polarisation states. As such, it is also proportional to the conditional probability of a photoelectron detection at the second polariser *given* that a detection has occurred at the first. Thus the probability of a coincident detection would be:

$$P(+,+) = \int cos^2(a-\theta)cos^2(a-b)d\theta = \frac{1}{2}cos^2(a-b) \tag{4}$$

where the integration over θ represents an average over all possible polarisation angles of the signal. (In the EPR setup, of course, asymmetry is considered, giving $\sin^2(a-b)/2$.) This result is fully in accord with the result from QM![22]

In view of all the above, it is natural to ask: where does Bell's analysis go wrong. On a fundamental level, Bell began by considering the correlation of two dichotomic random variables that were to represent spin measurements. As experiments on polarisation are less difficult and the models describing these phenomena are isomorphic (modulo a factor of 1/2), the fundamental launch point for Bell's analysis became Eq. (1) as applied to the symmetric but unpolarised emission of 'photons.' This expression implies that the coincidence probability must likewise factor because the coincidence is made up sums and differences of these probabilities. Thus, at the core of Bell's analysis, the fundamental item under consideration, is a coincidence probability which is dependent on three sets of variables, $P(a, b, \lambda)$. As a rule, such probabilities can be decomposed as follows:

$$P(a, b, \lambda) = P(\lambda)P(a|\lambda)P(b|a, \lambda), \tag{5}$$

where, $P(a|\,\lambda)$ is the *conditional probability* of a contingent on λ.[23] Note that at this point nothing has been added or assumed; Eq. (5) is a simple tautology that follows from basic definitions. It is always valid.

Exactly at this point Bell injected his physical arguments. In hypothesizing that in order to accommodate the requirement for local or causal interaction, the joint probability for the measurement events at stations A and B should take the form of the integrand in Eq. (1), he effectively (but evidently unknowingly while mislead by faulty notation) also assumed that

$$P(b|a, \lambda) \equiv P(b, \lambda) \tag{6}$$

holds. This can be so, however, only when the events are *statistically independent* with respect to all settings of the twin measuring device. That is, contrary to the fundamental assumption of EPR, Eqs. (1, 6) are true only when the twins are uncorrelated with respect to all settings of the measuring devices!

It is essential that one keep in mind that the issue here is *coincident* not independent events. In other words, the event space consists of *correlated pairs*, not individual detections. If at this juncture, one's view point is shifted, and single detections are considered, then it is possible to say that both detections must be fully independent of each other so that Eq. (6) makes sense. However, when the event space is correlated pairs, simple examples show that Eq. (6) does not hold. Consider for example, Bell's famous example, Professor Bertelmann's proclivity to wear unmatched socks; the conditional probability is clearly: $(1 - \delta_r^w)$, where δ is the Kronecker delta equal to one when the indices (i.e., socks) are identical and zero otherwise. God-like knowledge of hidden variables or even the control thereof does not alter this conditional probability; Eq. (6) remains false. In exactly the same way, the relative intensity of two polarisation measurements is dependant on the relative angular difference regardless of the polarisation of the incoming signal; i.e., the *conditional* probability is function of the paramenter of the distant measuring station and *not* the hidden variable.

In short, Eq. (1) is not valid for measurements of correlated events. Without Eq. (1), no Bell inequality can be derived.

Observe that it matters not at all how a correlation could arise; be it through nonlocal interaction, heredity or whatever. In order for Eq. (1) to obtain, no correlation can exist, all measurements of the the twins must be statistically independent. Thus, Bell inequalities do not pertain to correlated events. Their observed violation by phenomena selected because they are correlated, has therefore, no meaning for the question of the existence of an extended underlying theory of any character, with or without 'hidden variables.'

This is the raw form of the error in Bell's analysis. It consisting of implicitly selecting a different sample space in the middle of the analysis; and, it has pervasive consequences throughout the discussion of the fundamentals of QM. With this error hidden in the bowels of Bell's Theorem, the subsequent inconsistencies identified by this writer and others follow inexorably in one or another form. It makes it clear why, as Canals-Frau observed, there is no *physical basis* for Bell's Theorem; all empirical evidence pertains to *correlated* events; Bell's inequalities do not. Likewise, Lochak found that even if Eq. (1) is accepted, that the logical structure of the EPR experiment forces one to recognise that "the measurement process in general modifies the state of the observed system." Thus, in the end, the reasoning used by Bell to derive his inequalities does not pertain to measurement results; i.e., Eq. (6) is valid at best only for the unmeasured system. [24]

For some students of this question, the factorizability of the integrand of Eq. (1) is taken as the very definition of locality. To begin with, however, this is lexicographically illegitimate. Einstein preempted this deed with the definition of local interaction as that which does not occur over space-like displacements in Minkowski Space; local/causal interaction is restricted to the past light cone. Thus, a subsequent 'definition' of locality is in fact a claim of equivalence (a theorem) which requires proof. The implicit theorem in the identification of factorizability and locality is really the core of Bell's Theorem; and a one-to-one and on-to map of these disparate concepts had never been established—indeed, can not be; factorisation corresponds to noninteraction; i.e., *statistical independence of events from a different sample space.*

In subsequent analysis of Eq. (1), Bell, in his attempt to formulate a theory of "be-ables," gave extensive consideration to the exact role various independent variables, a, b, λ, etc., might play in the interpretation, in particular with respect to their relation to Einstein locality.[25] But, from the perspective of probability theory, as an abstract mathematical structure, the rôle that independent variables play in an interpretation is immaterial. Often in reality, theory and fact are reversed. In mechanics, for example, time is theoretically the independent, freely manipulable variable, and space is the dependent dictated variable; for mortals, however, rather the opposite is true. In any case, the structure of Eq. (5) must be respected no matter what rôle or interpretation is ascribed to the independent variables. Only correlated events in the original sample space matter .

Eq. (5) does not imply superluminal communication between stations A and B. It implies, simply, that the correlation built into the objects at their common birth result in correlated outcomes even at later times. The existence of such correlations is not a deep and mysterious ontological matter; it is just the consequence of a common cause. That the observation of hereditary correlations will also depend on conditions at the measuring stations is also not profound; indeed, the apparatus has to be set up correctly and turned on. Thus, the appearance of variables pertaining to station B in the *conditional* probability for a detection at station A is only logical; a *coincidence* can only be observed if both measuring stations are correctly set up and turned on. This is not the stuff of philosophy.

In this writer's opinion, Bell must have sensed that something was askew in his formulation of matters. Indeed, he eventually argued that by fixing those hidden variables accounting for correlations, the conditional probabilities pertinent to the outcomes at A and B become factorizable.[26] This is not correct, however, because it requires transferring the argument to consideration of a different sample space, that of single events.

3 Bell-Kochen-Specker Theorems

There are two basic formulations of Bell's hidden-variable no-go theorems: a version found first by Bell [27], but shortly thereafter refined and generalised by Kochen and Specker [28]; and a second version also found by Bell [29] and subsequently modified to accommodate experimental realities by various investigators.[18] The second version, discussed above, is the one most often encountered in physics literature. Both versions are false—as physics; as mathematics they are true but irrelevant to the actual application. The error in Bell-Kochen-Specker versions can be seen as follows:

First, recall some elementary facts concerning spin, which although well known, are essential below, and for convenience are repeated.[30]

Given a uniform static magnetic field **B** in the z-direction, the Hamiltonian is:

$$\mathbf{H} = +\frac{e}{mc}\mathbf{B}\sigma_z. \tag{7}$$

for which the time-dependent solution of the Schrödinger equation is:

$$\psi(t) = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{\frac{-i\omega t}{2}} \\ e^{\frac{+i\omega t}{2}} \end{bmatrix}, \tag{8}$$

and this in turn gives time-dependent expectation values for spin values in the x, y directions:

$$\langle\sigma_x\rangle = \frac{\hbar}{2}\cos(\omega t),\ \langle\sigma_y\rangle = \frac{\hbar}{2}\sin(\omega t). \tag{9}$$

where $\omega = eB/mc$.

What is here to be seen is that in the x-y plane, in contrast to the z (magnetic field) direction, expectation values can not be made up out of summed eigenvalues, as certain formal dicta would have it. Furthermore, what is usually meant by measuring spin in these directions requires reorienting the magnetic field so that in effect one is measuring σ_z in the new B - field direction.

Consider now the following version of the Bell-Kochen-Specker Theorem. At the onset, the system of interest is presumed prepared in a 'state' $|\psi\rangle$ and described by observables $A, B, C \ldots$. A hidden variable theory is then a mapping v of the observables to numerical values: $v(A), v(B), v(C) \ldots$ so that if any observable or mutually commuting subset of observables is measured on that system, the results of the measurement on it will be the appropriate values. Just how the values are determined is the substance of the particular theory.

If this theory is to be compatible with quantum mechanics, the observables must be then operators on a Hilbert space (an erudite way to say that the solutions of the Schrödinger Equation have the properties of those to a hyperbolic differential equation), and the measured values $v(A)$, etc. are the eigenvalues of these operators. It is simply a fact that if a set of operators all commute, then any function of these operators $f(A, B, C \ldots) = 0$ will also be satisfied by their eigenvalues: $f(v(A), v(B), v(C) \ldots) = 0$.

>From this point, the proof of a Kochen-Specker Theorem proceeds by displaying a contradiction. Surely the least complicated rendition of the proof considers two 'spin-$\frac{1}{2}$' particles. For these two particles, nine separate mutually commuting operators can be arranged in he following 3 by 3 matrix:

$$\begin{matrix} \sigma_x^1 & \sigma_x^2 & \sigma_x^1\sigma_x^2 \\ \sigma_y^2 & \sigma_y^1 & \sigma_y^1\sigma_y^2 \\ \sigma_x^1\sigma_y^2 & \sigma_x^2\sigma_y^1 & \sigma_z^1\sigma_z^2 \end{matrix} \;. \tag{10}$$

It is then a little exercise in bookkeeping to verify that any assignment of plus and minus ones for each of the factors in each element of this matrix results in a contradiction, namely, the product of all these operators formed row-wise is plus one and the same product formed column-wise is minus one.[18]

As for errors, it can be seen immediately that the proof depends on simultaneously assigning the [eigen]values ±1 to σ_x, σ_y and σ_z as measurables for each particle. (With some effort, for all other proofs of this theorem one can find an equivalent assumption.) However, above it was seen that if the eigenvalues ±1 are believable measurable results in the "B - field" direction, then in the other two directions the expectation values oscillate out of phase and therefore, can not be simultaneously equal to ±1.[31] Thus, this variation of a Bell theorem also is defective physics.

4 A local model for EPR correlations

If the proof that nonlocality is essential in QM is false, then it should be possible to model EPR correlations without recourse to it. Consider the following:

To begin, note that optical experiments invariably employ the photoelectric effect to measure optical field strengths. Photoelectron ejection is discreet whereas the optical fields are considered by theory to be continuous. A relationship between the count rate, C for these discreet events and the continuous field strength is given by an appropriate detector theory; e.,g., the semiclassical theory: $C = kE^2$, where k is a bundle of constants characterising the detector, and E^2, is the electric field intensity. For present purposes, it says that the count rate is proportional linearly to the field intensity. Thus, below, only field intensity is considered. (Note that 'digitisation by a photo detector occurs *after* squaring the electric field strength; this is the source of the 'special' character of quantum probabilities with respect to radiation—i.e., the addition of the 'square root of probabilities,' as electric fields.)

As an EPRB source, consider an atom, essentially a dipole consisting of an electron whirling about a neutron such that its motion results in unpolarised radiation on the average. In this spirit, therefore, let us take it that the signals generated in the optical version of the EPR gedanken experiment are in fact classical electromagnetic fields, not photons. The EPR source then can be seen as emitting an unpolarised signal in all direction. Thus, the geometric structure of electric field that reaches the A detector would be $E_A = \cos(\theta)\mathbf{x} \pm \sin(\theta)\mathbf{y}$, and that reaching detector B, $E_B = \sin(\theta + \phi)\mathbf{x} \pm \cos(\theta + \phi)\mathbf{y}$, where θ is the instantaneous polarisation angle and the ambiguous signs are to be chosen to account for the four channels. Also, factors of the form $\exp(i(\omega t + \delta(t)))$where $\delta(t)$ is a random function of t, have been suppressed as they all drop out with averaging. The probability of a detection, in the end necessarily a photoelectron, is proportional to the square of these fields. Thus, for these signals, $< A, B >=< E^2_{A,B} >= 1$ and $< A^2, B^2 >= 1$ so that in this case the rhs Bell's inequality, Eq. (3), is equal to 4. Because electrodynamics is linear at the field level and not the intensity level where the statistics enter, calculating Intensity correlations requires calculating forth order field correlations; i.e.,

$$< |AB| >= \frac{< (E_A \cdot E_b)(E_B \cdot E_A) >}{< E_A^2 + E_B^2 >} \tag{11}$$

For these signals, the individual probabilities are $P(++) = P(--) = \sin^2(\phi)/2$; $P(+-) = P(-+) = \cos^2(\phi)/2$, yielding the correlation is $-\cos(2\phi)$, the same result as given by QM. For this case the rhs of Eq. (3) is observed to be $\leq 2\sqrt{2}$, so that the the appropriately modified Bell inequality is satisfied. From this view point, the counting statistics of the EPR experiment are simply due to the geometrical interplay of polarisers and unpolarised radiation.

The numerator in Eq. (11) is the probability of a coincidence count; it is the product of the intensities of the separate signals, but in the form taught by coherence theory.[32] Often, intensity correlation calculations are based on the direct product of intensities, $\langle I_1 I_2 \rangle$; coherence theory, however, teaches that the correct form for this calculation is $\langle E_1 \cdot E_2 E_2 \cdot E_1 \rangle$. The effective difference is that the later form allows the phase to contribute to the calculation.

Intuitive justification for this approach is to be found in the fact that a coincidence is most likely to occur when both stations are simultaneously exposed to fields which are coherent with respect to each other. Instantaneous coherence between two signals is expressed by their product, and this must be squared to accommodate the 'square- law' detector. Note that all the information used in the calculation of the numerator of Eq. (11) is propagated to stations A and B from events in the past light cones of these events. The signals arriving at the measurement stations are, in this case, just classical electromagnetic signals for which there is no question of a violation of locality. Here it is seen clearly again that factorisation in Eq. (1) and locality are not isomorphic concepts.

The denominator in Eq. (11) is equal to the total intensity of both signals in both detectors and is, therefore, proportional to the total photoelectron count, again, for ideal detectors. The ratio of the numerator to the denominator then is by definition the probability of coincidence counts.

Taking all the above into consideration, provides the following expression for the coincidence count rate, $P(+,+)$:

$$\frac{\int_0^{2\pi}(\cos(\theta)\cos(\theta-\phi)+\sin(\theta)\sin(\theta-\phi))^2 d\theta}{2\int_0^{2\pi}(cos^2(\theta)+sin^2(\theta))d\theta}. \tag{12}$$

Evaluated, this expression gives the QM result:

$$P(+,+)=\frac{1}{2}\cos^2(\phi). \tag{13}$$

This model, comprising non quantum components, is local in Einstein's sense; and, it fully agrees with QM. In essence it is, given the vector character of electromagnetic radiation, just the angular analogue of the Hanbury-Brown—Twiss Effect. It stands as a counterexample to Bell's conclusion.

5 Conclusions

We see that the codification of locality as surmised by Bell is correct only for uncorrelated events. The factorizable form of the integrand of Eq. (1) can not pertain to the EPRB experiment. The consequences of this are not revolutionary. QM is in no way shown to be incorrect. Rather, with the invalidation of Bell's Theorem, there is no longer reason to accept that QM can not be extended as envisioned by de Broglie, Einstein, Schrödinger and many others. This will have extensive significance for the interpretation of QM and Philosophy (Ontology) done on the basis of QM. A visualisable interpretation for QM would be advantageous also for pedagogical purposes.

In addition, debunking Bell's Theorem expunges the only rationalisation for admitting nonlocality into science. It is this feature that fully justifies the question posed by Canals-Frau and quoted in the title above; it precludes all accusations of hyperbole. Nonlocality is profoundly antirational, antiscientific. It violates common sense, logic and relativity all at once, not to mention its total lack of empirical support. No previous "méprise" in the history of Physics, not the Ptolomeic solar system, phlogiston, cold fusion, or whatever, violates the fundamental precepts of logic, the essential tool of scientific pursuit, to the extent that the concept of nonlocality does. Allowing something so violently antirational to be a cornerstone of a fundamental *science* with little protest, or what’s worse, with wonderment, is beyond mind boggling. That nonlocality was not taken as a symptom of error, but celebrated as a profoundity, from the distance of future times will earn scorn.

This writer has never met John Bell, but from his writings judges him to have been among the most honest of the prominent students of QM in the later half of the twentieth century. He seemed uncowered by psychological warfare directed at dissenters from orthodox QM, especially in the middle decades. He never praised the glories or wonders of what stubborn rationalists consider nonsense. For unfathomable reasons, however, he did seem to regard nonlocality a real possibility; such, perhaps, are the exigencies of a career in big science. All in all, that he would have been delighted by a debunking of his theorem, is not unimaginable. It is, therefore, especially ironic, even sad that *his* contribution to this issue is so flawed; there are so many others, inveterate sycophants, who deserve this fate. Of course, the larger debate fostered by EPR is not closed just when Bell’s Theorem is refuted; a viable extension of QM must still be proposed. It is this writer’s conviction that the patron of this journal, Louis de Broglie, sowed the seeds that will yield that extension.[33, 34]

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