Open letter to Akira Tonomura : prediction of a crucial effect

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1 Physicality of a curless vector potential revisited.

By definition a curless vector potential $\boldsymbol{A}(\boldsymbol{r})$ is a gradient. If generated by an« autistic magnet »[1], it is the gradient :

$$\boldsymbol{A}(\boldsymbol{r}) = \nabla \boldsymbol{U}(\boldsymbol{r}) \tag{1.1}$$

of a multivalued superpotential $\boldsymbol{U}(\boldsymbol{r})$, the « period » of which is the trapped flux :

$$\Delta \boldsymbol{U} = \Phi = \oint \boldsymbol{A}(\boldsymbol{r}).\boldsymbol{dr}\,. \tag{1.2}$$

As this flux is quantized[2] in $h/2e$ units,

$$\Phi = Nh/2e, \tag{1.3}$$

the *potential action* $e\Phi$ of a test electron is quantized in $h/2$ units and its *potential phase* $\varphi(\boldsymbol{r})$ in π units.

That the periodic superpotential $\boldsymbol{U}(\boldsymbol{r})$ ot equivalently the periodic potential phase $\varphi(\boldsymbol{r})$:

$$\boldsymbol{U}(\boldsymbol{r}) = \boldsymbol{h}\varphi(\boldsymbol{r}) \tag{1.4}$$

is defined up to the gradient of a non periodic function precludes not that it will be observable as a function of **r**. Indeed, it will be so evidenced if a graduated grid is superposed upon the registration of the interference fringes.

In Tonomura's experiments there exists such a grid : the straight fringes pattern generated by the electron biprism.

Inasmuch, as the biprism is far ahead of the very small toroidal magnet, *a second gradient steps in* : the Fresnel fringes can be linkened to those generated by two interfering plane waves.

The overall fringe pattern then obeys the formula

$$\mathrm{h}\,\mathrm{d}\varphi(\boldsymbol{r}) = \{\boldsymbol{m}\Delta\mathrm{v} - \mathrm{e}\boldsymbol{A}(\boldsymbol{r})\}.\mathrm{d}\boldsymbol{r} \tag{1.5}$$

The constant vector $\Delta\mathrm{v}$, which is the difference between the group velocities of interfering waves, lies in the picture's plane, and the projection of $\boldsymbol{A}(\boldsymbol{r})$ is displayed as radial in this plane : *both vectors are gradients.*

Our prediction is : If, in Tonomura's beautiful experiments using a superconducting coating of the magnet, the registering film were *placed before rather than after the magnet and in contact with its plane face*, the vector potential $\boldsymbol{A}(\boldsymbol{r})$ would be contained in the picture's plane, *both gradients* $\mathrm{m}\Delta\mathrm{v}$ *and* $-\boldsymbol{eA}(\boldsymbol{r})$ *being displayed and measurable.*

In other words : The photographic registration will be similar to that obtained[3] with a magnet transparent to the electron wave.

Such is our prediction, contrary to feelings expressed to one of us (G.L.) during his visit at the Hitachi Company Laboratories. This is our chalenge to Dr Tonomura.

2 Additional remarks.

The potential phase $\varphi(\boldsymbol{r})$ of a test electron in presence of an autistic magnet is the product of the trapped flux Φ by a function of the magnet's geometry. *An explicit expression of it is easily found if the magnet is likened to a (not necessarily circular) filament carrying per line element a magnetic moment* $\boldsymbol{dM}(\boldsymbol{r}) = \Phi\boldsymbol{dl}$, Φ *constant. The magnet's potential then is :*

$$\boldsymbol{A}(\boldsymbol{r}) = -\Phi\int \boldsymbol{r}^{-3}\boldsymbol{r}\times \boldsymbol{dl} \tag{2.1}$$

The differential potential phase associated with a displacement $\boldsymbol{dr}$ is :

$$-e\boldsymbol{A}(\boldsymbol{r}).\boldsymbol{dr} = -e\Phi\int \boldsymbol{r}^{-3}[\boldsymbol{dl}\times\boldsymbol{dr}].\boldsymbol{r}\,, \tag{2.2}$$

namely, $-e\Phi$ times the varied solid angle $\Omega(\boldsymbol{r})$ through which the magnet is seen from $\boldsymbol{r}$. The potential phase (expressed in the source adhering gauge) then is :

$$\varphi(\boldsymbol{r}) = \mathrm{N}\Omega(\boldsymbol{r}) \tag{2.3}$$

with N denoting the number of trapped flux quanta $h/2e$. *An original proof of flux quantization in units* $h/2e$ then follows : in a cycle embracing once the magnet, $\Omega(\boldsymbol{r})$ varies by $\pm 4\pi$ and $\varphi(\boldsymbol{r})$ by $\pm 2N\pi$, this requiring (1.3) *as an integration condition*.

If the closed filament is plane the solid angle through which it is seen from a point $\boldsymbol{r}$ in its plane is 2π or 0 according as $\boldsymbol{r}$ is inside or outside the contour. So, in the experiment sketched above, *the straight Fresnel fringes outside the ring are the genuine Fresnel ones issuing from the biprism, those inside the ring being shifted by* $Nh/4e$ (that is, half the value resulting from the interference of the two beams far downstream).

Incidentally, if the point $\boldsymbol{r}$ is displaced along a straight axis going through the magnet this *open* contour yields the full $\pm 4\pi$ augmentation of the solid angle Ω.

Concluding : Our challenge to our friend Akura Tonomura is : Perform the flux quantization experience with the registering film placed before the shielded magnet, in contact with its plane face.

Références

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[1] O. Costa de Beauregard and J.M. Vigoureux, *Phys. Rev*. D9 (1974) 1626.
[2] A. Tonomura et *Alii., Phys. Rev. Lett*. **48** (1982) 1443.
[3] S. Olariu and I. Popescu, Rev. Mod. Phys. **57** (1985) 339.