Extended Particles Part II

On Klein's paradox

Nickola S. Todorov

Georgi Najakov Institute of Solid State Physics,
1784 Sofia, Bulgaria
E-mail : kolo@issp.bas.bg

ABSTRACT. Klein's paradox for both spin $\frac{1}{2}$ and spin 0 electrically charged particles is correctly formulated in the spirit of Klein's original consideration. On the basis of the interpretation of relativistic quantum mechanics set forth in Part I of this paper the paradox is then resolved as being due to a pointlike image of the relativistic microparticle that leads to unphysical results in the presence of very strong classical fields. Conventional theory (part of which is quantum field theory) is demonstrated to have achieved little success in resolving the difficulties, so Klein's paradox may prove to be one of the fundamental theoretical facts that could stimulate novel physical developments.

1. Introduction

The well known paradox of O. Klein [1] was formulated for spin $\frac{1}{2}$ particles (electrons) at the very beginning of the development of relativistic quantum mechanics (RQM). Klein examined one-dimensional electron scattering from a simple semi-infinite potential barrier $A^0(z)$ generating potential energy $V(z) = eA^0(z)$ of the form

$$V(z) = 0, z < 0; V(z) = V = const > 0, z \geq 0 \tag{1}$$

and noticed that in stationary states of eigenenergy

$$0 < E \equiv E(p) = (c^2p^2 + m^2c^4)^{\frac{1}{2}} < V - mc^2 \tag{2}$$

($p > 0$ standing for the momentum of the incident electron in the region $z < 0$) there exists not only a reflected wave of momentum -p in the said region but also a nonvanishing transmitted wave in the region $z \geq 0$. This is at complete variance with what we have in nonrelativistic QM where semi-infinite barriers $V > E$ are always impenetrable. Since the kinetic energy of the monochromatic electron at $z > 0$ turns out to be negative, it would appear that part of the incident electrons of mass $m > 0$ undergo a certain transmutation $m \to -m$ upon impinging on the barrier. The latter thus would be attractive for such electrons (positrons being then yet undiscovered), allowing their free motion in region $z > 0$ as well. (The existence of unphysical negative kinetic energies and hence masses within the conventional interpretation can be demonstrated even in a quasi-classical approximation [2]).

Sauter [3, 4]considered the paradox in the case of different potentials and noted that it tends to disappear for semi-infinite barriers in which the potential energy of a charged point varies about $z = 0$ by $\lesssim mc^2$ over distances $\gtrsim \lambda_C = \hbar/mc$, λ_C being the Compton wavelength. This result was perceived by Sommerfeld [5] as a resolution (for all practical purposes) of the difficulty both on the basis of an ad hoc postulate of Bohr forbidding the existence of classical fields of strength larger than that following from the above restriction and on arguments about inapplicability of classical electrodynamics in such conditions. Resolutions of this kind, however, were not universally felt as definitive.

Hund [6] was perhaps the first to notice that Klein's paradox exists in bosonic RQM too, the paradox range being once again defined by Eq. (2). (cf. also Winter [7]) and it was demonstrated in [8] that in the case of spin 0 bosons the paradox tends to die off in fields statisfying the same conditions as spin $\frac{1}{2}$ fermions. Hund [6] was also the first to study the paradox from the viewpoint of second quantization and pair production in very strong classical fields. The quantum field-theoretic viewpoint was taken up later by a number of authors (cf. e.g. refs. [9-15]) and became dominant in the attemps at resolution of the paradox. With the only exception of ref. [13], the latter papers assert a final resolution of the paradox.

The correct character of this claim, however, does not appear self-evident. Indeed, it is well known that the quantum field operators $\hat{\psi}(x) \equiv \hat{\psi}(t, \vec{x})$ are constructed in the usual field-theoretic approach with the aid of one-particle RQM states which represent appropriate solutions of the Klein-Gordon equation (KGE) and the Dirac equation (DE). (We shall restrict our consideration to the cases of spin 0 and spin $\frac{1}{2}$ electrically charged particles.) Therefore, one would have to determine first the properties of the RQM (first quantized) solutions of the KGE and the DE in the given physical conditions and then justify a particular representation of the field operators with the aid of a pertinent complete orthogonal set of such solutions. We shall demonstrate below that this task has not been resolved in the cited references and that Klein's paradox turns out to represent a profound difficulty for both RQM and quantum field theory (QFT) which employ the conception of local (point) interaction of charged (point) entities with classical (external) fields.

To this end we examine in Sec. 2. certain properties of determinate states by which term we designate stationary states which are obtained from the familiar scattering stationary states in quantum scattering theory with the aid of the operation of time reversal. (In this case of a short-range scattering potential the determinate states will be of the kind usually denoted as $\langle\vec{x}|\vec{p}-\rangle$ [16]). We then examine in Sec. 3 the wave functions in the case of interest Eqs. (1, 2) and demonstrate that determinate states have no relevance to the normal formulation of Klein's paradox. (The choice of the simplest case of semi-infinite "rectangular" barriers allows the most straightforward formulation of the main points without blurring them with unnecessary complications). In Sec. 4 we discuss the paradox within the frame of RQM and propose a resolution on the basis of the reinterpretation of the DE and KGE theories proposed by us [17], which treats particles as extended entities of size λ_C. The QFT resolutions of the paradox and their shortcomings are examined in Sec. 5. (These attempts largely rest, in particular, on the use of determinate RQM states.)

Problems as properties of certain anomalous bound states in very deep potential wells - which are sometimes regarded as variants of Klein's paradox - are not examined in this paper since these do not seem to offer different viewpoints.

2. A nonstochastic property of determinate states

As a first example of such a property examine the familiar states of the kind $\langle\vec{x}|\vec{p}+\rangle$ describing e.g. the scattering of an incoming nonrelativistic particle with momentum $\vec{p}$ by a short-range, shperically symmetric potential $V(\vec{x}) \equiv V(|\vec{x}|)$ [16]. The familiar asymptotic form of $\langle\vec{x}|\vec{p}+\rangle$ when $|\vec{x}| \to \infty$ is

$$\langle\vec{x}|\vec{p}+\rangle \propto e^{i\vec{p}\vec{x}/\hbar} + \frac{f(\theta)}{|\vec{x}|} e^{i|\vec{p}||\vec{x}|/\hbar} \quad ,$$

where $f(\theta)$, in an obvious notation, is the scattering amplitude. The invariance of nonrelativistic quantum dynamics under time reversal has as its consequence that the complex conjugate function $\langle\vec{x}|\vec{p}+\rangle^* = \langle\vec{x}|(-\vec{p})-\rangle$ will also be an admissible state of motion, so in particular the above divergent spherical wave representing the scattered flux will be replaced by the convergent wave $(f^*(\theta)/|\vec{x}|)e^{-i|\vec{p}||\vec{x}|\hbar^{-1}}$ in $\langle\vec{x}|\vec{p}+\rangle^*$. The interesting point in this trivial fact is a remarkable "asymmetry" in the interpretation of $\langle\vec{x}|\vec{p}+\rangle$ and $\langle\vec{x}|\vec{p}+\rangle^* = \langle\vec{x}|(-\vec{p})-\rangle$. Really, $\langle\vec{x}|\vec{p}+\rangle$ admits a stochastic interpretation in the sense that $d\sigma = |f(\theta)|^2 d\Omega$ ($d\Omega$ being an infinitesimal solid angle about a given direction (θ, ϕ)) gives the effective scattering cross section which is connected with the fortuitous character of scattering at different angles. In contradistinction with this, the time-reversed state $\langle\vec{x}|\vec{p}+\rangle^*$ contains only one outcome (and in this sense it may be termed determinate) : in this stage an incoming plane wave of momentum, say, $-\vec{p} = (0, 0, -p_z)$ interferes with the above-said convergent wave, the respective phase relations being such that, when $z \to -\infty$, we have a single outgoing plane wave of the same momentum $(0, 0, -p_z)$. That is, we have here a process of probability one in which two incoming fluxes - a plane wave and a convergent spherical wave - merge in a way leading to a pure outgoing plane wave, so this picture is of essentially different nature than the usual (stochastic) picture of scattering from a given external potential that contains, in particular, a single incoming particle flux.

It is worth giving one more example of a similar phenomenon within nonrelativistic QM that well imitates part of the subsequent RQM consideration. Examine the case of a one-dimensional potential barrier of the form

$$V(z) = 0, \quad z < 0; V(z) = const > 0, \quad 0 \leq z < a;$$

$$V(z) = 0 \quad a \leq z, \quad a > 0. \tag{3}$$

An incident plane wave $e^{ikz}(k = p/\hbar)$ of unit amplitude from the left will be partly reflected and partly transmitted by the barrier, the respective probabilities being $R > 0, T > 0 (R + T = 1)$. Correspondingly, the amplitude b of the reflected wave will be of smaller magnitude $|b| < 1$ than that of the incident wave in the region $z < 0$ (region 1). In the region $a \leq z$ (region 3) we shall have a transmitted single plane wave de^{ikz} in which $|d| < 1$, whereas the region $0 \leq z < a$ (region 2) is an intermediate range of little interest for us.

The operation of time reversal deletes the fortuitous element from the picture and changes the very nature of the latter. In the new picture we have two incoming waves : wave $d^* e^{-ikz}$ in region 3 and wave $b^* e^{ikz}$ in region 1. The first wave completely penetrates the barrier (region 2) without any backward scattering and interferes in such a way with the second wave in region 1 (and with the intermediate wave function in region 2) that the second incoming wave is totally reflected and the net result is an outgoing wave e^{-ikz} of unit amplitude in region 1. In other words, the phase relations within the time reversed state are such that we have a process of probability one in which the two incoming waves of initially opposite directions unite into a single overall wave e^{-ikz} in region 1. This is certainly a determinate state that is of different nature than the stochastic usual state containing a single incoming flux.

3. RQM states of motion employed in the formulations of Klein's paradox

With respect to potential energies $V(z)$ as defined by Eq. (1) practically all authors employing QFT for the resolution of Klein's paradox use one and the same set of E-eigenfunctions in the paradox range Eq. (2). For ease of comparison we will employ for these the notations of Hansen and Ravndal [14] in the system of units $\hbar = c = 1$:

$$p_1(z) = P_1, z < 0; \quad p_1(z) = \frac{1+\kappa}{2\sqrt{\kappa}} N_1 - \frac{1-\kappa}{2\sqrt{\kappa}} N_2, \quad z \geq 0 \tag{4}$$

$$p_2(z) = P_2, z < 0; \quad p_2(z) = -\frac{1-\kappa}{2\sqrt{\kappa}} N_1 + \frac{1+\kappa}{2\sqrt{\kappa}} N_2, \quad z \geq 0 \tag{5}$$

$$n_1(z) = \frac{1+\kappa}{2\sqrt{\kappa}} P_1 + \frac{1-\kappa}{2\sqrt{\kappa}} P_2, \quad z < 0 \quad ; \quad n_1(z) = N_1, \quad z \geq 0 \tag{6}$$

$$n_2(z) = \frac{1-\kappa}{2\sqrt{\kappa}} P_1 + \frac{1+\kappa}{2\sqrt{\kappa}} P_2, \quad z < 0 \quad ; \quad n_2(z) = N_2, \quad z \geq 0. \tag{7}$$

(In the notations of ref. [15] $P_1(z) = \psi_3, P_2(z) = \psi_4, n_1(z) = \psi_1, n_2(z) = \psi_2$). For bosons

$$P_1 = |p|^{-\frac{1}{2}} e^{ipz}, N_1 = |q|^{-\frac{1}{2}} e^{iqz} \tag{8}$$

and for fermions (in a transposed form)

$$P_1^T = \left(\frac{E+m}{|p|}\right)^{\frac{1}{2}} (1, 0, \frac{p}{E+m}, 0) e^{ipz} \ , \tag{9}$$

$$N_1^T = \left(\frac{E-V+m}{|q|}\right)^{\frac{1}{2}} (1, 0, \frac{q}{E-V+m}, 0)e^{iqz} \qquad (9')$$

(We examine only spin-up states since the spin-down results are identical). In these expressions p and q are positive,

$$p = (E^2 - m^2)^{\frac{1}{2}}, q = \left[(E-V)^2 - m^2\right]^{\frac{1}{2}} \qquad (10)$$

and $E = (p^2 + m^2)^{\frac{1}{2}} > 0$, and

$$\kappa \equiv \kappa_{\text{bos}} = \frac{q}{p} > 0, \quad \kappa \equiv \kappa_{\text{ferm}} = \frac{q}{p}\frac{E+m}{E-V+m} < 0 \qquad (11)$$

for bosons and fermions, respectively. P_2 and N_2 are obtained from P_1 and N_1 via the substitution $p \to -p$ and $q \to -q$. In the paradox range Eq. (2) the values of q are real and the solution in the region $z \geq 0$ has no tendency of dying off as $z \to \infty$.

However strange, there exists no agreement in the literature on which of the functions Eqs. (4)-(7) describe Klein's paradox and hence what is its essence in RQM. For instance, $n_1(z)$ is proposed in refs. [14] and [15] as the pertinent state that generates the paradox for both bosons and fermions in the case of RQM. The inappropriate character of such a choice in the case of fermions was already noted in the literature (cf. e.g. [13], [18]) ; the said choice coincide with that of Bjorken and Drell [19]. It is thus necessary to fix first the relevant RQM states.

The wave functions (4-7) statisty standard requirements for continuity at the plane $z = 0$: for fermions one demands continuity of the wave function ψ whereas for bosons continuity is demanded of both ψ and $\partial\psi/\partial z$ at $z = 0$. This guarantees continuity of the respective fluxes at $z = 0$:

$$j_{\text{ferm}} = \psi^\dagger \alpha_3 \psi, j_{\text{bos}} = -\frac{ie}{2m}(\psi^* \frac{\partial \psi}{\partial z} - \psi \frac{\partial \psi^*}{\partial z}) , \qquad (12)$$

where α_3 is the z-component of the familiar 3-vector $\vec{\alpha}$ in DE theory [19], j_{ferm} is the particle flux in the said theory, and j_{bos} is the electric flux in KGE theory (for known reasons the fluxes in question are to be treated exactly as particle and electric fluxes, correspondindly). The reflection and transmissions coefficients in both cases are

$$R = |j_{\text{ref}}|/|j_{\text{inc}}|, \quad T = j_{\text{trans}}/j_{\text{inc}}, \qquad (13)$$

where $j_{\rm inc}$, $j_{\rm ref}$ and $j_{\rm trans}$ stand for the fluxes connected with the incident, reflected, and transmitted waves, respectively.

We want to have an incident flux $\propto e^{ipz}$ of charged particles coming from the left ($-\infty \to 0$, region A). It is to be expected that in region A there will exist a backward wave $\propto e^{-ipz}$ as well whereas in the region $z \geq 0$ (region B) there can be only a transmitted wave $\propto e^{-iqz}$ (recall that p and q are always positive). The adequate RQM wave functions in this picture are therefore those of the kind $n_2(z)$ Eq. (7).

The necessity of the apparently strange choice of $-q$ as the momentum of the transmitted fermion wave was noted first by Klein himself [1]. The general validity of this choice can be demonstrated most easily by defining particle macroscopic velocities in plane waves via

$$v = j/\rho \quad , \tag{14}$$

with j from eqs. (12) and

$$\rho \equiv \rho_{\rm ferm} = \psi^\dagger \psi \quad , \tag{14}$$

$$\rho \equiv \rho_{\rm bos} = \frac{ie}{2m}(\psi^* \frac{\partial \psi}{\partial t} - \psi \frac{\partial \psi^*}{\partial t}) \quad , \tag{15}$$

respectively. In both cases we obtain that in the paradox range Eq. (2) positive velocities (transmitted waves) in region B correspond to negative "momenta" $-q$. (The more appropriate term in such cases appears to be pseudo-momenta [17]). Relations of this kind between velocities and (pseudo)momenta are characteristic of antiparticles (charge $-e$) to the "basic" particles (charge e), so we have here the first indication that Klein's paradox has to do with the existence of antiparticles in region B. A straightforward demonstration of this - which seems to be absent in the literature - is possible as well and is given below.

4. Discussion and resolution of the paradox within RQM

A "pictorial" demonstration of what we have in region B is obtained as follows. Examine the charge density $\rho_{\rm bos}$ Eq. (16) in our stationary states $n_2(z)$ for values ± 0 of z that are infinitely close to the origin $z = 0$:

$$\rho_{\rm bos}(z = -0) = \frac{eE}{m} |\psi(0)|^2$$

$$\rho_{\rm bos}(z=+0)=\frac{e}{m}(E-V)|\psi(0)|^2 \qquad (17)$$

Evidently, for barrier values $V > E > 0$, $\rho_{\rm bos}$ changes sign at $z = 0$, so bosonic RQM directly predicts the existence of anticharges to the right of $z = 0$ when $V > E$. At that, in the interval $E < V < E + m$, $\rho_{\rm bos}$ exponentially tends to zero as $z \to \infty$ whereas, in the paradox range $V > E + m$, $\rho_{\rm bos} \neq 0$ will be constant in the entire region B. This gives the idea that in the first case the particles, penetrating the barrier, can only tenporarily reside in it, at that in the form of anticharges, whereas in the second case the collision of the incident particle with the barrier can cause the real creation of charged boson-antiboson pairs, with paticles to the left and antiparticles to the right of $z = 0$. The pair creation interpretation is borne out by the properties of $j_{\rm bos}$: we have

$$R_{\rm bos}=\left(\frac{1+\kappa}{1-\kappa}\right)^2>1, T_{\rm bos}<0, R_{\rm bos}+T_{\rm bos}=1, \qquad (18)$$

so the backward electric flux in region A is larger than the incoming one. Comparison of the magnitude of the incoming, backward ("reflected") and transmitted electric fluxes is easily seen to show that all incoming charges are totally reflected from the barrier, whereas the amounts of the charges and anticharges created at the barrier are precisely equal.

The same values of $R_{\rm bos}$ and $T_{\rm bos}$ are obtained when one examines a picture containing an incident flux of *antiparticles* (wave $\propto e^{iqz}$) from the right ($\infty \to 0$, region B), a backward *antiparticle* flux $0 \to \infty$ (wave $\propto e^{-ipz}$, and a transmitted *particle* flux $0 \to -\infty$ (wave $\propto e^{ipz}$, region A). Clearly, the pertinent wave function now is $p_2(z)$.The existence of two adequate stationary eigenstates $n_2(z)$ and $p_2(z)$ is connected with the twofold degeneration of the eigenvalue E.

The stationary states $n_1(z)$ and $p_1(z)$ that are usually used in the literature are clearly determinate states which are obtained from $n_2(z)$ and $p_2(z)$, respectively, via time reversal $\hat{\theta}(\hat{\theta} \equiv \hat{\theta}_{\rm bos} = K, K$ denoting complex conjugation). The state $n_1(z)$, for example, corresponds to a picture in which an incoming antiparticle flux from the right, generated by the wave N_1, is completely annihilated at the origin $z = 0$ by an incoming particle flux (wave $((1 + \kappa)/2\sqrt{\kappa})P_1$), all the survinving particles in the leftt incoming flux being totally reflected by the barrier as a backward wave $((1+\kappa)/2\sqrt{\kappa})P_2)$. Evidently this has little in common (even in principle) with experimentally realizable scattering stituations in which one could test the paradoxical predictions.

We turn now to scattering states $n_2(z)$ in the fermion case. The conserved particle density $\rho_{\rm ferm}$ (Eq. (15)) is always positive in contradistinction with the charge density $\rho_{\rm bos}$. None the less, it is to be acknowledged that the entities in region B are antiparticles. Indeed, in region A we have, as usual, an incoming particle flux (wave $\propto e^{ipz}$ and a reflected particle flux (wave $\propto e^{-ipz}$), the respective coefficients being

$$R_{\rm ferm} = \left(\frac{1+\kappa}{1-\kappa}\right)^2 < 1, \quad T_{\rm ferm} > 0, \quad R_{\rm ferm} + T_{\rm ferm} = 1 \tag{19}$$

($\kappa \equiv \kappa_{\rm ferm} < 0$ - cf. Eq. (11)). A straightforward check-up shows that $R_{\rm ferm}$ is equal to Klein's reflection coefficient which in our system of units and notations is written as

$$R_{Klein} = \left(\frac{2mV}{V^2 - (p-q)^2}\right)^2 \tag{20}$$

In order to see what are the spin $\frac{1}{2}$ entities that penetrate region B with probability $0 < T < 1$ we examine a wave packet composed of states $n_2(z)$ within a narrow interval of E-values, coming from the left. When t is large enough ($t \to \infty$) the initially single wave packet will turn out split in two parts : a transmitted packet far to the right of the potential step at $z = 0$ and a reflected part far to the left of it. We now quickly ("instantaneously") switch off the force field at $z = 0$, trapping if necessary near the origin all particles and fields that could be generated by this act so as to prevent any possible perturbations of the picture by these. After the accomplishment of these manipulations and the fast subsequent removal of the auxiliary devices we are left with two distinct wave packets in free space that form a unique wave function. Due to the remoteness of these wave packet from the origin their physical nature, mathematical form and direction of motion could not be influenced in any way by the fast removal of the potential step at the initial moment. An easy computation of the pertinent Fourier coefficient shows that the right-hand wave packet contains only plane waves corresponding to negative eigenvalues of Dirac's free "Hamiltonian" (the appropriate terms being *pseudo-energy* eigenvalues and *pseudo-Hamiltonian* [17]). Employing the theory of ref. [17] (containing the operators $\widetilde{H}$ and $\tilde{e}$ of actual energy and actual charge, respectively) in which negative pseudo-energy states are in fact positive energy states of antiparticles to the "basic" particles, we see that the wave packet in region B respresents a positron state (if the "basic" particles in region A are electrons).

This completes the necessary RQM proof that in Klein's paradox we always have particles in region A and antiparticles in region B for both bosons and fermions.

As was the case with bosons we may also examine a $p_2(z)$-type fermion scattering state in which we have an incoming positron flux from the right. The flux now is partly reflected back in region B and partly penetrates region A, the particles in A being electrons and the reflected ones in B positrons. In analogy with the bosons case, the coefficients R_{ferm} and T_{ferm} in the state $p_2(s)$ are equal to those in the fermion states $n_2(z)$.

The fermion states of the type $n_1(z)$ and $p_1(z)$, used by many authors as a basis of the discussion of Klein's paradox, are once again simply seen to represent derterminate states containing two incident fluxes and obtained via the usual RQM operation of time reversal [19] from suitable "spin-down" scattering states. The state $n_1(z)$, say, represents an incoming "spin-up" positron flux from the right which completely penetrates region A, undergoing a transmutation into an electron "spin-up" flux at $z = 0$ and interfering with an incoming "spin-up" electron flux from the left in a way causing the complete reflection of the latter flux and the formation of an overall backward "spin-up" electron flux in A of absolute magnitude equal to the sum of the absolute magnitude of the two incoming fluxes. This is the essence in our interpretation of what Bakke and Wergeland call "Klein superparadox" [18] for fermions.

These charge $\leftrightarrow$ anticharge transmutations of fermions at the step $z = 0$ represent the catastrophic actual nature of the original Klein paradox since they give evidence about hidden charge nonconservation in DE theory. (An analogous property of the DE in the case of nonstatic potentials was considered in [17]). The situation with bosons appears to be better due to the explicit charge conservation in the picture (cf. also [17]) but this is just seeming.

Indeed, as pointed out in [17], a necessary condition for the existence of physically meaningful one-particle operators of actual energy $\tilde{H}(\tilde{H} = H\Lambda)$ and actual charge $\tilde{e}(\tilde{e} = e\Lambda)$ is the existence of a gap between positive and negative pseudo-energy states which demand makes the definition of the charge-sign operator Λ possible. When $V > 2m$) which is always the case with Klein's paradox - this condition is violated and

Λ cannot be defined. (For that reason we had the above states Eqs. (4-7) of indefinite charge content in the interval $-\infty < z < \infty$). The same applies to the actual energy E_a as a possible quantum number in the entire interval $-\infty < z < \infty$: despite the fact that the values of the "basic" charge and the pseudo-ernegy $E > 0$ are fixed, it is obvious that the anticharges in region B cannot have actual energies $E_a = E$ as is the case with the charge in A. Really their potential energy should be $-V$ rather that V, whereas their actual kinetic energy should be $E_{kin} = m(1 - v^2)^{-\frac{1}{2}}$, where $v < 1$ is defined in B with the aid of Eq. (14). Therefore, $E_a = E_{kin} - V = -E$ in B, as easily seen for both bosons and fermions, and $E_a = E$ in A, so the actual energy of any particle-antiparticle pair is exaclty zero. This means for bosons that under the impact of the incident particles the barrier can generate charged matter and antimatter "from nothing", i.e. without losing any energy in the process - which certainly is an unphysical result whose reason is the application of the one-particle KGE in situations in which it is invalid. In the case of determinate states we have the converse process in which matter is turned into nothing at the plane $z = 0$, which is equally unphysical.

The reason of the inapplicability of both the DE and the KGE in problems of the above kind is clear in the terms of our interpretation [17]. Indeed, only an actual charged point may be assigned e.g. potential energy $V = 0$ at $z = -0$ and a finite value $V \neq 0$ at $z = +0$. For entities of size $\lambda_C \sim 1/m$ this is an unphysical definition of the interaction term that would be expected to produce numerically unreliable results even outside the paradox range, whereas within that range it leads to the above spectacular paradoxes. The very idea to assign boundary conditions for states at a given point ($z = 0$) is an inexplicit expression of a point-particle outlook. Thus all the functions Eqs. (4-7) have in fact no relevance to physical reality. More generally, one-particle RQM becomes ineffective for any point-potential energies that vary too fastly (by $\gtrsim mc^2$ over distances $\lesssim \lambda_C$ and not necessarily in a jump-like fashion) in the sense of ref. [17]. The considerations in refs. [3, 4] and [8] represent particular illustrations of these natural variation limits of external fields. On the other hand, point-potential energies that vary by $\lesssim mc^2$ over distances $\gtrsim \lambda_C$ are good approximations of the interaction energy of an extended particle of size $\sim \lambda_C$ with the given field and the paradox tends to vanish.

The consideration in this Section offers a concrete viewpoint (hopefully nonfuzzy enough) on the nature of Klein's paradox and its resolution by assigning its existence to unwarranted applications of one-particle

RQM to extended objects in certain extreme conditions. We pass now to the evaluation of attemps at resolution of the paradox within QFT.

5. The QFT approach to Klein's paradox

The general conclusion in the QFT resolutions [9-12], [14, 15] of Klein's paradox is that the incident particles are always reflected by the barrier (which was our conclusion too for the particular case of bosons). The presence of antiparticles inside the barrier is attributed eitheir to spontaneous pair creation at $z = 0$ or to the existence of antiparticle fluxes from the right. We shall now examine the basic assumptions on which such considerations rest.

As well known, QFT is intrinsically connected with the one-particle solutions of RQM equations of motion. As a direct illustrations of this for the case of interest we examine the definition of the respective field operators in ref. [14]. (The consideration in ref. [15] is a less rigorous variant of the one in [14] that disagrees with the basic postulate of QFT stating that field operators should satisfy equations of the same form as those of RQM for wave functions [20]). Namely, Hansen and Ravndal propose the field operator (in obvious notations)

$$\hat{\psi}(x) = \sum_k (\hat{a}_{1k} p_{1k} + \hat{b}^\dagger_{1k} n_{1k}) \qquad (21a)$$

$$= \sum_k (\hat{a}_{2k} p_{2k} + \hat{b}^\dagger_{2k} n_{2k}) \qquad (21b)$$

(theirs Eqs. (31a) and (31b), where e.g. $\hat{a}_{1k}$ is the annihilation operator of an incoming particle with the wave function $p_{1k}, \hat{b}_{1k}$ is the creation operator of an incoming antiparticle with the wave function n_{1k} and analogously for $\hat{a}_{2k}$ and $\hat{b}_{2k}$ in the case of outgoing particles and antiparticles, $p_{1k}, \cdots, n_{2k}$ being functions of the kind (4-7), respectively. Another essential assumption in [14] (and [15]) is the existence of two different vaccum states $|0_{in} >$ and $|0_{out} >$ in the second quantized formalism.

1. Critique from a conventional viewpoint. One cannot insist that p_{1k}, for example, respresents only an incoming particle since, as we know, p_{1k} has an antiparticle content too one of whose components is an incoming antiparticle flux in the region $z \geq 0$. Similar remarks apply to all the other wave functions in Eqs. (21a,b) and to the interpretation of all the above operators $\hat{a}_{1k}, \cdots, \hat{b}_{2k}$. Besides, the possibility to introduce two different vacua in this problem is not obvious at all. Such a possibility appears to exist in the case of non-stationary external fields [21] but no proof of that appears to exist in the static case [11].

2. Critique from our viewpoint. The states (4-7) are unphysical and their introduction in the second-quantized formalism of QFT does not lead to physically understandable field operators.

Being aware of the conventional shortcomings of such formalisms [10, 11] Nikishov made an attempt at circumventing them in his basic work [10] on the theme by employing in the paradox range Feynman's non-second quantized variant of QFT (dealing only with RQM wave functions and causal propagators) for the case of static fields that may differ from (1) in the general case. In [11] he demonstrated that the results of [10] agree with those of a nonstationary adiabatic approach to the problem. In [10] he made use of a Green's function (asserted to be causal) in which the roles of z and t are interchanged from the view of the customary definition. This made it possible to fix the consideration to asymptotic values $z \to \pm\infty$ in analogy with the usual case in which one examines moments $t \to \pm\infty$.

1′. Critique from a conventional viewpoint. In his consideration, say, of reflection from the barrier Nikishov [10] employs for the role of a wave, which is to be completed reflected, the completely penetrating part of a determinate state in discord with the nature of such states. Besides, despite the fact that this approach takes into consideration only asymptotic values $z \to \pm\infty$, it would not be correct to describe without special justification (absent in [10]) spontaneous pair production by examining matrix elements containing a state with an analogous property in region B since any one of these states possesses an incoming component in the other region that vitiates the picture and whose role remains as unexplained as was the familiar case with [14]. And in the end, the agreement between static and adiabatic results is typical for QM and does not guarantee in itself correct physics.

2′. Critique from our viewpoint. Analagous to that of point 2 above : the RQM wave functions that are to be used in Feynman's approach (both in the role of states of motion and as a means of constructing causal operators) possess the unphysical properties of (4-7)-type states in very intense fields due to the absence of pseudo-energy gap.

6. Conclusion

Conventional quantum theory has proved inefficient for the resolution of Klein's paradox. The RQM wave functions which it offerts on the basis of its point-interaction conception and which have to be used either directly (conventional RQM or Feynman's approach) or as intermediate factors for the definition of field operators (QFT) have physically inadmissible properties in the case of very strong classical fields. The Klein paradox is one of those theoretical facts which can bring about the creation of new physics. It is possible that the elementary particles will enter the new picture in the form of extended entities with a well defined internal structure.

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