

## Bridge from Classical to Quantum Theory of Open Systems

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ABSTRACT. The paper consists in a short survey of some results concerning the physical interpretation of the basic equations for quantum open systems.

The concept of the continuous medium for quantum systems is introduced. From this point of view the question of completeness description and the hidden parameters (scales) for the quantum open systems is considered. The Heisenberg uncertainty principle in the physics of open quantum systems is represented. The question: Is the concept of "pure ensemble" justified in quantum mechanics" is also discussed.

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## 1 Introduction

In this paper we are going to pave the way for the quantum theory of open systems.

The simplest classical open system is the one-component rarefied gas of structureless particles (the Boltzmann gas). A natural expansion can be proceed for two particular models.

The first one is a rarefied electron-ion plasma, which is a three-component system comprised of gases of electrons and ions, and electromagnetic field.

The second is the system of atoms and field, which in the simplest case consists of two components. The gas component differs from Boltzmann gas in that the structure of atoms or molecules is taken into account. The second component is again the electromagnetic field.

A natural generalization of these two particular models is the so-called plasma-molecular system, which consists of at least four components [1]. An example of such system is provided by partially ionized plasma. Foundations of the kinetic theory of systems of this kind have been laid in [2-8]

A consistent description of plasma-molecular systems is only possible within the framework of quantum theory; hence the need of bridging the gap between the classical theory of open systems to the appropriate quantum theory. This is best done with the concrete examples of simple but real systems, such as the system of noninteracting hydrogen atoms or free electrons in electromagnetic field.

We shall use these examples to illustrate the transition from the reversible microscopic operator equations to irreversible equations for deterministic quantum distribution function, the matrix of density and the moments of fluctuation electromagnetic field. Such transition may be interpreted as replacement of a system of particles and field oscillators by a continuous medium.

In particular, this implies that Schroedinger equation of quantum mechanics for conventional (deterministic, not operator) wave function also describes the evolution of continuous medium, but ignores the dissipative terms. In this sense, there is an analogy between Schroedinger equation in quantum mechanics and Euler equation in hydrodynamics [9,10].

It would be natural to begin our study of quantum system of atoms and field with the microscopic equations, which give complete quantum

electrodynamic description of atoms (or free electrons) and electromagnetic field.

Known are two formulations of the initial equations of quantum electrodynamics. The first is the Schroedinger equation for the wave function of all variables, for instance, of the atoms and all field oscillators. This can be used for constructing the equation for quantum distribution function in, say, Wigner representation [11,12,3-5,13,9,10]. In the classical limit it will go over into the Liouville equation.

The second formulation is based on the set of reversible equations for operator wave function of electrons in atoms, and field operators. This is the method of secondary quantization. In the classical limit, the quantum operator equations correspond to equations in the microscopic phase density of atoms (or free electrons) and microscopic field strength. In the irreversible equations of statistical theory these functions are regarded as random.

Like in case of Boltzmann gas, we again are faced with important questions. What are the smallest scales on which the initial equations of quantum electrodynamics lose their reversibility? What is the cause of irreversibility?

Similar questions arise at different levels of description. We might ask, for instance, what are the smallest scales which allow going over from operator equations of quantum electrodynamics to the corresponding Schroedinger equation for the deterministic distribution function of, let us say, the atom of hydrogen.

The fact is that quantum mechanical Schroedinger equation is more coarse than the corresponding equations of quantum electrodynamics, since it only involves the mean field rather than the coordinates of individual field oscillators.

The problem consists therefore in finding the starting point of transition towards irreversible equations. This is mainly what the present paper is about. In this connection we should mention stimulating influence of the work of Ilya Prigogine [14,15], who for many years has been studying the possible generalization of the second law of thermodynamics to the microscopic level. The main role is assigned to the dynamic instability of motion of microscopic objects. Dynamic instability in quantum theory is manifested in significant changes of wave functions when the relevant initial conditions are varied even slightly.

Recall that in case of Boltzmann gas the procedure of smoothing

over physically infinitesimal volume (the "point" of continuous medium), which is necessary for going over from the system of particles to the approximation of continuous medium, reflects the existence of dynamic instability of motion. To put it differently, the very feasibility of smoothing over physically infinitesimal scales in the construction of irreversible equations of statistical theory is based on the existence of dynamic instability of motion of microscopic elements of the system. In this respect the dynamic instability of motion plays a constructive role in the statistical theory.

Since the theory of dynamic instability in quantum theory is still in its early stage [15-17], it will be expedient to start with the evaluation of the smallest scales on which smoothing is possible. This is necessary also for defining the limits of applicability of the initial microscopic equations of quantum electrodynamics.

This will provide us with a bridgehead for the second step, at which the conditions are clarified which allow going over from operator equations of quantum electrodynamics to equations in the relevant deterministic wave functions. This problem is closely associated with the structure of quantum kinetic equations. Examples of such equations will be analyzed.

The question of completeness of description of physical phenomena on the basis of quantum mechanics was the subject of the famous debate between Einstein and Bohr at the 5th Solvay Conference in 1927 (see [18]). Later this problem was discussed in the paper Einstein-Podolski-Rosen entitled "Can quantum-mechanical description of physical reality be considered complete?" [19], and also in the papers by de Broglie [24] and Bohm [20]. The issue of "deficiency of quantum-mechanical description" is closely related to the problem of the so-called hidden parameters in quantum mechanics.

Most physicists argued that quantum mechanical description is complete, and the problem of hidden parameters does not exist. This view was based for the most part on the book by John Neumann [25], in which he proved that hidden parameters are incompatible with the foundations of quantum theory. It was not mentioned, however, that Schroedinger equation of quantum mechanics is itself approximate.

Further theoretical and experimental studies were stimulated by the results of Bell [20,21], who formulated the condition of existence of hidden parameters as Bell's inequality. This was seen as a new possibility

of experimental verification: if quantum mechanical description is complete, then Bell's inequality does not hold.

In a recent review Belinskii - Klyshko [19] we read:

"The problem formulated many years ago by Einstein, Podolski and Rosen, by Bohm and Bell, still excite the new generation of physicists. To a large extent this is due to the fact that the contradiction between the prediction of quantum theory and the theory of hidden parameters can be settled in a convincing way (in favor of quantum theory, of course) by *experimentum crucis*, unlike most other quantum paradoxes. The theory of hidden parameters is closely related to the ensemble-statistical interpretation of quantum theory, and this is the reason why such experiments (real or imaginary) add serious evidence to the eternal debate between the advocates of statistical (Einstein) and orthodox (Bohr's or Copenhagen) interpretations, and their numerous modifications."

And in the next paragraph:

"It is yet possible that in the future this debate will be settled (perhaps in favor of a third way), and historian of science will be see it as a vivid example of fallacies which plagued even the brightest minds of the past"

So the authors do not rule out possibility of a third way.

The question put by Einstein, arises, naturally, and in the statistical theory of open systems. Thus incompleteness of the quantum-mechanical description appears by a consequence of inevitable transition from initial reversible equations of the quantum electrodynamics to approximative dissipative equations of the quantum theory in approximation of a continuous medium.

The part of the present work is devoted to of the statistical interpretation of Heisenberg's uncertainty relation.

The literature on basic questions of the quantum theory is very extensive. The references to a number of recent works can be found in the book [10]. In addition to them we shall specify, that recently were published the very interesting books Asher Peres "Quantum theory: Concepts and Methods" [23], Williams C., Clearwater S., "Explorations in Quantum Computing" [24], Ekert A., Josza R., and Penrose R., "Quantum Computation: Theory and Experiment" [25], and B.B.Kadomtsev "Dynamics and Information" [17]. These books release us from the necessity to consider here many basic questions of quantum theory. Only what

are necessary for understanding of the physical phenomena in quantum open systems will be considered.

Our treatment will be largely based on the ideas and methods described in detail in [2,4,9,10,39,40].

## 2 Microscopic and Macroscopic Schroedinger Equations

Imagine a quantum system which consists of an ideal gas of particles and fluctuation electromagnetic field. The role of particles is played by electrons which are either free or bound within hydrogen atoms. On the level of statistical description the electrons may be regarded as microscopic Brownian particles. The electromagnetic field will then act as the thermostat.

Microscopic description of quantum processes in such system starts with the reversible dynamic equations for particles and field. There are two possible approaches. One is based on the Schoedinger equation for the wave function of complete set of variables of particles and field. The other relies on the equation for quantum (operator) wave functions of electron and electromagnetic field. In both cases the initial dynamic equations give, in principle, an exhaustive quantum mechanical description of time evolution of the system under consideration. Such level of description corresponds to a *pure ensemble*.

For the sake of simplicity, we shall describe interaction with the electromagnetic field in the dipole approximation. By  $\hat{\Psi}(r, R, t)$  we denote the operator wave function of electron in hydrogen atom, and by the operator of strength of quantum electric field  $\hat{E}(R, t)$ . In the dipole approximation the equation for the operator wave function may be written as

$$i\hbar \frac{\partial \hat{\Psi}}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial R^2} \hat{\Psi} + U(r) \hat{\Psi} - d \hat{E}(R, t) \hat{\Psi}; \quad U(r) = -\frac{e^2}{r}. \quad (1)$$

where  $d = er$  is the dipole moment of electron. In the dipole approximation the vector  $R$  enters as a parameter. We shall also use the appropriate equation for the "quantum distribution function", the operator density matrix in Wigner's form,

$$\hat{f}(r, p, R, t) = \frac{1}{(2\pi)^3} \int \hat{\Psi}(r + \frac{1}{2}\hbar\tau, R, t) \hat{\Psi}^*(r - \frac{1}{2}\hbar\tau, R, t) \exp(-i\tau p) \frac{(2\pi\hbar)^3}{V} d\tau$$

$$\int \hat{f}(r, p, R, t) \frac{dr dp dR}{(2\pi\hbar)^3 V} = \int |\hat{\Psi}(r, R, t)|^2 \frac{dr dR}{V^2} = 1. \quad (2)$$

This equation follows from the Schroedinger operator equation and has the following form:

$$\begin{aligned} \frac{\partial \hat{f}}{\partial t} + v \frac{\partial \hat{f}}{\partial r} + \hat{E}(R, t) \frac{\partial \hat{f}}{\partial p} + \frac{i}{\hbar(2\pi\hbar)^3} \int \left[ U(r + \frac{1}{2}\hbar\tau) \right] \\ - \left[ U(r - \frac{1}{2}\hbar\tau) \right] \hat{f}(r, p' R, t) \exp [i\tau(p' - p)] d\tau dp' = 0. \end{aligned} \quad (3)$$

We also define the operator of polarization vector as

$$\hat{P}(R, t) = \frac{e}{V} \int r \hat{f}(r, p, R, t) \frac{dr dp}{(2\pi\hbar)^3} = \frac{e}{V} \int r |\hat{\Psi}|^2 \frac{dr}{V}. \quad (4)$$

Then the equation for field operator can be written in the form

Then the equation for field operator can be written in the form

$$\frac{\partial^2 \hat{E}(R, t)}{\partial t^2} - c^2 \frac{\partial^2 \hat{E}}{\partial R^2} = -4\pi \frac{\partial^2 \hat{P}^\perp}{\partial t^2}, \quad \text{div} \frac{\partial^2 \hat{E}(R, t)}{\partial t^2}. \quad (5)$$

Superscript denotes the curl component of the polarization vector. So, we have obtained a closed set of equations for operator wave function and field operator in the dipole approximation. These equations can be used for various quantum electrodynamic calculations in the dipole approximation.

However, owing to the nonlinearity of operator equations, averaging over the Gibbs ensemble results in a very sophisticated system of meshing equations in the moments of different order. Accordingly, we once again have to deal with the problem of closure, so as to obtain a closed system of approximate equations.

There is an analogy with the two approaches of the classical theory. Schroedinger equation for the wave function of the system of particles and field corresponds to the Liouville equation for the distribution function of variables of particles and field. Operator equations of quantum electrodynamics correspond to the classical system of equations for the microscopic phase density and microscopic field  $E^m(R, t)$ . For the system of noninteracting particles, in place of phase density one may use, like it is done in the theory of Brownian motion, the relevant dynamic



(microscopic) distribution function  $f^m(r, p, R, t)$ . The equation for this function follows in the classical limit from (3), and is

$$\frac{\partial f^m}{\partial t} + v \frac{\partial f^m}{\partial r} - \frac{\partial U}{\partial r} \frac{\partial f^m}{\partial p} E^m(R, t) \frac{\partial f^m}{\partial R} = 0, \quad (6)$$

whereas the equation for the microscopic field formally coincides with (5). As a result, we have a closed system of classical dynamic equations.

In quantum mechanics, equation (1) is replaced by Schroedinger equation for deterministic wave function of the electron:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial R^2} + U(r)\psi - dE(R, t)\psi; \quad \int |\psi|^2 \frac{dr dR}{V^2} = 1. \quad (7)$$

As compared with Schroedinger equation for the wave function of the system of particles and field, this equation is approximate but still reversible.

From (7) we may go over to the equation for deterministic quantum distribution function in Wigner's form,

$$\begin{aligned} f(r, p, R, t) = & \\ \frac{1}{(2\pi)^3} \int \psi(r + \frac{1}{2}\hbar\tau, R, t) \psi^*(r - \frac{1}{2}\hbar\tau, R, t) \exp(-i\tau p) \frac{(2\pi\hbar)^3}{V} d\tau & \\ \int f(r, p, R, t) \frac{dr dp dR}{(2\pi\hbar)^3 V} = \int |\psi(r, R, t)|^2 \frac{dr dR}{v^2} = 1. & \end{aligned} \quad (8)$$

Let us also write the corresponding reversible kinetic equation:

$$\begin{aligned} \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} + E(R, t) \frac{\partial f}{\partial p} + \frac{i}{\hbar(2\pi\hbar)^3} \int \left[ U(r + \frac{1}{2}\hbar\tau) \right. & \\ \left. - \left[ U(r - \frac{1}{2}\hbar\tau) \right] f(r, p', R, t) \exp[i\tau(p' - p)] d\tau dp' \right] & \\ = 0, & \end{aligned} \quad (9)$$

which in the classical limit becomes

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial r} + E(R, t) \frac{\partial f}{\partial p} - \frac{\partial U}{\partial r} \frac{\partial f}{\partial p} = 0. \quad (10)$$

We see that Schroedinger equation (7) for deterministic wave function corresponds in the classical limit to the kinetic equation for the

one-particle distribution function  $f(r, p, R, t)$ . Like Schroedinger equation, this equation is reversible since it does not take into account the dissipation due to the interaction of electron with fluctuation electromagnetic field.

Now, to facilitate the transition to the irreversible equations, let us refresh some points from Boltzmann's kinetic theory.

The kinetic Boltzmann equation differs from (10) in that it includes the "collision integral" which takes care of the dissipation due to redistribution of the particle velocities because of collisions between the particles. As before, by  $\tau$  and  $l$  we denote the relaxation parameters - the free path time and length. We also introduce the characteristic parameters of the problem  $T$  and  $L$ .

As we know, important are two extreme cases, corresponding to the approximations of gas dynamics and of free molecular flow. The gasdynamic approximation is used when  $\tau \ll T$ ,  $l \ll L$ . Then the kinetic Boltzmann equation may be replaced by the simpler equations in the gasdynamic functions  $\rho(R, t)$ ,  $u(R, t)$ ,  $T(R, t)$ .

In the opposite extreme, when  $\tau \gg T$ ,  $l \gg L$ , the dissipative term (the "collision integral") in the zero approximation may be dropped. This brings us to the reversible kinetic equation which formally coincides with (10).

Is it possible to exploit this analogy?

Recall first of all that the nature of description is changed dramatically when we go over from the Liouville equation, which carries all information about the motion of particles of Boltzmann gas, to the kinetic Boltzmann equation. To wit, from the system of particles whose motion is described by the reversible Hamilton equations, we go to a continuous medium in six-dimensional space of coordinates and momenta (or coordinates and velocities).

Naturally, the transition to the approximation of continuous medium is associated with restrictions from the side of small scales which depend on the size of the "point". Since the information about the motion of particles within the "point" is lost, the equations of continuous medium must be dissipative and therefore irreversible.

Accordingly, the transition to the approximation of continuous medium changes the entire time symmetry of the original system. In this situation the initial reversible equations of Hamiltonian mechanics for systems with a large number of microscopic degrees of freedom

(Boltzmann gas, or atoms and field) can only be regarded as the "starting point".

Of course, this transition should not be viewed as just a simplification. Rather than that, it is necessitated by dynamic instability of motion of microscopic elements of the system. It is the dynamic instability of motion, combined with uncontrollable small exertions from the surrounding world, that makes the transition to irreversible equations inevitable.

In this connection it is worthwhile to recall that dissipation is usually seen as damping of motion, scattering of energy, loss of information. At nonequilibrium phase transitions, however, which may result in the appearance of new structures (whose sequences form the processes of self-organization), the dissipation plays a constructive role. Persistent space-time dissipative structures are not feasible without dissipation.

This explains why the construction of dissipative equations is of crucial importance for the statistical theory of open systems. The first step in this direction consists in the revision of the concept of continuous medium. As we know, this requires giving a concrete definition of physically infinitesimal scales.

We know how to do this for Boltzmann gas. We have also defined physically infinitesimal scales for a plasma [3-5,10]. On this basis we defined Gibbs ensemble for nonequilibrium processes, and gave examples of kinetic equations for unified description of evolution on both kinetic and hydrodynamic scales.

How then do we apply this knowledge to the quantum system of atoms and electromagnetic field.

Let us return to the classical kinetic equation (10). This equation has many applications. For Boltzmann gas, for example, it can be used for describing free molecular flows, when the characteristic scales of the problem  $T$  and  $L$  are much less than the corresponding relaxation kinetic scales of rarefied gas. The approximation of continuous medium still holds, however, since equation (10) does not carry information about the motion of individual particles (atoms of the gas). This restricts the admissible values of  $T$  and  $L$  from below: they must be much greater than the relevant physically infinitesimal scales,

$$T \gg \tau_{ph}, \quad L \gg l_{ph}. \quad (11)$$

And yet, equation (10) is reversible. How does this comply with the above statement that the processes in continuous medium must be described by irreversible equations?

The answer is that equation (10) holds for Boltzmann gas only in the zero approximation with respect to a small parameter, for instance, to the ratio of the characteristic scale of the problem to the free path length. In the first approximation with respect to  $L/l$  the kinetic equation becomes irreversible. Moreover, the solution of (10) requires supplementing it with a boundary condition which will bring in irreversibility for any real system.

What is the situation in the quantum theory?

### 3 Continuous medium approximation in quantum theory

To take care of the dissipation, we must include into consideration the interaction of atom with fluctuation electromagnetic field. This will result in dissipative kinetic equations for the quantum distribution function (or density matrix) and the field. The appropriate "collision integrals" are determined by the small-scale fluctuations whose characteristic scales are much smaller than the characteristic scales of the kinetic equations.

In this way, the problem of the structure of "continuous medium" is brought up explicitly (or implicitly, in case of formal application of perturbation theory) in the derivation of kinetic equations.

This problem is closely associated with the definition of quantum ensemble. The initial microscopic description corresponds to "pure ensemble", when the entire statistical information is incorporated in the "exact" wave function of the system.

In the pure ensemble, the operator density matrix (for example, in Wigner's representation (see (2)) is expressed via the product of operator wave functions  $\hat{\Psi}(r, R, t)$ . Upon transition to the continuous medium, when the kinetic equations become irreversible, the pure ensemble is replaced by "mixed" ensemble. Then there is no representation in which the density matrix can be expressed via the product of wave functions.

It might seem that the above formulas defy this statement. Indeed, the quantum distribution function (8) for hydrogen atom is defined as the product of wave functions which satisfy Schroedinger equation (7). It is as if we returned to the "pure ensemble" again.

Naturally, there is a fundamental difference between these two definitions of pure ensemble. The first exactly renders the statistical prop-

erties of quantum mechanical description, whereas the second definition of pure ensemble corresponds to the "coarse" approximation of continuous medium. The approximation amounts to neglecting the dissipation altogether.

For the system of  $N$  particles this approximation corresponds to Hartree equation in quantum mechanics, or to self-consistent Vlasov [29,30] approximation in plasma theory, or to Euler approximation in hydrodynamics which disregards the dissipation due to viscous friction and heat conduction.

In order to define the structure of "continuous medium" in quantum theory, we must first of all introduce the characteristic scales of the system in question. The characteristic length and time (frequency) for hydrogen atom are given by

$$r_0 = \frac{\hbar^2}{ne^2}, \quad \omega_0 = \frac{me^4}{2\hbar^3}, \quad v_0 = \frac{e^2}{\hbar}. \quad (12)$$

The first of these defines the characteristic length scale of distribution  $|\Psi(r)|^2$  in the ground state (Bohr's radius  $r_0$ ); frequency  $\omega_0$  determines the energy of the ground state.

Additional parameters arise when we go over to the dissipative kinetic equations; we divide them into two classes.

The first class includes those parameters which characterize the process of relaxation towards equilibrium. They are similar to the relaxation scales  $\tau$ ,  $l$  of Boltzmann gas. In quantum theory, when the relaxation towards equilibrium proceeds from one energy level to another, we may introduce the time of relaxation  $\tau_{nm}$  for each transition. This time is defined by Einstein's coefficient [31,5] for spontaneous transitions,

$$\tau_{nm} = \frac{1}{A_m^n}. \quad A_m^n = \frac{4|d_{nm}|^2\omega_{nm}^3}{3mc^3} \sim \gamma(\omega_{nm}) = \frac{2e^2\omega_{nm}^2}{3mc^3} \quad (13)$$

The Einstein's coefficient  $A_m^n$  is proportional to the coefficient of radiation friction  $\gamma(\omega_{nm})$  at the transition frequency. The following estimate then holds good:

$$A_m^n \sim \mu^3\omega_{nm} \ll \omega_{nm}, \quad \text{where } \mu = \frac{e^2}{\hbar c}. \quad (14)$$

$\mu$  is the "constant of fine structure"

We see that the relaxation for the system of hydrogen atoms, governed by interaction with the fluctuation electromagnetic field, is a slow process compared to the characteristic time interval  $1/\omega_{nm}$ . At equilibrium, the population of levels is described by Boltzmann's formula. At zero temperature the atoms are in the ground state, and the density of distribution of electron's positions in the atom is isotropic. The most probable distance from the nucleus is characterized by Bohr's radius  $r_0$ .

The displacement of spectral lines due to the interaction between hydrogen atom and the fluctuation electromagnetic field is also determined by the same kinetic equations - such as, for instance, Lamb's shift of levels. By order of magnitude, the shift of energy levels is  $\Delta_n \sim \mu^3 |E_0|/n^3$ , where  $n$  is the principal quantum number. Accordingly, for the lower levels this effect is of the same order of smallness as the broadening of spectral lines because of spontaneous transitions.

Now let us try to answer the following question: does the distribution of electron's positions in the ground state display a finer structure than this would follow from Schroedinger equation (7)? What are the smallest times which the isotropic distribution takes to become established in the ground state?

It is obvious that the structure of "continuous medium" is defined by the scales smaller than those of (12). In classical and quantum electrodynamics there are two scales which are smaller than those of the theory of hydrogen atom:

$$r_e = \frac{e^2}{mc^2} \sim \lambda_C \quad ; \quad \lambda_C = \frac{\hbar}{mc} \sim \mu r_0 \ll r_0, \text{ where } \mu = \frac{e^2}{\hbar c} \quad (15)$$

The first is the "classical electron radius". In quantum electrodynamics, the main contribution to the effective cross section of scattering of photons by free electrons is proportional to  $r_e^2$  (see Sect.2).

The second parameter is Compton's length, which defines the shift of wavelength in case of scattering of  $x$ -rays by free electrons. Which of these characterizes the commencement of irreversibility, and thus defines the finest structure of "continuous medium" in quantum mechanics?

Observe that the study of the structure of "continuous medium" in quantum mechanics is stimulated, in particular, by those difficulties which are associated with the attempts to calculate quantum fluctuations, like fluctuations of velocity of free electron moving in a fluctuation electromagnetic field [9,10]. The solution of this problem will allow us

to define the smallest relaxation times over which the irreversibility sets in. This will also help us to define the new scales which are pertinent to the structure of "continuous medium" of atoms and field.

At the zero temperature the atoms are in the ground state, and the density of electron's position in the atom is isotropic. The most probable distance from the nucleus is characterized by Bohr's radius  $r_0$ .

We shall try to answer on the following question: does the distribution of electron's position in the ground state display a finer structure? What are the smallest times which the isotropic distribution takes to become established in the ground state?

It is obvious that the structure of "continuous medium" is defined by the scales smaller than those of (12). In classical and quantum electrodynamics there are two scales which are smaller than Bohr's radius  $r_0$ .

The first one is the "classical electron radius". In the next section we shall see that the main contribution to the effective cross section of scattering of photons by free electrons (the Thomson's scattering) is proportional to  $r_0^2$ .

The second parameter is Compton's length, which defines the shift of wavelength in case of scattering of  $x$ -rays by free electrons.

Which of these two characterizes defines the finest structure of "continuous medium" the size of "point" in quantum mechanics?

Remark that the study of the structure of "continuous medium" in quantum mechanics is stimulated, in particular, by those difficulties which is associated with the attempts to calculate quantum fluctuations, like fluctuations of velocity of free electron moving in a fluctuation electromagnetic field [9,10]. The solution of this problem gives us possibility to define the smallest relaxation time over which the irreversibility sets up. This smallest relaxation time is defined by the classical electron radius  $r_0$  :

$$\tau_{rel} \sim \tau_e = \frac{r_0}{c} \equiv \frac{1}{\Gamma}; \quad \Gamma = \frac{3mc^3}{2e^2} \sim \frac{c}{r_e}. \quad (16)$$

Now we shall show that the scales  $r_e, \tau_e$  characterize a motion of free electron in equilibrium electromagnetic field.

#### 4 Electron in Equilibrium Electromagnetic Field

We use the equation of motion of electron with radiation friction, external field, and Langevin force. The latter, as usual in the theory of

Brownian motion, reflects the structure of the medium. In the dipole approximation the forces are assumed to depend only on the time. Then

$$\frac{dv}{dt} - \frac{1}{\Gamma} \frac{d^2v}{dt^2} = \frac{1}{m} F(t) + \frac{1}{m} F_L(t), \text{ where } \frac{1}{\Gamma} = \frac{2e^2}{3mc^3} \sim \frac{r_e}{c} \quad (17)$$

is the time parameter which by order of magnitude is the time taken by photon to travel over the distance equal to the classical electron radius  $r_e$ . Traditionally, the dissipative term is defined by the radiation friction and is proportional to the third derivative. This leads to certain grave complications, and even to a contradiction with the second law of thermodynamics, as we shall see in the next section.

On the basis of equation (17) we are going to treat two problems:

- (1) scattering disregarding fluctuations, and
- (2) Brownian motion of free electron.

Recall that equation (17) only holds in the dipole approximation, when the wavelength  $\lambda$  is much greater than the electron radius:

$$\frac{\lambda}{2\pi} = \frac{c}{\omega} \gg r_e \sim 10^{-13} \text{ cm}, \text{ or } \omega \ll \frac{c}{r_e} \sim \Gamma. \quad (18)$$

Observe that these conditions do not involve Planck's constant: quantum effects become manifest on larger space-time scales. For Compton's effect, for instance, such scales are the Compton's length and the corresponding frequency:

$$\lambda_C = \frac{\hbar}{mc} \sim \frac{1}{\mu} r_e \gg r_e, \quad \omega_C = \frac{mc^2}{\hbar}. \quad (19)$$

Assume now that the Langevin force is absent, and the external force is harmonic,  $F(t) = eE \cos(\omega t)$ . Then the total effective cross section is given by the known formula

$$\sigma = \sigma_T \frac{1}{1 + \omega^2/\Gamma^2} \equiv \sigma_T \frac{1}{1 + 2\pi\sigma_T/3\lambda^2}, \quad \sigma_T = \frac{8\pi}{3} r_e^2. \quad (20)$$

Under conditions (18) this expression reduces to Thomson's formula for  $\sigma_T$ .

The effective cross section is defined by the classical electron radius, which is the smallest of the scales (15) of quantum electrodynamics and does not depend on Planck's constant. Thomson's cross section delimits



the region where the quantum theory ("wave mechanics") does not work. Indeed, the uncertainty relation does not allow for the existence of de Broglie waves with  $\lambda_B \sim r_e$ .

It follows that the scales smaller than or about equal to  $r_e$  must be excluded from quantum-theoretical consideration. This gives us grounds for accepting the classical electron radius for the minimum point size when using Schroedinger equation for deterministic wave function  $\psi$  or, in other words, when going over from the system of particles to the approximation of "continuous medium". Accordingly, the smallest in the hierarchy of physically infinitesimal scales may be defined as

$$l_{ph} \sim r_e \sim \mu \lambda_C \ll \lambda_C. \quad \tau_{ph} \sim \frac{r_e}{c} \sim \frac{1}{\Gamma}, \quad V_{ph} \sim r_e^3. \quad (21)$$

We see that they are  $\mu$  times as small as the relevant minimum scales in quantum theory for the system in question. These classical scales must be eliminated from the initial microscopic dynamic equations. It is the corresponding smoothing procedure that first brings irreversibility into the quantum theory.

Since the structure of effective cross section of photon scattering by electrons is so important, it will be worthwhile to consider another method of deriving the expression for the effective cross section. For this we take advantage of the linkage between the effective cross section and the imaginary part of the refraction index  $\kappa$ , or the coefficient of extinction  $h$ . The values of  $\kappa$  and  $h$  are linked by

$$h = 2\kappa \frac{\omega}{c} = \frac{4\pi}{\lambda_0} \kappa, \quad \lambda_0 = \frac{2\pi c}{\omega}. \quad (22)$$

We also use the linkage between  $h$  and the effective cross section,

$$h = n\sigma, \quad \text{where } n = \frac{N}{V} \quad (23)$$

is the number of particles in unit volume.

We can express the coefficient of extinction and the effective cross section from dielectric permittivity of the medium.

Return now to the problem of defining the effective cross section in terms of the "dielectric properties" of the system of electrons and field. Using the equation of motion (17) for free electron we arrive at the following expression for the permittivity:

$$\varepsilon = 1 - \frac{4\pi e^2}{mV} \frac{1}{\omega^2 + i\omega\gamma(\omega)}. \quad (24)$$

Now we write the appropriate equations for the real ( $n$ ) and the imaginary ( $\kappa$ ) parts of the refraction index:

$$\begin{aligned} n^2 - \kappa^2 &= 1 - \frac{4\pi e^2}{mV} \frac{1}{\omega^2 + \gamma^2(\omega)} \\ n\kappa &= \frac{2\pi e^2}{mV} \frac{\gamma(\omega)/\omega}{\omega^2 + \gamma^2(\omega)} \end{aligned} \quad (25)$$

and use (22) for  $\kappa$  and  $h$ . Since the refraction index is close to one in the pertinent range of frequencies for all realistic values of  $V$ , and the coefficient of damping  $\kappa$  is small, the coefficient of extinction may be written as

$$h = \frac{4\pi e^2}{mV} \frac{\gamma(\omega)}{\omega^2 + \gamma^2(\omega)} \frac{1}{c}. \quad (26)$$

Finally, using (23) and the definition of  $\Gamma$  (17), we find the desired expression for the effective cross section  $\sigma$ :

$$\sigma = \frac{4\pi e^2}{m} \frac{\gamma(\omega)}{\omega^2 + \gamma^2(\omega)} \equiv \frac{4\pi e^2}{m} \frac{\Gamma}{\omega^2 + \Gamma^2} \equiv \sigma_T \frac{1}{1 + \omega^2/\Gamma^2}, \quad (27)$$

which coincides with the expression obtained for the scattering of photons.

The first equation in (27) implies that the effective cross section in the dipole approximation is proportional to the coefficient of radiation friction. When the dissipation is completely disregarded, the effective cross section goes to zero.

So, we have calculated the effective cross section of scattering by free electrons, and the results seem to be quite reasonable. Fundamental difficulties arise, however, when the same equation (17) is used for calculating the fluctuation characteristics: the condition of statistical equilibrium in the system of electrons and field is violated.

## 5 Equilibrium State of System of Electrons and Field. Fluctuation-Dissipation Relation

Return to the equation of motion of electron (17) with radiation friction, assuming now that the external field is absent, and the fluctuations of the field are taken into account by introducing the Langevin force. Let us write the appropriate equation for the Fourier components:

$$(-i\omega + \gamma(\omega))v_\omega = \frac{f(\omega)}{m} = \frac{e}{m}E_\omega. \quad (28)$$

The spectral density of fluctuation field is given by Planck's formula

$$(EE)_\omega = 4\pi \frac{\omega^2}{c^3} k_B T_\omega, \quad k_B T_\omega = \frac{1}{2} \hbar \omega \coth \frac{\hbar \omega}{2k_B T}. \quad (29)$$

Hence, given the definition of the coefficient of radiation friction, we obtain the following expression for the spectral density of Langevin force:

$$(ff)_\omega = e^2 (EE)_\omega = 32m\gamma(\omega)k_B T_\omega, \quad \gamma(\omega) = \frac{2e^2\omega^2}{3mc^3}. \quad (30)$$

Then the frequency distribution of the mean kinetic energy of electron is

$$\frac{m \langle v^2 \rangle_\omega}{2} = \frac{\gamma(\omega)}{\omega^2 + \gamma^2(\omega)} 3k_B T_\omega. \quad (31)$$

For the system of electrons and field we have obtained the quantum Nyquist's formula; the dissipation is characterized by the coefficient of radiation friction.

However, relations (30), (31) lead to a contradiction with the second law of thermodynamics (9,39,40), since the condition of equilibrium of "Brownian particle" and field is violated. Indeed, at zero temperature, for instance, we get the following for the mean kinetic energy of electron at equilibrium:

$$\frac{m \langle v^2 \rangle}{2} = \frac{3}{4\pi} \int \frac{2\gamma(\omega)}{\omega^2 + \gamma^2(\omega)} \frac{1}{2} \hbar \omega d\omega = \frac{3}{4\pi} \hbar \Gamma \ln \left( 1 + \frac{\omega_{\max}^2}{\Gamma^2} \right). \quad (32)$$

We see that at the state of equilibrium the mean kinetic energy is proportional to the dissipative factor  $\Gamma$ . This is a contradiction with the second

law of thermodynamics, which states that the mean energy characteristics are functions of state and therefore cannot depend on dissipative parameters.

This calls for a revision of the structure of the coefficient of radiation friction, and the coefficient of friction in the field equation. Traditionally, the coefficient of radiation friction is found from the lagging solution of the field equation in dipole approximation. The first nonvanishing term with odd time derivative of the polarization vector in the expansion with respect to lagging defines the dissipative term in the equation of motion of electron (17).

Observe that by this method the dissipation is found from solution of the reversible field equation. The mechanism of dissipation remains unknown. In particular, it is not clear what are the scales on which the irreversibility sets in. As a matter of fact, the above-mentioned fundamental difficulties result from the loose definition of dissipation in the field equations (and hence in the equation of motion of electron).

Let us try to introduce dissipation into the equation of motion of electron from a somewhat different standpoint.

The main assumption will consist in the following.

Dissipation arises when small scales are eliminated – the scales within  $r_e$  and  $\tau_e$ , which cannot be included in the equations of quantum mechanics. Recall that the characteristic non-quantum time interval  $\tau_e$  is linked with the dissipative factor  $1/\Gamma$  by (16). Smoothing, as will be shown in the next section, brings the dissipative term  $\Gamma v$  into the equation of motion of electron in place of the traditional term with radiation friction. So, instead of (17), we get the following equation of motion:

$$\frac{dv}{dt} + \Gamma v = \frac{1}{m} F(t) + \frac{1}{m} F_L(t), \text{ where } \Gamma = \frac{3mc^3}{2e^2}. \quad (33)$$

The effective cross section is now given by

$$\sigma = \frac{4\pi e^2}{m} \frac{\gamma(\omega)}{\omega^2 + \gamma^2(\omega)} \equiv \sigma_T \frac{1}{1 + \omega^2/\Gamma^2}. \quad (34)$$

Note that this expression for  $\sigma$  coincides with (27) established on the basis of equation (17) with radiation friction. The advantages of the new approach become obvious in the calculation of fluctuations.

Let us quote the relevant results for the fluctuations of velocity.

Using the Langevin equation (33) instead of (31), we get the following expression for spectral density of fluctuations of velocity:

$$\frac{m\langle v^2 \rangle_\omega}{2} = \frac{\Gamma}{\omega^2 + \Gamma^2} 3k_B T; \quad \frac{1}{2\pi} \int \frac{m\langle v^2 \rangle_\omega}{2} = \frac{3}{2} k_B T \quad (35)$$

Now there is no contradiction with the second law of thermodynamics, since these equations express the existence of equilibrium between the "Brownian particle" (electron) and the electromagnetic field. The mean kinetic energy is then determined only by the temperature, and does not depend on the dissipative parameters. This result agrees with the fact that the equilibrium solution of the corresponding quantum kinetic equation is the Maxwell distribution.

Observe that for a free electron the result (35) has the classical form. Because of the existence of the so-called relict radiation (background fluctuation electromagnetic radiation with the temperature  $T = 2.7K$ ), the intensity of the electron velocity fluctuations is always nonzero.

The above-introduced characteristic frequencies form a chain of inequalities

$$\Gamma \gg \omega_{\max} \gg \gamma_{\max} \sim \frac{c}{r_0} \gg \omega_0 \gg \gamma(\omega_0), \quad (36)$$

and the relevant inequalities for the characteristic lengths are

$$r_e \ll \lambda_C \ll r_0 \ll \lambda \ll \frac{c}{\gamma(\omega_0)}. \quad (37)$$

These inequalities allow us to distinguish between the fast (small-scale, or fine-grained) and the slow (large-scale, or coarse-grained) fluctuations. The line of demarcation may be defined, for instance, by the time interval  $1/\omega_{\max}$  and the Compton length  $\lambda_C$ . Of course, the separation is to a large extent a matter of convention, and will depend on the particular problem under consideration. In any case, however,  $V_C = \lambda_C^3$  is the minimum volume which may be regarded as the "point" of "continuous medium" in quantum theory. This volume is much larger than the "classical volume of electron"  $V_e$ . Their ratio defines the corresponding small "density parameter"

$$\varepsilon_e = \frac{V_e}{V_C} \sim \mu^3. \quad (38)$$

In the parlance of the kinetic theory of gases and plasmas, the small-scale fluctuations may be referred to as "collisionless", since their characteristic scales are much less than the characteristic scales of "collision integrals".

So, using the example of calculation of the effective cross section of scattering of photons by free electrons, and the electron velocity fluctuations, we have demonstrated the efficiency of dissipation introduced by means of smoothing with respect to "non-quantum" small scales  $\tau_e$ ,  $r_e$ . Accordingly, these scales act as "hidden parameters" (or "hidden scales") in quantum theory.

As indicated above, the feasibility and the expedience of hidden parameters for many years have been a matter of debate. Let us reproduce here some passages from de Broglie's paper [24].

"Bohr – a Rembrandt of contemporary physics, who sometimes displays a kind of taste for chiaroscuro – said that a particle is a "loosely defined entity within finite space-time limits." [...]

"In classical theories, such as the kinetic theory of gases, the probability laws are regarded as the result of our ignorance of the perfectly deterministic though erratic and sophisticated movements of innumerable gas particles. Had we known the positions and velocities of each molecule, we could have made a precise calculation of evolution of the gas. In practice, however, we have to rely on probabilities because we do not know these hidden parameters. [...]

"The purely probabilistic formulation of quantum mechanics, however, rejects such interpretation of probability laws: these laws are due not to the fact that we do not know the hidden parameters, which evidently are the coordinates and velocities of particles, but rather to the non-existence of hidden parameters, since a particle may only be detected with definite coordinates and definite velocity at the instant of observation or measurement. Thus, in quantum mechanics the probability is no longer the result of ignorance. [...]

"A couple of decades ago, Neumann has proved that the experimentally tested probability laws in the form accepted in wave mechanics are incompatible with the existence of hidden parameters. For a long time I regarded Neumann's theorem as irrefutable. Now I have some reservations. [...] His proof is mainly based on the assumption that all probability distributions allowed by wave mechanics are physically realized irrespective of whether the experiment which realizes one of such

states is carried out or not.”

Then de Broglie turns to analyzing Bohm’s paper:

”Like Bohm, one can assume that while the probabilistic interpretation is good for predicting the events on the atomic scale (from  $10^{-8}cm$  to  $10^{-13}cm$ ), it may become inadequate on the nuclear scale ( $10^{-13}cm$ ). [...]

”Einstein and Schroedinger have flatly refused to accept the probabilistic interpretation of quantum mechanics. As far as I know, there has been no satisfactory answer to Einstein’s objections. [...]

”Bohr’s answers are characterized by that same ”chiaroscuro” of which I have spoken above”.

”It seems that physics urgently needs the possibility to introduce the electron ”radius”, like in the old Lorentz theory. [...] One must avoid the danger of too much faith in the purely probabilistic interpretation of quantum mechanics eventually making this possibility futile. [...] Wave mechanics, the way it is currently taught, has to a large extent exhausted its ability to explain things. [...]

”The progress of science is continually harassed by the tyranny influence of certain concepts which in the course of time have become dogmas. Because of this, the principles which have been recognized as final must be subjected to most thorough revision.”

After this digression into history, let us return to formulas (27), (34) for the effective cross section of scattering of photons by free electrons. From these expressions it follows that the time dispersion is only observable in the ultrarelativistic domain, since the ratio

$$\frac{\omega^2}{\Gamma^2} \sim \left( \frac{\hbar\omega}{mc^2} \right)^2 \mu^2, \quad \mu = \frac{e^2}{\hbar c} \quad (39)$$

is very small.

We see that even in the relativistic region, when the ratio  $\hbar\omega/mc^2$  is of the order of unity, the cross section is practically independent of the frequency and is determined by the Thomson cross section. Quantum electrodynamic calculation of the effective cross section leads to the known Klein–Nishina formula. In the linear approximation with respect to dimensionless parameter  $\hbar\omega/mc^2$  we get

$$\sigma = \sigma_T \left( 1 - 2 \frac{\hbar\omega}{mc^2} \right). \quad (40)$$

The correction is proportional to the ratio of the Compton length to the wavelength,  $\lambda_C/\lambda$ . In the non-quantum approximation, when Planck's constant is zero, we return to the classical Thomson formula.

So, the "non-quantum" Thomson formula for the effective cross section holds not only for the optical, but also for the x-ray frequency range—that is, for the domain of quantum mechanics. This seemingly paradoxical result reveals certain limitations of quantum mechanics (or, more precisely, wave mechanics) on the scales of the order of the classical free electron radius  $r_e \sim 10^{-13}cm$ , in harmony with de Broglie's statement quoted above.

Now, however, there is a possibility of giving a more concrete definition of "hidden parameters" ("hidden scales"): they are those small scales which fall beyond the scope of applicability of quantum mechanical Schroedinger equation regarded as an equation of continuous medium neglecting the dissipation. Elimination of these small scales results in dissipative equations for the density matrix with appropriate "collision integrals".

We shall make just one step on the way towards construction of dissipative equations of quantum theory. Namely, we shall see how the structure of the initial microscopic equations of quantum theory is affected by smoothing with respect to small but non-quantum scales.

In first, we shall show that the scales  $r_e, \tau_e$  define the fine structure of the atom's ground state.

## 6 Establishment of ground state in atoms-oscillators

### 6.1 Bohr's scales for open systems

Consider a gas of noninteracting atoms in equilibrium electromagnetic field. Following to Thomson model, we regard the atom as a sphere of radius  $r_0$  which carries a uniformly distributed positive charge. The electron vibrates with respect to the center of the sphere. The frequency of oscillations is found from the expression for the elastic force

$$e\mathbf{E} = -\frac{e^2}{r^3}\mathbf{r} = -m\omega_0^2\mathbf{r}; \quad \omega_0 = \sqrt{\frac{e^2}{mr^3}}. \quad (41)$$

For the sake of simplicity we consider one-dimensional oscillations of the atom-oscillators.



In the equilibrium state the mean kinetic and potential energies are defined by expressions:

$$m \langle v^2 \rangle = m\omega_0^2 \langle x^2 \rangle = kT_{\omega_0} = \hbar \frac{1}{2} \hbar \omega_0 \coth \frac{\hbar \omega_0}{2kT}. \quad (42)$$

For atom-oscillators at room temperature the  $\coth \hbar \omega_0 / 2kT$  is close to one, and the amplitudes  $v_0, x_0$  are therefore defined by the zero-point energy  $\hbar \omega_0 / 2$ . So we have the following expressions for the frequency and amplitude of atom-oscillators which are in equilibrium with electromagnetic field:

$$\omega_0 = \sqrt{\frac{e^2}{m x_0^3}}; \quad m v_0^2 = m \omega_0^2 x_0^2 = \frac{1}{2} \hbar \omega_0. \quad (43)$$

We find from them the expressions the amplitude, frequency and velocity of atom-oscillator [9,10]

$$x_0 = \frac{\hbar^2}{m e^2}, \quad \omega_0 = \frac{m e^4}{\hbar^3}, \quad v_0 = \frac{e^2}{\hbar}, \quad (44)$$

which are known in quantum mechanics as Bohr's parameters. Now these parameters are not consequence of Schroedinger equation for atom-oscillators, but they represent the condition of equilibrium between atom-oscillators and electromagnetic field.

## 6.2 Physically infinitesimal scales. Fast and slow diffusion

To establish a kind of the Langevin and Fokker-Plank equations, we shall enter physically infinitesimal scales for considered system of atoms-oscillators in a thermal electromagnetic field. For this purpose before let's remind definition of the appropriate scales for a plasma and considered system.

For plasma physically infinitesimal scale of length was defined by the Debye radius  $r_D$ . To  $r_0$  is comparable here the Bohr radius  $x_0$ , as both of them are the minimal scales, of the kinetic theory. Really, just these scales define the minimal volumes, which structure is not described by the appropriate equations.

To volume come on one particle of plasma  $v_0 = 1/n$ , is comparable volume  $\lambda_C^3$  with Compton's length  $\lambda_C$ . It is justified by that, that the

Compton length is the minimal *quantum scale*. Accordingly, to this number of particles in the Debye sphere  $N_D = nr_D^3 \approx r_D^3/v_0$  we compare a number of the minimal quantum volumes in sphere to the Bohr radius

$$N_{\hbar} = \frac{r_0^3}{\lambda_C^3} \approx \frac{1}{\mu^3}; \quad \mu = \frac{e^2}{\hbar c}. \quad (45)$$

Thus, number "of particles" - number of the minimal quantum volumes in the sphere with the Bohr radius in inverse proportion cube of constant thin structures  $\mu^3$ . Thus, order of a grade  $\mu^3$  corresponds to the plasma parameter.

Let's remind, at last, that in plasma the physically infinitesimal temporally interval is defined by time of spatial diffusion on volume with the Debye radius. By analogy we shall define the physically infinitesimal interval for considered system by expression  $\tau_{ph} = r_D^2/D_{(x)}$ . Here  $D_{(x)}$  - uncertain while a diffusion coefficient on coordinate  $x$ . It is necessary to determine physical sense order a grade  $\tau_{ph}$  (or  $D_{(x)}$ ).

It is natural to determine  $\tau_{ph}$  in the minimal relaxation time for system atoms - fields  $\tau_e$ . This time interval is possible to determine and as the attitude of the Bohr period corresponding to Bohr frequency  $\omega_0$  to number of the minimal quantum volumes in sphere with Bohr radius  $N_{\hbar}$ .

In result we come for considered quantum system to the following definition for physically infinitesimal scales and the diffusion coefficients on variable  $x$

$$\tau_{ph} \approx \tau_e \approx \frac{r_e}{c} \approx \frac{1}{\Gamma}; \quad l_{ph} \approx x_0; \quad D_{(x)} \approx \frac{\omega_0^2 x_0^2}{\gamma(\omega_0)} \approx \Gamma x_0^2. \quad (46)$$

Let's also remind, that  $\gamma(\omega_0)$  - the coefficient of radiating friction on the Bohr frequency

$$\gamma \equiv \frac{\omega_0^2}{\Gamma} \approx \frac{2e^2\omega_0^2}{3mc^3}. \quad (47)$$

From the formula for the diffusion coefficient  $D_{(x)} \approx \omega_0^2 x_0^2 / \gamma(\omega_0)$  follows the appropriate expression for the diffusion coefficient on speed

$$D_{(v)} = \gamma(\omega_0) v_0^2 = \gamma(\omega_0) \frac{kT\omega_0}{m}. \quad (48)$$

Thus, spatial diffusion is the most fast relaxation process. Opposite, the diffusion on speeds is the most slow relaxation process. The appropriate characteristic time

$$\tau_{D(v)} \sim \gamma^{-1} \gg \Gamma^{-1}. \quad (49)$$

For definition of the Fokker–Planck equation for the distribution function in a phase space  $(x, v)$  we shall write in first the appropriate Langevin equations.

### 6.3 Langevin equations for atoms in a thermal field

The given expressions for the diffusion coefficients define a intensity of the appropriate Langevin sources. With it we have, following, system of the Langevin equations:

$$\begin{aligned} \frac{dx}{dt} + \Gamma x = y_{(x)}; \quad \langle y_{(x)} \rangle = 0; \quad \langle y_{(x)}(t)y_{(x)}(t') \rangle = 2D_{(x)}\delta(t-t'); \\ \frac{dv}{dt} + \gamma v + \omega_e^2 x = y_{(v)}(t) \quad (50) \\ \langle y_{(v)} \rangle = 0; \quad \langle Y_{(v)}(t)y_{(v)}(t') \rangle = 2D_{(v)}\delta(t-t'). \end{aligned}$$

Introducing here of  $\delta$ - functions is supposed, that correlations time  $\tau_{cor}$  of a random source  $y_{(x)}$  in the equation for the coordinates are less than time of fast relaxation  $\tau_{cor} \ll \tau_e \sim R_e/c$ . The existence of such inequality is supposed already in the name for  $\tau_e$ : "the physically infinitesimal time". It will be excluded from final kinetic equations equations. Their role will be shown only to demonstration that spatial diffusion on volume with Bohr radius, i.e. the establishment of the ground state occurs on small times  $\tau_e \sim r_e/c$ .

Width of  $\delta$ - functions in a source  $y_{(v)}$  is defined by larger time correlation  $\tau_{cor} \sim \tau_e \sim 1/\Gamma$ .

Let's consider now the appropriate the Fokker-Planck equation.

## 7 Fokker-Planck equation for atoms in a thermal field

### 7.1 Fast and slow processes

The transition from the Langevin equations to the Fokker–Planck equation is carried out on by the standard method. In result we come to the equation for the distribution function  $F(x, v, t)$

$$\begin{aligned} \frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \omega_0^2 x \frac{\partial f}{\partial v} = \frac{\partial}{\partial v} \left[ D_{(v)} \frac{\partial f}{\partial v} \right] + \frac{\partial}{\partial v} [\gamma v f] + \\ \frac{\partial}{\partial x} \left[ D_{(x)} \frac{\partial f}{\partial x} + \frac{\omega_0^2 x}{\gamma} f \right]. \end{aligned} \quad (51)$$

Thus, the second dissipative term in the Fokker–Planck equation is defined by spatial diffusion. Let's remind, that for physically infinitesimal time is accepted the diffusion time on volume with Bohr radius. The Bohr radius, similarly to the Debye radius in plasma, plays here a role of the physically infinitesimal length.

The Fokker–Planck equation describes relaxation processes on all temporal scales. As a result of complete relaxation the equilibrium state with following distribution is established:

$$f(x, v) = C \exp \left( -\frac{mv^2/2 + m\omega_0^2 x^2/2}{kT_{\omega_0}} \right), \quad kT_{\omega_0} = \frac{1}{2} \hbar \omega_0 \coth \frac{\hbar \omega_0}{2kT} \quad (52)$$

Let's consider now the possibility of simplification of the Fokker–Planck equation.

## 7.2 Exclusion of fast relaxation

Let's take advantage of that the time of spatial diffusion it is a lot of less not only of slow time relaxation on speeds, but also for period of fluctuations, i.e., has a place a double inequality.

$$\tau_{(v)} = \frac{v_0^2}{D_{(v)}} = \frac{1}{\gamma} \gg \frac{1}{\omega_0} \gg \tau_{(x)} = \frac{x_0^2}{D_{(x)}} = \frac{1}{\Gamma}. \quad (53)$$

"The Force of inequalities" is expressed through the constant of thin structure. It is possible, thus, to allocate "fast" and "slow" stages of evolution to the full equilibrium state with the distribution function (52). It gives the basis to proceed from the equation (51) to the more simple equation. By excluding of fast movement it is possible to make as in the equations Langevin equations, so and in the Fokker–Planck equation. Let's go on the first ways. Shall present a dependence coordinate and velocity on time in the form:  $x = x(t, \varepsilon t)$ ;  $v = v(\varepsilon t)$ .

Carry out integration in the first Langevin equation on a time interval such, that

$$\frac{1}{\omega_0} \gg \Delta t \gg \tau_{(x)} = \frac{x_0^2}{D_{(x)}} = \frac{1}{\Gamma} \quad (54)$$

and also we shall enter the notation for the smoothed function

$$\tilde{x} = \int_0^{\Delta t} x(t-t') \exp(-\Gamma T') \frac{dt}{\Delta t}; \quad \tilde{v}(t) = v(t). \quad (55)$$

Let's carry out the appropriate smoothing the derivative  $dx/dt$ . Using only the basic term. Then

$$\int_0^{\Delta t} \frac{d}{dt} x(t-t') \exp(-\Gamma t') \frac{dt}{\Delta t} \approx -\Gamma \tilde{x}. \quad (56)$$

Let's carry out the appropriate operation of smoothing and for Langevin sources. According to the given definition of correlation times, we have:

$$\tilde{y}_{(x)}(t) = 0; \quad \tilde{y}_{(v)}(t) = \int_0^{\Delta T} x(t-t') \exp(-\Gamma t') \frac{dt}{\Delta t} = y_{(c)}(t). \quad (57)$$

In result we obtain the following more simple system of the Langevin equations

$$\frac{d\tilde{x}}{dt} = v; \quad \frac{dv}{dt} + \gamma v + \omega_E^2 \tilde{x} = y_{(v)}(t) \quad (58)$$

The moments of the random source  $y_{(v)}(t)$  are defined former expressions. The appropriate equation Fokker-Planck equation for more smooth function has the following form (mark "  $\sim$  " above  $\tilde{X}$  is omitted):

$$\frac{\partial f}{\partial t} + v \frac{\partial f}{\partial x} - \omega_0^2 x \frac{\partial f}{\partial v} = \frac{\partial}{\partial v} \left[ D_{(v)} \frac{\partial f}{\partial v} \right] + \frac{\partial}{\partial v} [\gamma v f]. \quad (59)$$

The diffusion and friction coefficient are defined by the above mentioned formulas. Let's notice, that this equation does not contain scales, accepted for the physical infinitesimal scales. Let's pay the special attention, that in this equation the dynamic terms remain by classical. The quantum character of system is shown only in expression for the diffusion coefficient. Such description, though it and popular in the literature (see, for example, [32,33], are rather simplified. It is necessary, certainly, at take into account quantum effects and in dynamics. For this purpose

it is necessary to take into account, that the quantum distribution function satisfies with two equation (see. Addition to ò.1). From inequalities (53), in particular, we get the inequality  $\tau_{(v)} \gg 1/\omega_0$ , which allows to carry out the further simplification of the equations (59) by averaging over the period of fluctuations. In result we obtain even more simple and more symmetric on  $x, v$  the Fokker–Planck equation:

$$\frac{\partial f}{\partial t} = \frac{1}{2}D_{(v)} \left( \frac{\partial^2 f}{\partial V^2} + \frac{1}{\omega_0^2} \frac{\partial^2 f}{\partial x^2} \right) + \frac{\partial}{\partial v} (\gamma v f) + \frac{\partial}{\partial x} (\gamma x f) \quad (60)$$

It describes only process of slow temporary relaxation with characteristic time  $1/\gamma \gg 1/\omega_0$ . Thus, description of Brownian motion of atoms-oscillators on a basis the Fokker-Planck equations it is possible to carry out with a different degree of details. The most detailed information contains in the equation (51). For it the lost spatial - temporary scales lay in limits  $\tau_e, r_0$ . At transition to the equation (59) the information on spatial - temporary scales, smaller period of oscillation is lost. At last, at transition to the equation (60) the information and on the period of fluctuations is lost So, the spherically symmetric area with the Bohr radius - the ground state, is formed in result of the fast spatial diffusion process in open system of atoms-oscillators in the thermal electromagnetic field. We shall show now, that at a substantiation of the Heisenberg relation of uncertainty, the representation about system of atoms, as about open system, allows better to understand the contents of this fundamental relation, of the quantum theory.

## 8 Statistical presentation of Heisenberg uncertainty principle

### 8.1 Two interpretations of the Heisenberg principle of uncertainty

Let's address to one of the most popular book of the quantum mechanics [34]. Symbolically, that first in it is the paragraph " a *Principle of uncertainty* ". On page 14 of this book is readable:

" Thus, mechanics, with which submit to the nuclear phenomena, - so-called *quantum* or *the wave mechanics*, should be based on representations about movement, essentially differ from representations of the classical mechanics. In the quantum theory there is no concept trajectories of a particle. This circumstance makes the contents so-called of a *principle of uncertainty* - One of the basic principles of the quantum mechanics, open by Heisenberg in 1927.

Rejecting usual representations of the classical mechanics, the principle of uncertainty has, it is possible to tell, by the negative contents. It

is natural, that in itself is completely insufficient for construction on its basis of the new mechanics of particles. In a basis of such theory should lay, of course, any the positive statements, which will be are considered below”.

It is possible to approach to a question of a substantiation of the Heisenberg principle of uncertainty from other position. It is based on representation of the Schroedinger equation, as one of the equations of the mechanics of continuous medium. In this respect, it, as already it was indicated above, analogues to the Euler equation in hydrodynamics, to the Vlasov equation in the theory of plasma and to the Boltzmann equation for a free molecular flow.

As well as any equations of the mechanics of continuous medium, the Schroedinger equation is approached, as in it the information on movements on scales inside points of continuous medium is lost. All equations of continuous medium with inevitability are dissipative.

However, as well as in the numerical above examples of the equations of continuous medium, a dissipation in the Schroedinger equation in attention is not accepted. It can to be justified only in some special cases. However, without taking into account a dissipation it is impossible to describe, for example, emission and absorption of radiation by atoms, evolution to an equilibrium condition. For this purpose it is necessary to use, as it also is made already in during of many years, the quantum dissipative kinetic equations for a matrix of density or appropriate distribution functions. The attempts (see, for example, [17,18,35]) are known, to use and the appropriate Langevin equations directly for wave function.

At use of those or other equations of continuous medium, for example, of Fokker-Planck or Einstein-Smoluchowsky equations, concept of a particle and its trajectory loses sense: in continuous medium there are no particles!.

Let's return to the Heisenberg uncertainty relation.

So, as we already saw above in the theory of gases and plasma and in the theory of Brownian motion, that the equations of continuous medium are a dissipative kinetic equations. On their basis there can be accounted, in particulars for an equilibrium condition, dispersions of coordinates and momenta, which enter into the Heisenberg relation of uncertainty. In what then quantum specific of the Heisenberg relation of uncertainty? On what preconditions the conclusion of this fundamental principle is

based.

Two moments here are essential.

The quantum theory is obliged by the its existence to opening by Planck the quantum of action  $\hbar$ . The length of a wave de Broglie is  $\lambda_B = \hbar/mv$ . Under conditions, when  $\lambda_B$  there is of order or more characteristic scale ( $\lambda_B \geq L$ ), arises the necessity of replacement of numerical representation of physical quantities by the appropriate operators.

Representation of physical quantities as the operators and the existence of the wave function, with which help are accounted the average values of the physical quantities appropriate themes or other operators, enough, as we let's see, for an establishment of the Heisenberg relation of uncertainty. Thus kind of the equation for wave function, i.e. kind of the Schroedinger equation, not is essential. Existence of wave function is important only.

The computation of dispersion of coordinate and momenta can be carried out on a basis of the kinetic equations can be carry out and at differ from zero of temperature  $T$ . In what is the difference from the Heisenberg relation?

The product of dispersions for of coordinate and momenta at  $T \neq 0$  depends, for example for system atoms - oscillators, not only from a constant  $\hbar$ , but also from frequency of oscillators and temperature. In the quantum theory this product remains finite and within the limits of zero temperature. It is defined at  $T = 0$  only by  $\hbar$  and does not depend on parameter of system.

Just in it consists the generality of the Heisenberg relation of uncertainty. It restricts the product not only coordinate and momenta, but also any pair not commuting physical quantities.

Thus, at  $T = 0$  the Heisenberg relation of uncertainty not depends on characteristic parameters of system and, thus, is the general principle of the quantum theory of open systems.

It reflects a condition of balance of system atoms - fields in approach of continuous medium at zero temperature. This state is characterized by differ from zero value of entropy. It defines by a constant, to which tends entropy in a limit  $T \rightarrow 0$ . This result serves an illustration of the Nernst theorem, which (in the formulation of Planck!) says:

At  $T \rightarrow 0$  the entropy tends to constant. At it all processes, occurring at zero temperature, go without change of entropy.



Before to pass to an establishment of the Heisenberg uncertainty principle, we shall consider briefly question concerning of area of scales, in which the quantum description is necessary. We shall see, that it is restricted as with the side small, so and large scales.

## 9 Two "exits" from domain of quantum theory to classical one

### 9.1 "Exit" in the side of the large scales

We saw that in quantum range of frequencies (visible light and  $X$  rays) the effective cross section for the Thomson scattering does not depends on Planck's constant and, therefore, can be obtained on the basis of a classical theory. Thus, for example, all calculations of scattering for laser diagnostics of nonrelativistic are classical. At the same time, the characteristic scales  $r_e$  and  $\tau_e$  are so small that the Heisenberg uncertainty relations are violated.

Thus, there are two "exit" from the domain of quantum theory. One correspond to the region of large scales. The description of transition from the quantum equations to, appropriate, classical ones is not simple problem. It is the consequence, in particular that processes in complex systems are characterized by several de Broglie wavelengths. For example, for system of hydrogen atoms there are, at least two lengths, accordingly, for electrons and atoms, as whole. They are essential differ. It is possible therefore "partial" transition from the quantum description to classical one, when, for example, the movement of atoms as whole, is classical, but the movement of electrons in atoms is described by quantum equations.

Thus, for transition from the quantum description to the classical one it is necessary, that characteristic length  $L$  would be much more than de Broglie wavelength for electrons. This condition restrict "a field of action" of the quantum theory from the side of the large scales.

Naturally, that and in the classical domain it is possible to make the calculations on the basis of the quantum theory, and in final results to do the limiting transition  $\hbar \rightarrow 0$ . It means, that the quantum theory is more general(common) or, otherwise, the classical theory is limiting case for quantum on at  $\hbar \rightarrow 0$ .

9.2 "Exit" in the domain of small scales. "Classical quantum of action"

There is a question: whether are restrictions on the applicability of the quantum theory from the side of small scales? On this question hardly it is possible now to give the complete answer. It is possible to make, nevertheless, some useful remarks [9,10].

Being based on the Bohr's scales  $x_0$  and  $1/\omega_0$ , through constant of thin structure  $\mu = e^2/\hbar c$  introduce smaller scales of length and time:

$$r_e \approx \mu \lambda_C \approx \mu^2 x_0; \quad \tau_E \approx \mu \frac{\lambda_C}{c} \approx \mu^2 \frac{1}{\omega_0}. \quad (61)$$

We see, that from the Bohr's parameters it is possible to make "two steps" in the direction of smaller scales. However, from the appropriate speed in the direction of large speeds it is possible, because of finite quantity of speed of light, to make only one step:

$$\frac{v_0}{\mu} = \frac{e^2}{\mu \hbar} = c. \quad (62)$$

Thus, for parameters of length and time on a first step we come to the Compton scales. It allows on the "first step" to make on the basis the Heisenberg relation two estimations:

$$x_0 m v_0 \sim \lambda_C m c \sim \hbar. \quad (63)$$

On the following step already it is impossible to satisfy with the Heisenberg relation, so as thus scale of length decreases, but the speed already on a first step is equal speed of light and, hence, further to grow can not. In result we come to an opposite inequality - product of dispersions of coordinate and momenta it is a much less of the Planck constant:

$$r_e m c \sim \frac{e^2}{c} \sim \mu \hbar \ll \hbar. \quad (64)$$

We see, that thus there is appeared the classical combination of universal constant dimensionality of action, which is a much less of the Planck constant.

Thus, there is an exit in domain "of the classical theory" in the side of small scales. The words classical theory are put in brackets, as theory is also is quantum, however, the classical quantum of action  $e^2/c$  it is a much less of the Planck constant.

## 10 The oscillated form of Heisenberg relation

As well known from the text books on quantum mechanics, the Heisenberg uncertainty principle follows from a inequality

$$\int \left| \frac{x}{L}\psi + L\frac{d\psi}{dx} \right|^2 \frac{dx}{L} \geq 0; \quad \int |\psi|^2 \frac{dx}{L} = 1. \quad (65)$$

Here  $L$  is any length parameter. Let us  $f(x, p, t)$  is a quantum distribution function - Wigner function (8) then the last inequality we can present in the following form:

$$\int \left( \frac{x^2}{L^2} + \frac{L^2 p^2}{\hbar^2} \right) f(x, p, t) \frac{dx dp}{2\pi\hbar} \geq 1. \quad (66)$$

The left side of this inequality we can present as mean value of energy for harmonic oscillator with the proper frequency is defined relation

$$\omega_0 = \frac{\hbar}{mL^2}; \quad \frac{\hbar^2}{2mL^2} = \frac{1}{2}\hbar\omega_0. \quad (67)$$

Thus

$$\int \left( \frac{m\omega_0^2 x^2}{2} + \frac{p^2}{2m} \right) f(x, p, t) \frac{dx dp}{2\pi\hbar} \geq \frac{1}{2}\hbar\omega_0. \quad (68)$$

This means that the mean value of a harmonic oscillators can not be less than the corresponding zero energy

$$\frac{m\omega_0^2 \langle x^2 \rangle}{2} + \frac{\langle p^2 \rangle}{2m} \geq \frac{1}{2}\hbar\omega_0. \quad (69)$$

The inequality are presented here it is possible to write in the followings forms:

$$L^4 - \frac{\hbar^2}{\langle p^2 \rangle} L^2 + \hbar^2 \frac{\langle x^2 \rangle}{\langle p^2 \rangle} \geq 0; \quad (70)$$

and

$$\omega_0^2 - \frac{\hbar}{m \langle x^2 \rangle} \omega_0 + \frac{\langle p^2 \rangle}{m^2 \langle x^2 \rangle} \geq 0. \quad (71)$$

From this inequality the Heisenberg uncertainty relation follows

$$\langle x^2 \rangle \langle p^2 \rangle \geq \frac{\hbar^2}{4}. \quad (72)$$

In a general case parameters  $L$  and  $\omega_0$  have arbitrary values. For the sign "=" these parameters are not - there are some restrictions on its

$$L^2 = \frac{\hbar}{m\omega_0} = 2\langle x^2 \rangle = \frac{\hbar^2}{2\langle p^2 \rangle} \quad (73)$$

or in other form

$$\frac{\langle p^2 \rangle}{m} = m\omega_0^2 \langle x^2 \rangle = \frac{\hbar^2}{2mL^2} = \frac{1}{2}\hbar\omega_0. \quad (74)$$

### 11 The sign "=". Distribution functions

For the sign "=" the equation (65) has the following solution

$$|\psi(x)|^2 = \frac{1}{\sqrt{2\pi\langle x^2 \rangle}} \exp\left(-\frac{x^2}{2\langle x^2 \rangle}\right). \quad (75)$$

The corresponding solution of the equation (66) it is possible to present as the Wigner distribution for the harmonic oscillator with the proper frequency  $\omega_0$

$$f(x, p) = \frac{\hbar}{\sqrt{\langle x^2 \rangle \langle p^2 \rangle}} \exp\left(-\frac{x^2}{2\langle x^2 \rangle} - \frac{p^2}{2\langle p^2 \rangle}\right). \quad (76)$$

The dispersions  $\langle x^2 \rangle, \langle p^2 \rangle$  are defined by relations (74).

### 12 S-theorem for quantum systems. Relative ordering of states "=", ">"

Of all macroscopic functions, only entropy  $S$  possesses a combination of properties that allow it to be used as a measure of uncertainty in the statistical description of processes in microscopic systems. Entropy being the sole function with properties of a measure of chaos, there is but one option. It is necessary to redefine entropy so that the average energy remains constant in the course of evolution. But evolution in time it is equally possible to consider the evolution of stationary states in open systems at slowly changing of control (governing) parameters. It is for this type of evolution, that criterion was introduced [36,6,10]. This criterion was for the first time formulated for specific cases and called "S-theorem" (Klimontovich, 1983, 1984 (see in [36,10])). Later, its general formulation was suggested, to make possible to direct comparison between the relative degree of order from experimental data [10,36].

Here we shall consider the evolution of quantum states corresponding, accordingly, to the sign " $=$ " and " $>$ " in the Heisenberg relation. In general case, the degree of order of the distinguished state differs, which account for one of them being more chaotic than the other. Let us term it "physical chaos". As a rule, this state is nonequilibrium and more ordered than the equilibrium state. Let us, by assumption, the quantum state corresponding to the sign " $=$ " is most chaotic. For this state the quantum distribution function  $f_0(x, p)$  is determined by the expression (76). The corresponding entropy

$$S_0[x, p] = - \int f_0(x, p) \ln f_0(x, p) \frac{dx dp}{2\pi\hbar} = - \int f_0(x) \ln f_0(x) \frac{dx}{L} - \int f_0(p) \ln f_0(p) \frac{L dp}{2\pi\hbar} \equiv S_0[x] + S_0[p] \quad (77)$$

The mean energy for this state is defined by the zero energy

$$\langle E \rangle = \frac{m\omega_0^2 \langle x^2 \rangle}{2} + \frac{\langle p^2 \rangle}{2m} = \frac{1}{2} \hbar\omega_0. \quad (78)$$

From the inequality (69) the mean energy more then this. But according the S-theorem to determine the relative degree of order it is necessary to compare the states at equal values of the mean energy To satisfy this condition it is necessary to replace the distribution function by the renormalized one

$$f_0(x, p) \rightarrow \tilde{f}_0(x, p). \quad (79)$$

The renormalized distribution function is also Gaussian but with renormalized values  $\langle \tilde{x}^2 \rangle$ ,  $\langle \tilde{p}^2 \rangle$  for dispersions

$$\tilde{f}_0(x, p) = \frac{\hbar}{\sqrt{\langle \tilde{x}^2 \rangle \langle \tilde{p}^2 \rangle}} \exp\left(-\frac{x^2}{2\langle \tilde{x}^2 \rangle} - \frac{p^2}{2\langle \tilde{p}^2 \rangle}\right) \geq 0. \quad (80)$$

To change the mean value of energy we introduce some the non zero temperature  $T$

$$\frac{\langle \tilde{p}^2 \rangle}{m} = m\omega_0^2 \langle \tilde{x}^2 \rangle = k_B T_{\omega_0} = \frac{1}{2} \hbar\omega_0 \coth \frac{\hbar\omega_0}{2k_B T} \geq \frac{1}{2} \hbar\omega. \quad (81)$$

Let us the quantum distribution function  $f(x, p, t)$  characterizes any non-stationary state with the sign " $>$ " in the Heisenberg uncertainty relation. The quantum distribution function  $f(x, p, t)$  may have and negative

values, but the corresponding distribution functions separately for coordinates and momenta in any cases are positive

$$\int f(x, p, t) \frac{Ldp}{2\pi\hbar} = f(p, t) \geq 0, \quad \int f(x, p, t) \frac{dx}{L} = f(x, t) \geq 0. \quad (82)$$

The corresponding entropy

$$S[x, p] = - \int f_0(x, p) \ln f_0(x, p) \frac{dx dp}{2\pi\hbar} = - \int f(x) \ln f(x) \frac{dx}{L} - \int f(p) \ln f(p) \frac{Ldp}{2\pi\hbar} \equiv S[x] + S[p]. \quad (83)$$

To find the necessary value of temperature  $T$  we must solve the equation

$$\int \left( \frac{m\omega_0^2 x^2}{2} + \frac{p^2}{2m} \right) \tilde{f}_0(x, p) \frac{dx dp}{2\pi\hbar} = \int \left( \frac{m\omega_0^2 x^2}{2} + \frac{p^2}{2m} \right) f(x, p) \frac{dx dp}{2\pi\hbar} \equiv \int \frac{m\omega_0^2 x^2}{2} f(x) \frac{dx}{L} + \int \frac{p^2}{2m} f(p) \frac{Ldp}{2\pi\hbar}. \quad (84)$$

The solution of this equation

$$T(t) \geq 0 \quad (85)$$

therefore the choice of the state with the sign "=" in the Heisenberg uncertainty relation as the more chaotic state is correct. In the (84) variable  $t$  for nonequilibrium states plays the role of parameter. Using the expression (80) for the renormalized distribution function  $\tilde{f}_0(x, p)$  and the constancy condition (84) for the average energy, the expression for the entropy difference of states with signs "=", ">" can be present as inequality

$$\tilde{S}[x, p] - S[x, p] \equiv \tilde{S}[x] - S[x] + \tilde{S}[p] - S[p] = \int f(x, t) \ln \frac{f(x, t)}{\tilde{f}(x)} \frac{dx}{L} + \int f(p, t) \ln \frac{f(p, t)}{\tilde{f}(p)} \frac{Ldp}{2\pi\hbar} \geq 0 \quad (86)$$

Thus, the state with the sign "=" in the Heisenberg relation is the most chaotic. The last expression serves as the quantitative measure for relative degree of order any quantum state - stationary or nonstationary - and most chaotic state which corresponds to sign "=" in the Heisenberg uncertainty relation. It is necessary remember that the oscillatory model

was exploited above only in the special case concern of real physical oscillator. The model was considered in much more general. The parameter  $L$  in the previous formulas is some general length parameter. If  $L$  is the size of the system then the relation  $L$  and  $\omega_0$  allows to use "as example" the oscillatory model for description of a free particle motion [6,17,10,39,40].

### 13 About some eternal questions of quantum mechanics

We saw that the concept of "pure ensemble" is clear defined only in the case of complete quantum mechanical description system of atoms and electromagnetic field. In quantum mechanics, however, the term "pure ensemble" is used also in those cases when the description is based on the Schroedinger equation (8) for the deterministic wave function  $\psi(r, t)$  of only of the variables of particles. This approach corresponds to approximation of continuous medium and when the dissipation is not taking into account. On this ground it is impossible to describe transitions between stationary levels accompanied by emission of radiation. Since the description based on the Schroedinger equation is not complete, there exist "hidden parameters" ("hidden scales") which are revealed the use of a more realistic approach to problem under consideration. We have illustrated this possibility with a concrete example of the system of atom-oscillators and field. By introducing the scales of "continuous medium" we were able to take into account the small-scale fluctuation in quantum Fokker-Planck equation. The calculation of small-scale fluctuations is also of interest by itself: these fluctuations, for instance, are definitive for the effective cross section of a scattering of photons by free electrons [9,10]. It is important that the effective cross section in the quantum domain (visible and  $x$ -ray ranges) does not depend on Planck's constant, and can therefore be found from classical calculations. The relevant scales  $\tau_e, r_e$  are so small that Heisenberg's uncertainty relation does not hold. In this way, there are two "exits" from quantum theory. One of these corresponds to the domain of large scales and slow a spatial variations. The other is associated with the transition to the scales which are much smaller than any of the quantum scales of the system in question, which brings us into the realm of "hidden parameters" ("hidden scales"). The second question of this section which was the subject of the famous debate between Einstein and Bohr is "Is Quantum Mechanical Description Complete?" - the title of the famous paper (Einstein, Podolski, Rosen [22]). In the light of arguments developed above, the answer to this ques-

tion is negative. The fact is that quantum mechanical description based on the reversible Schroedinger equation can always be supplemented by the inclusion of fluctuations. Then, owing to the existence of fluctuation-dissipation relations, the dissipation is inevitable. Consequently, any kind of quantum mechanical description is in practice incomplete, and the concept of "pure ensemble" is just an abstraction. The above arguments score in favor of Einstein's standpoint. In this connection it would be interesting to note that the problem of incompleteness of quantum mechanical description had been actually solved by Einstein long before the emergence of quantum mechanics as such. As early as 1916 Einstein formulated the concept of *spontaneous emission* for the case of two-level atom interacting with the equilibrium electromagnetic field [28]. This term emphasizes the inevitability of energy loss by radiation (Einstein 1916). The dissipation thus being inseparable from real processes. The Schroedinger equations for hydrogen atom and other mechanical systems can only be regarded as a useful idealization. At this point we would like to quote Ilya Prigogine, who said [14]: *"...Or, on the contrary, should we argue that nobody has ever seen an atom that would not decay when brought into an excited level? The physical reality then corresponds to systems with continuous spectra, whereas standard quantum mechanics appears only as a useful idealization, as a simplified limiting case"*. Prigogine does not belong to the founding fathers of quantum mechanics. His statement, however, closely echoes the words of Louis de Broglie. De Broglie expressed doubt in the completeness of quantum mechanical description even at the heyday of the quantum theory. His scepticism was not, however, shared by his contemporaries, and so he also abandoned this attitude, becoming one of the most brilliant advocates of the "Copenhagen" formulation of quantum mechanics. He recalled this period in the following words (de Broglie 1952 [24]): *"Some people, remembering that I abandoned my first attempts and used the interpretation of Bohr and Heisenberg in all my works for twenty-five years thereafter, will accuse me of being inconsistent when they see that I am again doubtful and ask myself whether my initial orientation had been right after all. Should I feel like joking, I can reply in Voltaire's words, "Only foolish people never change their minds" "The answer, however, can be more prudent. "The progress of science is continually harassed by the tyrannical influence of certain concepts which in the course of time have become dogmas. Because of this, the principles which have been recognized as final must be subject to most thorough*



revision". At that time these words were the voice of one crying in the wilderness. This can be illustrated with a quotation from the article published in the influential American newspaper:

"The principle of uncertainty has eventually made all contemporary physicists (with the exception of Dr.Einstein) recognize that there is no causality or determinism in nature. Dr.Einstein in majestic solitude has held out against all these concepts of quantum theory" (The New York Times, 30 March 1952).

Yes another pertinent passage is taken from Dirac's paper published shortly before his death (Dirac, 1978 [38]):

"I think it might turn out that ultimately Einstein will prove to be right, because the present form of quantum mechanics should not be considered as the final form. There are great difficulties (which I shall mention later} in connection with the present quantum mechanics. It is the best that one can do up till now. But, one should not suppose that it will survive indefinitely into future. And I think that it is quite likely that at some future time we may get an improve quantum mechanics in which there will be a return to determinism and which will, therefore, justify the Einstein point of view. But such return to determinism could only be made at the expense of giving up some other basic idea which we now assume without question. We would have to pay for it some way which we yet cannot guess at, if we are to reintroduce determinism." We do not wish to comment on these statements:the clarity and boldness of the classics can only be admired. Their words encourage further studies in the quantum statistical theory of open systems, which will be continued in the next part of this Lecture. We tried to define the author's position with regard to the fundamental problems of quantum theory. I shall try go on by this way.

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