

Early quantum theory genesis in the intertheoretic context*

RINAT M. NUGAYEV

Tatarstan Academy of Science,
Kazan 420044, Volgogradskaya 49, Russia

ABSTRACT. Genesis of the early quantum theory represented by Planck's 1897-1906 and Einstein's 1905-1907 papers in intertheoretic context is considered. It is argued that in both cases the first quantum theoretical schemes were constructed as crossbreed ones composed from ideal models and laws of Maxwellian electrodynamics, Newtonian mechanics, statistical mechanics and thermodynamics. Deriving black-body radiation law Max Planck had to take the experimental evidence into account but it forced him not simply to deduce from phenomena but to use more theory also. Planck's and especially Einstein's theories represented the stages of ambitious programme of Maxwellian electrodynamics and statistical mechanics reconciliation.

Introduction

Many introductory textbooks on quantum mechanics and many historical and philosophical accounts of the early quantum theory genesis (see, for instance, Norton, 1993) are based on the assumption according to which the theory under consideration was invented as a "deduction from phenomena". Early quantum theory in academic history of science have been taken as paradigmatic of a case where theory is determined by the evidence. According to the well-known narratives, the "critical experiments" of Lummer and Pringsheim and Rubens and Kurlbaum refuted the classical theory of black body radiation leading to Planck's and Einstein's theories as to direct generalisations of new empirical data

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obtained. However a more thorough analysis arrives on the following conclusions.

(i) At the end of the 19-th century there was no classical theory of production and transformation of radiation in many-particle systems. For instance, there was no such a thing as *classical theory of black-body radiation*. It appeared only *after* the quantum papers of Planck due to Rayleigh's and Jeans's efforts and became an established one due to Lorentz only in 1908. But Planck's radiation law was elaborated already in 1899. Einstein's main impact to the early quantum theory was made in 1905 - 1907. Wien's classical formula describing the dependence of density of blackbody radiation upon its frequency was a phenomenological one obtained in an ad hoc way and was to be corrected by the works of Max Planck. The latter applied the joint apparatus of electrodynamics, thermodynamics and statistics to rederive it as a theoretical law and to improve Wien's half-phenomenological results.

(ii) On the other hand Einstein's efforts to introduce light quanta were not the attempts to save the phenomena and to explain the "hard facts" that could not be explained by classical theories. Although one often reads the statement that in 1905 Einstein was concerned with an explanation of the photoelectric effect, the study of 1905a ("light quanta") paper reveals that this was not the case. The measurements of the effect at that time were not sufficiently accurate to point without any doubt to a violation of classical behaviour (Ter Haar, 1967). Einstein was worried not so much by the evidence dealing with photoeffect and appealed to fluorescence, photoelectricity and photoionization data only as to *indirect* evidence in favour of his light quantum thesis.

(iii) The history of the early quantum theory in recent literature was shown in no ways to be an unproblematic, paradigmatic case where theory is definitely determined by the evidence. On the contrary, there is a certain portion of theoretical undetermination there.

At first, in 1969-1984 Timothy H. Boyer (see, for instance, Boyer 1975, 1984) convincingly demonstrated that most of the early and old quantum theories' phenomena (beginning with Planck's radiation law) can be explained in a strictly classical way in his "stochastic electrodynamics". The presence of random classical electromagnetic zero-point radiation with a Lorentz-invariant spectrum modifies the ideas of classical electron theory so as to provide three *classical* derivations of Planck's spectrum.

Secondly, Robert Hudson (1997) showed that, contrary to Einstein, Jeans and Poincare (in a company with Klein, Kuhn and Norton), the Rayleigh-Jeans law is *not* a law one should expect from a classical physicist since the derivation of this law requires an ad hoc hypothesis on equipartition . And many classical physicists (like Boltzmann) did not in fact accept the hypothesis and classical equipartition theorem.

Hence the aim of the paper is to make a step towards a more complete description of the Quantum Revolution and to bring some light on the intertheoretic relations between the early quantum theory and other physical theories of the day , stressing their role in its genesis and acceptance. The real origins of the revolution lied not only in the clashes of classical theories with “facts” but in the series of clashes of 3 classical theories between each other also. Experimental evidence did not play the dominant role. At least part of the genuine innovations occurred as a result of the efforts to reconcile the three research programmes of classical physics.

A few words should be said on the historiographical basis of the present work. Though not completely, it is based on Thomas Kuhn’s 1978 book and papers on quantum discontinuity, not to forget Rene Dugas’s (1959), Martin Klein’s (1962,1966), Max Jammer’s (1966), Stephen Brush’s (1967) and Hans Kangro’s (1970) impacts. Though I am not quarrelling with the facts presented in their accounts I am discontent with some *interpretations* of them and especially with the selection procedures picking up the important facts and disregarding the unimportant ones. (For instance I disagree with Kuhn’s underestimation of Planck’s impact into early quantum theory). In full accord with Imre Lakatos’s (1971) recipes my rational reconstruction presents a selection of facts governed by theory-change model lying in the heart of the matter.

1. Max Planck’s black-body theory genesis

At the end of 19-th century three research programmes were to interact consequentially in Max Planck’s work : thermodynamics, electromagnetic theory and statistical mechanics. Before 1900 he has made important contributions to all three but their weight diminished in his creativity from thermodynamics through Maxwell’s theory to Boltzmann’s approach. Thermodynamics was a paragon of scientific theory for him. Planck’s work in it was well-known and highly appreciated before he first turned, with doubts and hesitations, to electrodynamics. Statistical mechanics entered Planck’s research later and against much

resistance. For him the role of electrodynamics and statistical mechanics initially was instrumental. Maxwell's equations and Boltzmann's technique provided conceptual tools to solve the problem of irreversibility first and then the problem of blackbody radiation.

It should be pointed out from the very beginning that **“classical theory of black-body radiation”** before Planck's efforts did not exist at all. But what did exist? - A plenty of phenomenological and half-empirical laws obtained either at the expense of generalisation of empirical data or either due to physically wild and ad hoc assumptions inserted into the body of knowledge.

For instance, in 1879 Josef Stefan, as an extrapolation from preliminary experiments, showed that the dependence of the black-body radiation density u upon temperature T is described by

$$u = \sigma T^4$$

Then Langley, W. Michelson, H. Weber, F. Paschen produced expressions for black-body distribution function $u(\nu, T)$ derived from experiments until Wien tried to obtain the expression for $u(\nu, T)$ from

theoretical considerations in a highly speculative way. For Wien (1896) a heated gas served as the source of blackbody radiation. Following the line of thought of Russian physicist Wladimir Michelson, Wilhelm Wien showed that in the gas the number of molecules with velocities in the range between v and $v + dv$ is, by Maxwell's distribution law, proportional to $v^2 \exp(-v^2/\alpha^2)$ with α proportional to the gas temperature T . If one makes the extremely unnatural assumption that both the frequency and the intensity of the radiation from a given molecule are functions only of that molecule's velocity, then the distribution of radiation must obey the equation

$$u(\nu, T) = F(\nu) \exp(-f(\nu)/T)$$

After determining F and f Wien arrived at the expression (in modern notation, using more exact values for the constants)

$$u(\nu, T) = 8\pi h\nu^3/c^3 \exp(h\nu/kT)$$

Of course as a product of theory the Wien distribution law had little authority until Planck rederived it by a very different route in 1899. His

efforts transformed an empirical law obtained in a dubious way into a theoretical one, with the domain of validity established :

$h\nu/kT \gg 1$. Hence Planck's works were the first effort to construct genuine *theory* of black-body radiation, i.e. to rederive the radiation density $u(\nu, T)$ from the 'first principles' of thermodynamics and electrodynamics as opposed to phenomenological efforts to guess the right expression through experimental results. Naturally Planck met many obstacles on this way, deriving theoretical laws, comparing them with experimental results, correcting the laws, etc.

Planck was a theoretician par excellence with thermodynamics considered as a paragon of scientific theory. In Planck's times thermodynamics became an established, carefully thought and composed, respectable theory with a plenty of practical applications. When he came to a conclusion that thermodynamics is insufficient, he turned to Maxwellian electrodynamics - young and dubious at that times, whose empirical corroboration just begun. However, Planck, in his quest for irreversibility, had to insert resistanceless vibrating resonators in his theoretical scheme - tiny oscillating currents governed by Maxwell's equations. Planck's oscillators had nothing to do with experiments. They had nothing to do with molecules, atoms or even electrons being a kind of theoretical device for establishing thermodynamical equilibrium between matter and radiation.

In 1897 Planck published the first paper of a five-series entitled "On Irreversible Radiation Processes" (Planck, 1897a). In all five papers (Planck, 1897, a, b, c, 1898, 1899), as well as in the major article, which recapitulated their results for the "Annalen der Physik" in 1900 (Planck, 1900b), he investigated the properties of an *ideal model* consisting of a system of resonators interacting with an electromagnetic field. However, after he had read his first paper to the Academy, a critique by Boltzmann was presented to the same audience. Boltzmann argued that, though Planck's expressions for resonator absorption and emission were correct, the programme for which they were designed should necessarily fail. Both Maxwell's equations and the boundary conditions on their solution are invariant under time reversal.

When the application of Maxwell's equations turned out to be insufficient, Planck had to use statistics - quite unwillingly, step by step, under the constant pressure of experimental results. The following story is of special importance here (Dugas, 1959; see also De Broglie, 1962,

p.139). Planck, who intended to broaden the domain of validity of statistical thermodynamics, developed (with a help of classical continuous notions) thermodynamics of electromagnetic radiation and tried to introduce entropy of radiation by analogy with its energy. Being an admirer of famous Ludwig Boltzmann, Max Planck informed the founder of statistical mechanics about the investigations done, and presented one of his papers to Boltzmann's judgement. However, Boltzmann answered that Planck **would never be able to create a correct theory of statistical thermodynamics of radiation without introducing previously unknown element of discontinuity into processes of radiation.**

Even in 1872, 28 years before Planck's 1900b paper, Boltzmann in his paper "Further investigations of thermal equilibrium between gas molecules" applied the notions of discrete energy to the exchange processes. Deriving the second law of thermodynamics from statistical point of view, Boltzmann arrived at energy atoms in the processes of molecules' interactions. The notion of the finite energy amounts that can be exchanged by colliding molecules lead Boltzmann to calculations of the number of collisions with the help of combinatorials. However, due to the dominant worldview, Boltzmann considered the notions of energy quanta just as an auxiliary mathematical device.

"Boltzmann's heritage" was controversial and debatable. At first Planck tried to get through by using an electromagnetic analogue of Boltzmann's H-theorem, and when it appeared to be insufficient, he had to apply the probability calculus and the combinatorial definition of entropy which he whole-heartedly disliked most of all. So, "by mid-winter 1897-98, at the latest, Planck was studying Boltzmann's version of the second law with care, was exploiting suggestions he found there, and had abandoned or all but abandoned his resistance to Boltzmann's approach. Unfortunately for historians, he did not explicitly acknowledge his change of mind for almost two years, a delay that has reinforced the almost universal impression that his conversion to a statistical viewpoint was intimately associated with his introduction of a quantum hypothesis at the end of 1900" (Kuhn, 1978,p.78).

However, one has *direct* evidence of Planck after the events. In a letter to Robert Wood (Planck, 1931) he confessed that "Boltzmann explained the existence of thermodynamical equilibrium through statistical equilibrium; if his considerations are applied to equilibrium between matter and radiation, one arrives at the conclusion that the transformation of all the energy into radiation [demanded by classical physics]

can be avoided by the supposition that energy should exist from the very beginning in some discrete portions". Planck remembered later that all his life, either in publications or in correspondence, Boltzmann was cool with him. And only in his last years, when Planck told him about the atomistic justification of his radiation law, Boltzmann changed his attitude radically and became very friendly to Planck.

So, by the beginning of 1900, only one aspect of Boltzmann's treatment of irreversibility was still absent from Planck's approach, the use of combinatorials, and by the end of the year, Planck had embraced that aspect, too. But what led him to do so was no longer the problem of irreversibility. It was rather the search for a radiation law that could pass the test of new, more refined experiments.

The single obvious imperfection of the derivation of the Wien distribution law that Planck submitted to the "Annalen der Physik" in November 1899 was the lack of a uniqueness proof for the function he had defined as oscillator entropy. But, pointing on their measurements of the frequency distribution of radiation from a new piece of experimental apparatus, the first laboratory black cavity, O. Lummer and E. Pringsheim proposed a new formula for the blackbody radiation, deviating from Wien's law. That difficulty was eliminated by Planck in a paper submitted for publication in February 1900. In that paper he claimed to have derived, rather than defined, oscillator entropy for the first time, supporting Wien's law again.

But experiment proved stubborn. In a paper, reported to the Physics Section of the Naturforscherversammlung on the 18 September 1900, Lummer and Pringsheim declared that the Wien-Planck distribution law did not represent their measurements on black radiation in the low frequency region. In this range, achieved only with the aid of recently developed techniques, the discrepancies between experiment and theory were near 50% and could not be due to experimental errors.

The evidence was entirely convincing, but Planck was well prepared to meet it. In a paper presented to the Physical Society on 19 October he referred to the proof of the Wien law he had submitted to the "Annalen der Physik" in March, and at once pointed its shortcoming. The entropy of n oscillators must, he said, depend not simply on their total energy, as was supposed, but on the energy U of a single oscillator. The expression $\delta^2 S / \delta U^2 = -\alpha / U$ earlier found is too simple and should be exchanged by a more sophisticated form. And Planck had found an equation that "is the simplest by far of all the expressions which yield S as a logarithmic

function of U (a condition which probability theory suggests) and which besides coincides with the Wien law for small values of U ". If the equation for S is regarded as the first term $(-U/\alpha)$ in a power series expansion of $(\delta^2 S/\delta U^2)^{-1}$, his new form follows directly by addition of a term proportional to U^2 .

With $\delta^2 S/\delta U^2 = -\alpha/U(U + \beta)$, two integrations, the standard condition $\delta S/\delta U = 1/T$ and an application of the displacement law yield a new distribution law $U = b\nu/\exp(\alpha\nu/T) - 1$. This radiation formula, Planck stated, "so far as I can see by quick inspection, represents the hitherto published observational data just as satisfactorily as the best previously proposed distribution function ...I therefore feel justified in directing attention to this new formula, which, from the standpoint of electromagnetic radiation theory, I take to be the simplest excepting Wien's "

New measurements quickly showed the last equation to be superior to all the other distribution laws. But Planck had to find the route to the formula that was less ad hoc. "On the very day", Planck says, "when I first formulated this law, I began to devote myself to the task of investing it with a real physical meaning, and that issue led me of itself to the consideration of the relationship between entropy and probability, and thus to Boltzmann's line of thought "(Planck,1909). As Thomas Kuhn correctly pointed out, those remarks have regularly been read as recording Planck's initial conversion from a phenomenological to statistical thermodynamics. But that turn-about had occurred at least a year, and more probably three years, before. When Planck referred to "the relation between entropy and probability" he did not have in mind the statistical approach in general but only, as his words suggested, Boltzmann's combinatorial definition of entropy. Planck, who must have discovered the combinatorial definition in Boltzmann's "Gas Theory", appears to have been the first man to acknowledge even its existence.

Planck's initial derivation of the Wien law contained an internal contradiction. The n resonators he considered were required to be independent, but his argument depended on supposing that their total energy Un was distributed equally among them. An improved argument would consider the various ways in which that energy might be divided between resonators as Boltzmann, in his combinatorial arguments, had divided the total energy of a gas among its molecules. Expression for Planck's distribution law $U = b\nu/\exp(\alpha\nu/T) - 1$ can be manipulated

to yield $1/T$ as a function of U and ν , and $1/T$ is just $\delta S/\delta U$. After integration one gets

$$S = (b/a) \log(1 + U/b\nu)^{1+U/b\nu} / (U/b\nu)^{U/b\nu} + \text{const}$$

Planck should have been encouraged by its clear resemblance to Boltzmann's expression for the logarithmic relation between entropy and probability. Yet the equation applies only to a single resonator with energy U in equilibrium with a radiation field and is not suitable for interpretation in probabilistic terms.

So let us imagine N independent resonators of frequency ν in equilibrium with their radiation field. Their total entropy should be equal to NS , and their total energy should be equal to NU . *If combinatorials are to be introduced, the total energy must be subdivided into P elements of size ε , so that $P\varepsilon = NU$.* Multiplying Planck's equation for s by N and substituting $P\varepsilon/N$ for U yields

$$S_N = (b/a) \log(N + P\varepsilon/b\nu)^{N+(P\varepsilon/b\nu)} / N^N (P\varepsilon/b\nu)^{P\varepsilon/b\nu} + \text{const}$$

To obtain an expression involving only integers, the size of the energy element ε must be set equal to $b\nu$. The quantity then reduces, for large N and P , to $(N+P-1)!/(N-1)!P!$. But that expression is the standard expression for the number of ways in which P indistinguishable elements can be distributed over N distinguishable boxes. The first stages in Planck's utilisation of Boltzmann's relation between entropy and probability were completed.

But the problem still remained. The combinatorial expression discovered by working backwards from Planck's distribution law was very different from the one Boltzmann had developed in deriving the equilibrium distribution of gas molecules. Hence Planck had to show that it is proportional to the probability appropriate to equilibrium radiation. Planck's problem was to compute the entropy of particular distribution of the total energy E over N resonators and then to discover its maximum with respect to variation of the distribution of the total energy over frequency. To compute the entropy of an arbitrary distribution Planck had to introduce combinatorials and hence he followed Boltzmann in subdividing the energy continuum into elements of finite size. "We must now give the distribution of energy over the separate resonators of each group, first of all the distribution of the energy E over the N resonators

of frequency ν . If E is considered to be a continuously divisible quantity, this distribution is possible in infinite many ways. We consider, however, - this is the most essential point in the whole calculation - E to be composed of a very definite number of equal parts and use there the constant of nature $h = 6,55 \times 10^{-27}(\text{erg}\cdot\text{sec})$. This constant multiplied by the frequency, ν , of the resonator yields the energy element ε in ergs, and, dividing E by ε , one obtains the number, P , of energy elements to be distributed over the N resonators" (Planck, 1900b; translated by Ter Haar, 1967, p.83).

Planck next defines a "complexion" (an expression, he stresses, "used by Boltzmann for a similar concept") as a particular specification of the set of numbers, which fixes the number of elements e attributed to the various resonators in the set of N . The total number of possible complexions is R . To find the equilibrium distribution one has to maximise R or $\log R$ by varying the energies at the various frequencies. Straightforward manipulations show that the entropy ($\log R$) will have a maximum if

$$U_\nu = h\nu / \exp(h\nu/kT) - 1$$

Corresponding distribution for the field

$$u_\nu = (8\pi\nu^2/c^3)U_\nu$$

It should be specially pointed out that both in his original derivation papers and far more clearly, in "Lectures" (1906), Planck's radiation theory is incompatible with the quantization of the resonator energy. That theory only require fixing the size of the small intervals into which the energy continuum is subdivided for purposes of combinatorial computation. In Planck's theory, resonator emission and absorption are governed in full by Maxwell's equations. Planck did repeatedly write expressions like $U_N = Ph\nu$. But U_N is the total energy of N resonators. Restricting it to integral multiples of $h\nu$ does not impose any similar restriction on the energy of an individual resonator, which may vary continuously.

Thus, though Planck constantly turned to the experimental results, the role of experiment should not be over-rated. Black-body experiments played the role of the factor, that forced Planck to apply statistics in the growing rates. In the lack of experimental data Planck would not

use Boltzmann's combinatorials in full rate since he did dislike them whole-heartedly. The latter was based on introducing special and dubious hypotheses that Planck, an admirer of classical thermodynamics, tried to avoid.

One cannot declare that Planck's distribution law was a simple generalisation of experimental results. On the contrary, Planck's route to it was from top to the bottom. Of course he had to take the experimental evidence into account but it forced him not to "deduce from phenomena" but to use more theory instead. As one of the first pure theoreticians in physics, a theoretician "par excellence", a leader of German theoretical physics, Planck can in no ways be described as a boy lucky to find a law the significance of which he did not understand. As a professional theoretician, Planck was extremely sensitive to the importance of the problem he tried to solve and to the emergence of taking it in the intertheoretic context. He clearly understood the origin of the problem lying in the deep contradictions between mechanics, statistics, electrodynamics and thermodynamics.

"Nowadays, (the following) two significant fields are set against each other : mechanics and electrodynamics, or, as they are sometimes called, the physics of matter and the physics of ether. The first includes acoustics, heat and chemical processes ; the second includes magnetism, optics and radiant heat. Is this subdivision final ? I don't think so, mostly because neither of these fields of investigation is divided by strict and firm lines. For instance, does radiant heat belong to mechanics or to electrodynamics ? Or to which field must the law of electron movement be attributed ? At first sight one can state that it should be attributed to electrodynamics since ponderable matter plays no role for electrons. But let us direct our attention on the movement of electrons in metals. Studying Lorentz's works one can arrive at a conclusion that the laws of such motions are more appropriate for kinetic theory of gases than to electrodynamics" (Planck, 1910, p. 616).

And more definitely in 1906 "Lectures on Heat Radiation" Planck uses the same arguments as Einstein in 1905a paper :

"Introduction of probabilistic notions into electromagnetic theory of heat radiation means introduction of completely new element that is align to the fundamentals of electrodynamics. Hence from the very beginning the principal question occurs on the justification and necessity of such notions. At first sight one can easily come to a conclusion that theory of probability has no place in theory of pure electrodynamics.

Indeed, since electromagnetic field equations together with initial and boundary conditions are known to definitely determine the duration of electromagnetic process in time, all the considerations disconnected with field equations seem to be basically unjustified and at least not necessary. Indeed, either they lead to the same results given by the main equations of electrodynamics - and they are needless, - or either they lead to other results and hence they are false.

In spite of the unsolvable dilemma, there is a gap in the above considerations. Indeed, trying to get deeper into the heart of the matter, one can see that both "initial and boundary conditions" and "time duration" of some process, as we understand them in electrodynamics, are not the same in thermodynamics. To understand the problem situation better let us turn to a concrete example considered in the last chapter - to hohlraum radiation that is homogeneous in all directions. From thermodynamical point of view, the state of radiation is completely determined if the intensity J_ν of the monochromatic radiation for all the frequencies ν is given. But from the point of view of electrodynamics all these is insufficient; from this point of view to determine the state one has to determine each of six magnetic and electric components of the field in all the space points... " (Planck, 1906, p.105).

It was the fact of origin of early quantum theory from the clash between classical electrodynamics and statistical mechanics that was indicated by one of the leading Russian theorists of the 20-th century beginning :

"But the most curious thing is that the quantum idea should be born half a century ago, when the kinetic theory of matter was created, since this idea is intimately connected with molecular structure of matter and is a specific reflection of this structure" (Goldammer, 1911).

The works of Michelson, Wien, Boltzmann, Planck and Einstein were the stages of penetration of statistical methods and concepts into radiation theory, the stages of considering the electromagnetic field from statistical point of view (Bucklayev, 1957). In the sense one can speak of the Boltzmann-Wien-Planck synthetic programme of reconciling electrodynamics and statistical mechanics. (See more on synthetic and reductionist research programmes in Nugayev, 1999). Subsequent versions of the programme are characterised by ad hoc and non ad hoc reactions to the experimental results. The programme provided constant empirically progressive problemshift in comparison to its rival - the reductionist programme of James Jeans. Its first ideal model was presented in the

paper of Wladimir Michelson who tried to explain an analogy between the form of experimental curve describing the frequency dependence of black-body radiation energy and Maxwell's distribution curve in theory of gases. Though Michelson's ideal model was elaborated by Wien, the hypotheses that provided the shift from initial version of the programme to the second one, appeared to be so ad hoc, that in fact the first real theoretical derivation of the black-body radiation spectrum in the high-frequency limit was given by Max Planck.

However, reconciliation of these theories on the crossbred-level forced to changes in the foundations of the theories in future. In particular, inconsistency in the energy distribution between oscillators further brought to individual oscillator energy quantization.

The irrefutable hard core of the programme was definitely outlined in the given long quotation (Planck, 1906) and consisted of three constructively-independent basic theories : electrodynamics, thermodynamics and statistical mechanics. The positive heuristic, as opposed to Jeans's programme, consisted in the assumption that **the electromagnetic radiation in some aspects shows particular properties** and hence can be described by the laws and principles of statistical mechanics. The protecting belt of auxiliary hypotheses was constructed by a system of models M_1, M_2, \dots, M_k that gave more and more exact "pictures of reality". Some of the resulting theories were ad hoc, but were later exchanged by the better ones that corrected their faults. For instance, some of Planck's theories were ad hoc, and his distribution law was obtained in an ad hoc manner first but was corrected soon.

The hard core of Jeans's reductionist programme consisted of classical electrodynamics. The positive heuristic of the programme consisted in the assumption that the physical situations studied in black-body experiments are not cases of equilibrium at all. Hence the negative heuristic tells one that thermodynamics and statistical mechanics are inapplicable in these cases. Protecting belt of the programme was able to avoid the ultraviolet catastrophe : millions of years might be required to transmit energy from the lower to higher modes of vibration. A genuine equilibrium might never be achieved. Hence Maxwell-Boltzmann equipartition theorem is inapplicable.

The history of the competition of the synthetic programme with its rival, especially with the reductionist programme of Jeans, is interesting in itself. But we will not pay much attention to it here. The main trouble with Jeans's programme was that not only experiments on the

distribution of radiant energy became inexplicable. By denying that these experiments dealt with equilibrium situations, Jeans also denied the relevance of thermodynamic arguments to them. New derivations would have to be found for Kirchhoff's law, the Stefan-Boltzmann law, and the Wien displacement law. Though Jeans provided interesting and skillful explanation of the Stefan-Boltzmann law in his programme, it was quickly forced out by the development of the quantum programme with its successes in the domain of specific heats first of all.

Contrary to Jeans, quantum synthetic programme provided the construction of the crossbred objects constructed from the basic objects of electrodynamics, thermodynamics and statistical mechanics. All the models constructed within the synthetic programme under consideration were crossbred ones but in different extents. They *became* more and more crossbred under the influence of experiments.

So, Planck did notice that introduction of $\varepsilon = h\nu$ was caused by a gap between statistical mechanics and electrodynamics. Real elimination of the cross-contradiction consisted in creation of quantum electrodynamics, in construction of quantum theory of radiation that took electromagnetic particles as Boltzmann's molecules that can gain energy under collisions with usual molecules and resonators (Landau, 1958). The other man who noticed it was Albert Einstein.

2. Einstein's route to discontinuity in intertheoretic context

Although one often reads the statement that in 1905 Einstein was concerned with an explanation of the photoelectric effect, the study of 1905a paper reveals that this was not the case. The measurements of the effect at that time were not sufficiently accurate to point without any doubt to a violation of classical behaviour (Ter Haar, 1967). Einstein was worried not so much by the evidence dealing with photoeffect and appealed to fluorescence, photoelectricity and photoionization data only as to indirect evidence in favour of his thesis. Rather, Einstein was concerned mostly with contradictions between classical mechanics and classical electrodynamics. Look at the beginning of his 1905a paper : "There exists a profound formal difference between the theoretical conceptions physicists have formed about gases and other ponderable bodies, and Maxwell's theory of electromagnetic processes in so-called empty space". What does this difference consist in ? - "While we conceive of the state of a body as being completely determined by the positions and velocities of a very large but nevertheless finite number of atoms

and electrons, we use continuous spatial functions to determine the electromagnetic state of a space, so that a finite number of quantities cannot be considered as sufficient for the complete description of the electromagnetic state of a space. According to Maxwell's theory, energy is to be considered as a continuous spatial function for all purely electromagnetic phenomena, hence also for light, while according to the current conceptions of physicists the energy of a ponderable body is to be described as a sum extending over the atoms and electrons. The energy of a ponderable body cannot be broken up into arbitrarily many, arbitrarily small parts, while according to Maxwell's theory (or, more generally, according to any wave theory) the energy of a light ray emitted from a point source of light spreads continuously over a steadily increasing volume". (Translated by Anna Beck, 1989). But this difference can give rise to a situation where "the theory of light, which operates with continuous spatial functions, will lead to contradictions with experience when it is applied to the phenomena of production and transformation of light". Hence "it *seems to me* that the observations regarding "black-body radiation", photoluminescence, production of cathode rays by ultraviolet light and other groups of phenomena associated with the production or conversion of light can be understood better if one *assumes* that the energy of light is distributed discontinuously in space". (I would like to stress the words "seems to me", "assumes" reflecting the level of indeterminacy of Einstein's thesis).

And in the first part of his 1905a Einstein discloses that the joint application of mechanical and electrodynamical "theoretical pictures" for description of black-body radiation leads not only to contradiction with experiment (his paper did not cite the results of Lummer and Pringsheim and Rubens and Curlbaum), but to *paradox* that cannot be eliminated by usual methods. To demonstrate it Einstein uses *gedankenexperiment* with the abstract objects of three theories : electron theory, Maxwell's electrodynamics and statistical mechanics. He considers a cavity containing free electromagnetic field, gas molecules and Hertz's resonators. As a result one can conclude that joint application of mechanics and electrodynamics leads unavoidably to Raleigh-Jeans law for energy density of black-body radiation. But "this relation, obtained as the condition of dynamic equilibrium, not only fails to agree with experience but it also states that in our *model* a definite distribution of energy between ether and matter is out of the question", since "the wider the chosen range of the resonators' frequencies, the larger the radiation energy of the space, and we obtain in the limit

$$\int_0^{\infty} \rho_{\nu} d\nu = (R/N)(3\pi/L^3) \int_0^{\infty} \nu^2 d\nu = \infty$$

Thus, Einstein pioneered in demonstrating how the cross contradiction of mechanics and electrodynamics led to “ultra-violet catastrophe”.

How did Einstein intend to eliminate the contradiction in his 1905a ? - To answer the question one should turn to his first papers published in the “Annalen”. All the Einstein’s papers from 1901 to 1905 have one trait in common : statistico-thermodynamics approach. Thomas Kuhn correctly pointed out that what brought Einstein to idea of photon was a coherent development of a research programme started in 1902, a programme “so nearly independent of Planck that it would almost certainly have led to the black-body law even if Planck had never lived” (Kuhn,1978,p.171).Einstein’s first two papers, published in 1901 and 1902, studied intermolecular forces by applying phenomenological thermodynamics. From the start of his career Einstein was deeply impressed, as Martin Klein has emphasised, by the simplicity and scope of classical thermodynamics. But for him thermodynamics included the statistical approach he had learned from Boltzmann’s works, and he began to develop statistical thermodynamics. The result was a series of three papers published in 1902,1903 and 1904.They provide the clue for understanding his 1905a on quanta,1905b work on Brownian motion and 1905c paper on STR. In describing 1902-1904 papers I’ll follow Kuhn’s 1978 excellent study.

The first important result consisted in the fact that for physical systems of extraordinary general sort Einstein has produced, by the summer of 1903, both a generalised measure for temperature and entropy, containing some universal constant χ .By the time he finished his 1903 paper, Einstein had recognised that χ could be evaluated in terms of the values of the gas constant and of Avogadro’s number. But the theory that had led him to the constant was, however, applicable to systems far more general than gases. It should therefore have a correspondingly general physical basis. The basis should reflect statistico-mechanical nature of the approach that led him to the constant, explaining not only its role as a scale factor for temperature, but also its position as a multiplier in the probabilistic definition of entropy. Physical significance of χ was the central problem attacked in Einstein’s third statistical paper, submitted

to “Annalen” in the spring of 1904. Solution of the problem consisted in the phenomena of energy fluctuations. Einstein demonstrated that

$$\overline{\varepsilon^2} = 2\chi T d\overline{E}/dT,$$

where $\overline{\varepsilon^2}$ is a measure of thermal stability of the system. And it was recognition of the constant ε physical role that directed his attention to the black-body problem. “The equation just found would permit an exact determination of the universal constant ε if it were possible to determine the energy fluctuation of the system. In the present state of our knowledge, however, that is not the case. Indeed, for only one sort of physical system can we presume from experience that an energy fluctuation occurs. That system is empty space filled with thermal radiation” (Einstein, 1904, p.360; translated by Kuhn, 1978).

At least one more step in development of the programme of statistical thermodynamics was needed, and Einstein took it in a famous paper published in the following year, in 1905. Its content strongly suggests that Einstein had begun to seek a black-body law of his own, that he had quickly encountered the paradox - contradiction between statistical mechanics and Maxwellian electrodynamics - and that he had dropped the search for the law in favour of an exploration of the paradox itself. This is clear from the very beginning of his paper which was already quoted. The first part of 1905a ended by revelation of “ultraviolet catastrophe”. How did Einstein resolve the paradox?

In the second part of his 1905a Einstein applies thermodynamics ($dS=1/T$), statistical mechanics ($S=k \log W$) and Maxwellian electrodynamics ($E = V \int \rho_\nu d\nu$) to describe the domain of empirical reality covered by Wien’s radiation law. Einstein takes $\beta = h/k = Nh/R$ as undefined constant in 1905a paper and hence he writes $R\beta/N$ everywhere instead of h . Joint application of the three fundamental theories enables Einstein to arrive at apparently deductive argument: if monochromatic radiation of frequency ν and energy E is enclosed in the volume V_0 , then the probability W that at any moment all the radiation energy will be found in the partial volume V of the volume V_0 is given by

$$W = (V/V_0)^{E/h\nu} \quad (i)$$

Yet in the same paper Einstein demonstrates that in the case of n independently moving particles enclosed in a volume V_0 the probability of finding them all momentarily in the subvolume V is

$$W = (V/V_0)^n \quad (ii)$$

Comparing (i) and (ii), Einstein comes to a conclusion that “monochromatic radiation of small density behaves in thermodynamic respects as though it consists of distinct independent energy quanta of magnitude hn ”. As had been pointed out by Igor Yu. Kobzarev, “applying Boltzmann’s rule, Einstein could think that the main principles of classical mechanics are valid for field of heat radiation, but Maxwell’s equations - are not...In general, all the situation appeared to Einstein as development of classical atomistic. Maxwell and Boltzmann had introduced atoms instead of continuous media; now the same was to be done for electromagnetic field which is something like the gas consisting of interacting photons” (Kobzarev, 1979, p.18). Many of Einstein’s contemporaries described genesis of early quantum theory in the same way.

For instance, Arthur Haas in a sequence of lectures delivered in Wien in winter 1919-1920 and in summer 1920 in Leipzig had pointed out that “quantum theory was created because of intention to extrapolate the atomistic principle, that appeared to be very fruitful in the domains of matter and electricity, on more abstract questions. Classical theory of heat and classical electron theory applied atomistic principle to the carriers of physical phenomena; the quantum theory transferred the atomistic principle to the domain of physical processes themselves” (Haas, 1924, p.68).

Dmitry Goldammer (1923, pp.118-120) wrote that “it is very curious that the idea of quantum should be born fifty years ago, when kinetic theory of matter was created, since this idea is inseparably connected with molecular structure of matter and is nothing more than manifestation of this molecular structure...By analogy explanation of physical properties of bodies from the point of view of atomic and molecular motions for a long time was independent of explanation of light and sound radiation. In both domains, from a well-known point of view, the explanations were simple. But science brought them together, and quanta of energy occurred as a result. In both cases the situations became more complicated, at least at first sight; but, due to idea of quanta, these phenomena, that stood independently, were unified, so to say, under the common flag of molecularity of energy and matter”. Thus, the conclusion that radiation in the cavity consists of independent energy quanta follows directly from application of general principles of thermodynamics and statistical mechanics to processes of radiation. It should be specially pointed out that the quantum hypothesis was derived not only by “inference from phenomena” but by inference from the first principles

of statistical mechanics and Maxwellian electrodynamics too. The only additional information used was half-phenomenological classical Wien's law derived in 1896. Though Einstein was aware of Planck's radiation law he did not actually use it in 1905a. Moreover, in his 1906 "On the Theory of Light Production and Light Absorption" he had to confess that "at that time it seemed to me that in certain respect Planck's theory of radiation constituted a counterpart to my work". However new considerations, presented in 1906 paper, enable him to demonstrate that "the theoretical foundation on which Mr. Planck's radiation theory is based differs from the one that would emerge from Maxwell's theory and the theory of electrons, precisely because Planck's theory makes implicit use of the aforementioned hypothesis of light quanta".

Our thesis can be independently confirmed by studying J. Larmor's "Baikerian" lecture "On the Statistical and Thermodynamical Relations of Radiant Energy" that was read in 1909. The lecture contained an analysis of the papers of Planck and Lorentz on black-body radiation. But the crux of the lecture was the study of the well-known paradox according to which application of consecutive principles of classical statistical mechanics to energies of Planck's oscillators is possible only under the condition $h \rightarrow 0$. And what were Larmor's methods of resolving the paradox?

"The motive of this present discussion is the conviction expressed at the beginning, that the statistical method, in Boltzmann's form, must in some way hold the key of the position, no other mode of treatment sufficiently general being available. The writer has held to this belief, with only partial means of justification, ever since the appearance in 1902 of Planck's early paper extending that method to radiation. In the "British Association Report", 1902, p. 546, there is a brief abstract of a communication "On the Application of the Method of Entropy to Radiant Energy", in which it was essayed to replace Planck's statistics of bipolar vibrators by statistics of elements of radiant disturbance. "It was explained that various difficulties attending this procedure are evaded and the same result obtained, by discarding the vibrators, and considering the random distribution of the permanent elements of the radiation itself, among the differential elements of volume of the enclosure, somewhat on the analogy of the Newtonian corpuscular theory of optics" (cf. "British Association Report, 1900") (Larmor, 1909, p. 95).

In 1906 Ehrenfest and Einstein were among the first to recognise that Planck's blackbody law could not be derived without restricting the

resonator energy to integral multiples of $h\nu$. Ehrenfest's conversion to quanta was not accidental. In 1899-1900 Ehrenfest attended Boltzmann's lectures on the mechanical theory of heat. His first publication was a paper dealing with a small point in the theory of gases, which Boltzmann presented to the Royal Academy of Sciences on July, 1903. Ehrenfest's thesis "The Motion of Rigid Bodies in Fluids" Boltzmann characterised as "very fundamental", "diligently and cleverly worked out". When Ehrenfest cited Boltzmann's work in a particularly complete way, Boltzmann remarked: "If only I knew my own work that well" (quoted from Klein, 1970, p.48). Ehrenfest's interests were in statistical mechanics; so, at least initially, quantum theory seemed to him to be a branch of statistics. Even in his 1911 paper "Which Features of the Hypothesis of Light Quanta Play an Essential Role in the Theory of Thermal Radiation?" Ehrenfest explained why energy quanta were proportional to frequency. The origin consisted in the requirements of the second law of thermodynamics in its statistical form.

Until 1908, Einstein's and Ehrenfest's demonstrations had little apparent impact (Einstein's photon paper was the first sympathetic response to Planck's blackbody investigation). But the paper, presented by Lorentz in 1908, caused a profound change in the attitude of the community towards the quantum: "... one cannot escape Jeans's conclusion, at least not without profoundly modifying the fundamental hypothesis of the theory" (Lorentz, 1908, p.160). The Raleigh-Jeans law and the "ultraviolet catastrophe" did not initially pose problems for more than two or three physicists. But finally they became central in physics due to their repeated rederivation by a variety of different technique. Lorentz's paper appeared in the early spring of 1908. By the end of the following year, Lorentz, Wien and Planck himself had been persuaded that radiation theory demanded quanta. Arnold Sommerfeld and James Jeans were moving towards that position in 1910, the year Lorentz provided particular clear and widely appreciated arguments for it. After 1910 leadership in quantum investigations passed to specific heats at low temperatures. Up to 1911, the Roentgen radiation, photoeffect (Stark's and Barkla's experiments), luminescence, atomic theories became important domains of application of the early quantum programme. They all had provided constant empirically progressive problemshift. A serious success became Nernst's 1911 confirmation of Einstein's 1907 specific heats formulae. "If Planck's theory strikes to the heart of the matter", then one should, according to Einstein, make a fundamental change in

the foundations of statistical mechanics. Quantum discontinuity appeared to be connected not only with interaction of matter and radiation. But what about the oscillators that appear in the molecular theories? - They too must obey the quantum restrictions in direct contradiction to classical statistical mechanics. Einstein found confirmation in the departures of some specific heats from the Dulong-Petit rule, that went against the equipartition theorem. Thus, Einstein's specific heat theory was a statistical-mechanical one, independent of electrodynamics. He has quantized the energies of neutral atoms also. In 1907 Wien's theory was confirmed. He considered the radiation of moving charged particles with the help of Planck's theory. In the same year, Wien used his theory to analyse the Roentgen spectra. His predictions were confirmed in 1912, when X-ray diffraction was found. At the same time Paul Ehrenfest demonstrated that if one is to describe radiation by particle representation, then the "particles" one uses must have properties substantially different from those of any particles previously used in physical theories. The new particles are not independent, but must show a kind of correlation. Ehrenfest's results agreed with that of Einstein obtained in 1909.

The first Solvay Congress (1911) definitely enough revealed the inability of classical mechanics and classical electrodynamics to solve the problems concentrated in the radiation theory. "The result of the discussion of these questions seems to be a general acknowledgement of the inadequacy of the classical electrodynamics in describing the behaviour of systems of atomic size" (Bohr, 1913, p.2).

Black-body theory and specific heats were the two quantum topics well established by the end of the period 1911-1912. However, Roentgen radiation, luminescence, Bohr spectra became new important areas of the development of the Quantum Subprogramme. The Bohr theory was one of the last blows for ether-based wave theory. His paper "On the Constitution of Atoms and Molecules" begins by stating the "inadequacy of the classical electrodynamics in describing the behaviour of systems of atomic size". It contains lots of expressions like "the failure of classical mechanics", "obvious contrast to the ordinary ideas of electrodynamics", etc. One of Bohr's main conclusions consisted in that "the intention, however, has been to show that the sketched generalisation of the theory of the stationary states possibly may afford a simple basis of representing a number of experimental facts which cannot be explained by help of ordinary electrodynamics, and that the assumptions used do not seem to be inconsistent with experiments on phenomena for which a

satisfactory explanation has been given by the classical dynamics and the wave theory of light" (Bohr,1913,p.13). In 1924 S.Bose came to first derivation of Planck's blackbody radiation formula by endowing light quanta with new statistical properties. On the basis of new statistics Einstein in 1924 and 1925 had quickly developed quantum theory of monatomic ideal gases. In 1924 Louis De Broglie in the doctoral thesis came to conclusion that matter should possess certain wave properties. In spite of the successes, the quantum theory of radiation raised even more difficult problems, connected to its relations to classical radiation theory, classical statistical mechanics and classical thermodynamics. Mutual interpenetration of these theories led to the creation of quantum mechanics, quantum electrodynamics and quantum field theory.

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