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The Structure of the Photon

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Introduction

This paper is dedicated to the celebration of Georges Lochak's 70th birthday. Naturally we cannot give a complete appreciation of all of his merits, his work and activities. Rather, we confine ourselves to that aspect of his activities which is connected with our cooperation and joint interests. This cooperation has tradition: already de Broglie himself was in contact with members of the University of Tübingen, in particular, with G.Möllenstedt. Whilst the topic of the dialogue of de Broglie and Möllenstedt was electron diffraction, our present cooperation is concerned with the further development of de Broglie's fusion theory. Such a development is a matter of actual interest because it could lead to a better understanding of the difficulties of quantized theories and the interpretation of high energy processes with gauge bosons, in particular with photons.

At present the generally accepted theory of light is quantum electrodynamics, and this theory is embedded into the more comprehensive theory of the Standard model. In spite of its success it seems necessary to further discuss the meaning and the limits of this model as by no means all problems are solved. One of the problems is the role which play the gauge bosons in the physical world. Understanding the role of the photon leads to understanding the role of all other gauge bosons and then to the decision whether or not gauge theories are fundamental pieces for the description of nature. In this paper we discuss the aspects of an approach to a quantum theory of light with composite photons which was initiated by de Broglie and further pursued by many authors.

Corresponding to the modern theoretical points of view we consider de Broglie's idea and formalism within the framework of a quantum field theory, i.e., we discuss the question whether quantum electrodynamics can be replaced by a more subtle quantum field theory of composite photons. In any case such an approach modifies the conventional role of the photon, and recently Georges Lochak [1] and Olivier Costa de Beauregard [2] discussed consequences of such a new concept for photons on the quantum mechanical level. In addition, Georges Lochak supported the quantum field theoretic treatment of these problems and participated in this line of approach [3].

This paper is organized as follows: first we discuss difficulties of conventional quantum electrodynamics, then we look for experimental and theoretical hints which support de Broglie's approach. Finally we present the general framework for quantum fields, which allows to formulate composite particle theories, in particular for composite photons [4]. The theoretical basis of this approach is provided by a nonperturbatively regularized nonlinear spinorfield model, and it is demonstrated how the problem of probability interpretation can be solved for composite photon states which can be exactly calculated within this model.

1 Critique of quantum electrodynamics

On account of its unsurpassed agreement between theory and experiment quantum electrodynamics is frequently considered as the best theory which has ever existed in physics, cf. for instance the results with respect to spectroscopic observations, Lohrmann [5], Milonni[6]. However, in spite of these successes many outstanding physicists of the pioneer generation as for instance Dirac, Bohr, Born, Fock, Heisenberg, Pauli, Peierls, Schwinger, and many other leading theoretical physicists came to the conclusion that "the failure of quantum electrodynamics at high energies would require a revolutionary break with current theory"; see the review by Prugovecki [7]. But by the majority of theorists this criticism was ignored in favor of a "conventionalistically instrumentalist approach to quantum physics (quantum engineers)". Nevertheless in the course of time even the high precision experiments and calculations were doubted, see Jaynes [8]. Whilst the criticism of the pioneer physicists offered no hint how to proceed in the further development of the theory, more substantial advices came from the side of constructive quantum field theory. In this domain mathematicians and physicists attempted to construct a continuum quantum field theory as a limit of corresponding lattice theories. They discovered that a resulting ϕ^4 field theory will be trivial. And they formulated "These arguments apply equally to the four-dimensional Yukawa and electrodynamic interactions. If these interactions are all trivial, it would mean that a short distance cut off resulting from the quark interaction is essential to a theory of protons, photons, mesons and electrons as elementary particles" and : "Such a short distance cut off set at the proton radius, for example, would not violate experimental facts of physics" (Glimm and Yaffe [9]). Still another finding is of utmost interest: "The (mathematically rigorous) research revealed unsuspected mathematical anomalies in the treatments underlying mathematically rigorous formulations of the Gupta-Bleuler formalism that have emerged from the adaption by Strocchi and Wightman of the Whightman's axioms to the case of massless quantum fields." (Prugovecki [7]).

To solve these difficulties Prugovecki and others proposed the incorporation of Heisenberg's fundamental length into the theory, which in its modern version is formulated as a quantum geometry. This kind of solution is a highly formal approach, which results in the manipulation of the interaction terms etc. in order to achieve finite values. Another way to get rid of these undesirable infinities is the finite quantum electrodynamics of Scharf [10] based on methods of Stueckelberg, Bogoliubow and Shirkov and Epstein and Glaser who modified the evaluation of the S matrix.

So the question arises: are these highly refined mathematical methods the true solution of the problem or is there any physical idea which admits to focus the attention in another direction?

Indeed such an idea exists. Formulating quantum electrodynamics in an algebraic Schrödinger representation, Stumpf, Fauser and Pfister [11] showed that the boson field commutator of the conjugate electromagnetic field variables is fixed by the system dynamics and the fermionic anticommutator. This means that the quantum behavior of the electromagnetic field is determined by the quantum properties of the fermion field. It is therefore consistent to assume that the electromagnetic field has its origin in fermionic fields, or with other words: the further evolution of quantum electrodynamics leads back to de Broglie's theory of light and requires the adaption of this theory to a modern quantum field theoretic formulation. But before discussing this topic we look for experiments which possibly could confirm this approach.

2 Photons with partonic structure

The parton model was introduced by Feynman [12], Bjorken and Paschos [13] and other authors in order to explain the results of deep inelastic electron-proton scattering. In this model a hadron consists of a collection of partons (pointlike objects) with partly well defined quantum numbers but without reference to a specific dynamical law. After putting forward quantum chromodynamics partons were identified with quarks, but until now quantum chromodynamics has not completely replaced the parton model, and for many applications both kinds of models do coexist.

To understand the partonic structure of photons it is advisable to study the substructure of protons first, as it is revealed by deep inelastic electron proton scattering. As far as this process is concerned one can visualize it by considering the proton as the fixed target of a high energy electron microscope, where the scattered electrons lead to a diffraction spectrum which contains the information about the structure of the proton. In this picture the laboratory system, where the proton is at rest, is a distinguished frame of reference. In this frame the differential cross-section is defined by the exchange of a virtual photon between electron and proton. It reads, see Leader and Predazzi [14]

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{1}{2}\frac{\alpha^2}{m_N q^4}\frac{E'}{E}L_{\alpha\beta}(e,e')W^{\alpha\beta}(N) \tag{2.1}$$

with the following definitions:
$p \ := (m_N, 0)$ initial proton momentum
$k \ := \ (E, \mathbf{k})$ initial electron momentum
$k' := (E', \mathbf{k}')$ final electron momentum
and $q = (k-k')$, $d\Omega := 2\pi \, d\cos\Theta$, where Θ is the angle of the scattered k'-vector with respect to the initial k-vector, while $L_{\alpha\beta}$ and $W^{\alpha\beta}$ are to be calculated by means of the electronic or hadronic currents respectively. Obviously (2.1) describes the diffraction pattern of the scattered electron which is given by the variation of the beam intensities in dependence on the variables E' and Ω or Θ respectively. The parton model is brought into play if one tries to find a physical interpretation of the experimental results. To perform this interpretation we refer to the evaluated form of (2.1) which reads (in laboratory frame) [14]:

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{4\alpha^2(E')^2}{q^4}\left(2W_1^{eN}\sin^2\frac{1}{2}\Theta + 2W_2^{eN}\cos^2\frac{1}{2}\Theta\right) \tag{2.2}$$

where in general the structure functions W_i^{eN} should depend on $\nu = E - E'$ and $Q^2 = -q^2$, as independent variables. But experiments show that not ν or Q^2, but rather

$$x = \frac{Q^2}{2m_N \nu} \tag{2.3}$$

is the independent variable of W_1^{eN} and W_2^{eN}. This 'scaling' behavior can be explained if the nucleon is assumed to be composed of point-like spin half constituents (partons) and if the structure functions for deep inelastic reactions can be viewed as built up from an incoherent sum of elastic scatterings of the virtual photon on these constituents. In this case one finds a dependence of W_1^{eN} and W_2^{eN} upon only the variable x as desired.

However: if one whishes to elaborate this concept in more detail, by theoretical considerations one is forced to change the reference system in order to give an appropriate description of such reactions. Therefore, clearly, the first step in such a theoretical discussion must be a reformulation of (2.2) in terms of covariant variables. Whithout going into details we give the result of such a reformulation. The invariant varables are given by $q^2 = -Q^2$ and by $s = (p+k)^2$ and

$$x = -\frac{q^2}{2(pq)} \; ; \; y = \frac{(pq)}{(pk)} \tag{2.4}$$

and with the transformation

$$\frac{d^2\sigma}{dxdy} = \frac{2\pi m_N y E}{E'} \frac{d^2\sigma}{dE' d\Omega} \tag{2.5}$$

the covariant equivalent of (2.2) reads [14]

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4} \left\{ xy^2 F_1 + \left(1 - y - \frac{xym_N^2}{s}\right) F_2 \right\} \tag{2.6}$$

where F_i depend on x and Q^2. By means of (2.6) one is free to choose any reference frame for the description of the scattering process which seems to be appropriate from the theoretical point of view. Astonishingly this appropriate system turns out to be the infinite momentum frame S^∞. In this frame the nucleon's four momentum is given by [14]

$$p = \left(\left(P^2 + m_N^2\right)^{\frac{1}{2}}, 0, 0, -P \right) \tag{2.7}$$

where the speed of the reference frame as seen in the laboratory frame is

$$\beta = \frac{P}{(P^2 + m_N^2)^{\frac{1}{2}}} \approx 1 \tag{2.8}$$

for sufficiently large P, i.e., in the limit of infinite P. In this case it is possible to perform quantitative calculations using the parton model. It is not our intention to describe these calculations. Rather we observe that S^∞ is a natural frame of reference for photons and this brings us to the relationship between partons and photons.

The photon is the simplest of all gauge bosons mediating electromagnetic interactions. As such it couples to the elementary charged constituents of matter, like leptons and quarks, in a well defined way, which makes it a very good probing tool for the structure of more complicated objects like hadrons. This can be used to measure the parton distributions of the nucleon in deep inelastic scattering processes as was shown above. However, the point like nature of the photon, assumed in the Standard model, has to be confronted with the fact that the photon may also exhibit properties similar to those of normal hadrons. This (strange) behavior can be observed in various high energy reactions, but for simplicity we concentrate on the example of e^+e^- interactions leading to two-photon reactions. In this case, when one of the photons with very small virtuality (quasi-real photon) interacts with the other one with high virtuality, the interaction can be thought of as a deep inelastic scattering of one photon on the other, in which case the situation is similar to the probing of a nucleon by a highly virtual photon. It is therefore natural to introduce the notion of the photon structure function in analogy to the well known nucleon case, see Abramowicz, Charchula, Krawczyk, Levy and Maor [15].

The following notation for the kinematical variables will be used:

p	$=$	$(E, 0, 0, E)$	initial electron momentum
p'	$=$	$(E', 0, E'\sin\Theta', E'\cos\Theta')$	final electron momentum
k	$=$	$(E_\gamma, 0, 0, -E_\gamma)$	momentum of the target photon
q	$=$	$(p - p')$	momentum of the probing virtual photon

In p it is $E \gg m_e$ and thus m_e is neglected. The standard scaling variables are the same as in the nucleon case and the cross-section can be formulated in the following way [15]:

$$\frac{d^2\sigma^\gamma}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}\left\{xy^2 F_1^\gamma + (1-y)\,F_2^\gamma\right\} \tag{2.9}$$

A comparison of (2.6) with (2.9) shows: the similarity is complete. Hence we can use the diffraction pattern of the outgoing electron as a high energy microscope for the target photon. Therefore $F_i^\gamma, i = 1, 2$, can be treated as the structure functions of the target photon. The virtual probing photon is assumed to be a point-like object. Although the virtual photon may also develop a nontrivial structure, viewing it as a point-like objekt is a very good approximation at large Q^2.

How are these results to be explained and interpreted? The common interpretation runs as follows, see Drees and Godbole [16], Erdmann [17]: the uncertainty tells us that for a short period of time a photon can fluctuate into a pair of charged particles. Apart from the bare photon state, the photon can fluctuate into quark-antiquark pairs without forming a hadronic bound state (anomalous) or form a vector meson. The photon can therefore interact directly or through its resolved states, and in this case the partons are identified with quarks. This interpretation immediately opens the possibility to treat the photon structure by means of QCD. Fluctations of the photon into virtual two-lepton states are well understood, and are in fact a crucial ingredient of the quantitative success of QED. Fluctuations into quark-antiquark pairs are much more problematic, however. Whenever the life time of the virtual state exceeds about 10^{-25} sec, the virtual $q\bar{q}$ pair has sufficient time to envolve into a complicated hadronic state that cannot be described by perturbative methods only. Even if the life time is shorter hard gluon emission and related processes complicate the picture substantially. Therefore it is not surprising but a bit embarrassing that many reactions involving (quasi) real photons are much less well understood, both theoretically and experimentally [16].

3 Conclusions

Before going into details of the calculations of photon structure functions, let us first ask whether the fluctuation model of the photon is in accordance with general principles of quantum electrodynamics. For instance we consider elastic electron-muon scattering using kinematical variables analogously to section 2. The lowest order scattering graph results from the amplitude

$$\mathcal{M} := -e_0^2 \langle k' \mid j_\mu^e \mid k \rangle \frac{g^{\mu\nu}}{q^2} \langle p' \mid j_\nu^M \mid p \rangle \tag{3.1}$$

where j^e_μ or j^M_ν are the electron or muon currents respectively. In the next order of perturbation theory one obtains the amplitude, see Griffiths [18]

$$\mathcal{M}' := -ie_0^4 \langle k' \mid j^e_\mu \mid k\rangle \left(\frac{g^{\mu\nu}}{q^2} - \frac{i}{q^4} I_{\mu\nu} \right) \langle p' \mid j^M_\nu \mid p\rangle \tag{3.2}$$

where $I_{\mu\nu}$ contains a contribution of the vacuum polarization, i.e. the virtual fermion-antifermion pairs. This part can be calculated, and in this approximation the total amplitude is given by the coherent superposition of M and M' i.e.

$$\mathcal{M}^r = \mathcal{M} + \mathcal{M}' =$$
$$e_0^2 \langle k' \mid j^e_\mu \mid k\rangle \frac{g^{\mu\nu}}{q^2} \left\{ 1 - \frac{e^2}{12\pi^2} \left[\ln\left(\frac{\Lambda^2}{m^2} \right) - f\left(\frac{-q^2}{m^2c^2} \right) \right] \right\} \langle p' \mid j^M_\nu \mid p\rangle \tag{3.3}$$

where m is the bare electron mass and the function f is defined in [18] (7.180). To this order we define the renormalized charge e_r by [18]

$$e_r := e_0 \left(1 - \frac{e_0^2}{12\pi^2} \ln\left(\frac{\Lambda^2}{m^2} \right) \right)^{\frac{1}{2}} \tag{3.4}$$

where Λ^2 is a suitable cutoff. In terms of e_r the amplitude (3.4) can be rewritten as [18]

$$\mathcal{M}^r = e_r^2 \langle k' \mid j^e_\mu \mid k\rangle \frac{g^{\mu\nu}}{q^2} \left\{ 1 + \frac{e_r^2}{12\pi^2} f\left(\frac{-q^2}{m^2c^2} \right) \right\} \langle p' \mid j^M_\nu \mid p\rangle \tag{3.5}$$

which is equivalent to (3.3) up to the fourth order in e_0.

This procedure can be continued to all orders of perturbation theory and leads to renormalized charge and dressed propagators. Obviously renormalization of charge and dressing of the propagators is by no means a time dependent process. Rather, in the systematic version of renormalization the dressing concept is elaborated in a self consistent way so that the initial, i.e., the ingoing, and the final, i.e., the outgoing particles, are dressed ones. Hence, at any time these particles are surrounded by their polarization cloud, i.e., they are permanently dressed. This result is a clear contradiction to the idea that the bare particles fluctuate temporarily into fermion-antifermion pairs. Therefore we conclude that from the foundations of quantum electrodynamics the photon should be a dressed, i.e., a composite particle for all times. This interpretation is supported by a critical discussion of special calculations in the quark-parton picture of the photon.

It was Witten [19] who showed that in this picture the structure functions can be computed exactly, at least in the so-called 'asymptotic' limit of infinite Q^2. Including next-to-leading order, the result can be written as

$$F_2^\gamma\left(x, Q^2\right) = \alpha\left[\frac{1}{\alpha_s\left(Q^2\right)} a(x) + b(x)\right] \tag{3.6}$$

where $a(x)$ and $b(x)$ are calculable functions of x, and $\alpha_s\left(Q^2\right)$ is the value of the QCD scale parameter.

Unfortunately this calculation leads to divergent results for $x \to 0$, namely

$$a(x) \sim x^{-0.59} \ ; \ b(x) \sim x^{-1} \quad . \tag{3.7}$$

The coefficient of the $1/x$ pole in b is negative; equation (3.6) therefore predicts negative counting rates at small x. The divergence is worse in the next-to-leading order contribution b than in the leading order term a. It can be shown that this trend continues in yet higher orders, i.e., the asymptotic prediction for F_2^γ rapidly becomes more and more divergent for $x \to 0$ as more higher order corrections are included. And these are not the only difficulties, but for a detailed discussion we refer to Drees and Godbole [16]. What are their conclusions from this disaster? They refuse the resolution of the process into perturbative subdiagrams and state that the only meaningful approach seems to be that suggested by Glück and Reya [20]. In this approach one does not attempt to compute the absolute size of the quark densities inside the photon. Rather one introduces input distribution functions $q_i^\gamma\left(x, Q^2\right)$ into

$$F_2^\gamma\left(x, Q^2\right) = 2x \sum e_{q_i}^2 q_i^\gamma\left(x, Q^2\right) \tag{3.8}$$

at some scale Q_0^2 and tries to calculate about a wider range of Q^2. *But this means nothing more than the acceptance that the photon is a composite particle.* In our conclusion we go one step further: it is not very sensible nor logically consistent to consider the photon as a composite particle and the other gauge bosons as elementary point like entities. In a radical departure from the gauge philosophy by the arguments given above one is justified to consider all gauge bosons as composites.

But then calculations of the photon structure functions by means of QCD become doubtfull, and a new approach is necessary. Some aspects of such an approach are discussed in the following sections.

As a side remark let us mention that in the low energy region modern quantum optical experiments measure photon field correlations with increasing accuracy, c.f., Georgiades, Polzik and Kimble [21], Lloyd [22], Mandel and Wolf [23]. One may expect also from this side defined results on the internal photon structure.

4 The photon equation and its solutions

In sections 1 and 2 we collected and summarized theoretical and experimental evidence in favor of the introduction of the concept of composite photons. Of course this concept must be realized within the framework of a relativistic quantum field theory, and of course such a concept should not be confined to the photon only. But in any case the conventional quantum field theoretic calculation schemes do not suffice to master this problem. Therefore, a new method of nonperturbative quantum field theory was developed with the aim of deriving effective theories for the dynamics of composite particles or fields, respectively. The method is based on the Hamiltonian formalism in combination with elements of algebraic representation theory of quantum fields and Heisenberg's equation of motion. The field dynamics is formulated by functional equations in functional spaces which are isomorphic to the original state spaces of the quantum fields under consideration, c.f., Stumpf and Borne [4], Borne, Lochak, Stumpf [3]. As far as the physical content of the theory is concerned, the following assumptions are made.

By generalization of the ideas of de Broglie's fusion theory the bosons and fermions of the Standard model are considered as composite particles. A suitable ansatz for a quantitative formulation of this assumption is a NJL–like spinorfield model for subfermions which is regularized by a new nonperturbative Pauli–Villars regularization. Composite particle states are defined by means of solutions of generalized de Broglie–Bargmann–Wigner equations, and by Weak Mapping the spinor field theory can be transformed into a SU(2)×U(1) unbroken effective local gauge theory describing the dynamics of composite bosons and fermions. After symmetry breaking this yields the electro weak sector of the Standard model for the first generation.

In this paper we cannot describe this extensive and comprehensive formalism, rather we concentrate upon the photon equation and its interpretation which can be derived within the general formalism. But before beginning a detailed discussion it should be emphasized that in the framework of such a theory the existence of composite photon states is not in contradiction to 'no go' theorems, because these theorems do not take into account the consequences of regularization.

We start with the hard core equations for composite photon states which are incorporated into generating functional states. For details of its derivation, etc., we refer to references [3] and [4]. We use a highly symbolic notation in order to obtain clearly organized expressions. In particular we define the super indices:
$I := (Z, \mathbf{r})$ in energy equations
$I := (Z, x)$ in covariant equations
with $\mathbf{r} \in \mathbb{R}^3, x \in \mathbb{M}^4$, and $Z = (i, \kappa, \alpha)$ where $i =$ auxiliary field index, $\kappa =$ superspin-isospin index, $\alpha =$ Dirac spinor index.

Let $\varphi^{(2)}(I_1 I_2)$ be the covariant, antisymmetric state amplitude of the composite photon. Then within the general formalism for $\varphi^{(2)}$ the following set of covariant equations can be derived:

$$\begin{aligned} K_{I_1 K_1} \varphi^{(2)}(K_1, I_2) &= 3 W_{I_1 K_2 K_3 K_4} F_{K_4 I_2} \varphi^{(2)}(K_2, K_3) \quad , \\ K_{I_2 K_1} \varphi^{(2)}(I_1, K_1) &= -3 W_{I_2 K_2 K_3 K_4} F_{K_4 I_1} \varphi^{(2)}(K_2, K_3) \quad , \end{aligned} \tag{4.1}$$

with $D^\mu := \gamma^\mu_{\alpha_1 \alpha_2} \delta_{\kappa_1 \kappa_2} \delta_{i_1 i_2}$ and $m := m_{i_1} \delta_{\alpha_1 \alpha_2} \delta_{\kappa_1 \kappa_2} \delta_{i_1 i_2}$.

Furthermore, we have

$$K_{I_1 I_2} = \left[D^\mu_{Z_1 Z_2} \partial_\mu(x_1) - m_{Z_1 Z_2} \right] \delta(x_1 - x_2) \ , \tag{4.2}$$

$$W_{I_1 I_2 I_3 I_4} = U_{Z_1 [Z_2 Z_3 Z_4]} \delta(x_1 - x_2) \delta(x_1 - x_3) \delta(x_1 - x_4) \ , \tag{4.3}$$

$$F_{I_1 I_2} = -i \lambda_{i_1} \delta_{i_1 i_2} \gamma^5_{\kappa_1 \kappa_2} \Big[(i \gamma^\mu \partial_\mu(x_1) + m_{i_1}) C \Big]_{\alpha_1 \alpha_2} \triangle (x_1 - x_2, m_{i_1}) \tag{4.4}$$

where the superindex κ is defined below, and where $\triangle (x_1 - x_2, m_{i_1})$ is the scalar Feynman propagator. The meaning of the index κ can be explained by decomposing it into $\kappa := (\Lambda, A)$ with Λ superspin index (spinors and charge conjugated spinors) and A isospin index with the following correspondence

$$\kappa = \begin{cases} 1 & \text{for} \quad \Lambda = 1, A = 1 \\ 2 & \text{for} \quad \Lambda = 1, A = 2 \\ 3 & \text{for} \quad \Lambda = 2, A = 1 \\ 4 & \text{for} \quad \Lambda = 2, A = 2 \; . \end{cases} \tag{4.5}$$

Then we have for the vertex (4.3) $U_{Z_1 Z_2 Z_3 Z_4} = \lambda_{i_1} B_{i_2 i_3 i_4} V_{\substack{\kappa_1\kappa_2\kappa_3\kappa_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4}}$ where $B_{i_2 i_3 i_4}$ indicates the summation over the auxiliary field indices and V is given by

$$V_{\substack{\kappa_1\kappa_2\kappa_3\kappa_4 \\ \alpha_1\alpha_2\alpha_3\alpha_4}} := \frac{g}{2} \sum_{h=1}^{2} \Big\{ v^h_{\alpha_1\alpha_2} (v^h C)_{\alpha_3\alpha_4} \delta_{\kappa_1\kappa_2} \big[\gamma^5 (1 - \gamma^0) \big]_{\kappa_3\kappa_4} \Big\}_{\text{as}[2,3,4]} \tag{4.6}$$

with $v^1 := \delta_{\alpha_1\alpha_2}$ and $v^2 := i\gamma^5_{\alpha_1\alpha_2}$.
It is remarkable that the field theoretic formalism leads to equations (4.1) which for $g \equiv 0$ yields de Broglie's original fusion equations for local $\varphi^{(2)}$. Therefore equations (4.1) can be considered as generalized de Broglie-Bargmann-Wigner equations. Equations (4.1) admit exact solutions, see Pfister, Rosa and Stumpf [24]. In particular their corresponding secular equation leads to finite eigenvalues which are given by the invariant mass of the boson $k^2 = \mu^2$. We give only the results of such calculations. Let $\varphi^{(2)}_{I_1 I_2}$ be a solution of (4.1). Then $\varphi^{(2)}_{I_1 I_2}$ describes a vector boson with momentum k, if it is given by

$$\varphi^{(2)}_{\substack{i_1 i_2 \\ \kappa_1\kappa_2 \\ \alpha_1\alpha_2}}(x_1, x_2) = T^a_{\kappa_1\kappa_2} \exp\Big[-i\frac{k}{2}(x_1 + x_2)\Big] A_\mu \chi^\mu_{\substack{i_1 i_2 \\ \alpha_1\alpha_2}}(x_1 - x_2 | k) \tag{4.7}$$

with the relative wave function

$$\chi^\mu_{\substack{i_1 i_2 \\ \alpha_1\alpha_2}}(x|k) := \frac{2ig}{(2\pi)^4} \lambda_{i_1} \lambda_{i_2} \int d^4p \; \exp\big[-ipx\big] \quad \times \tag{4.8}$$

$$\Big[S_F\Big(p + \frac{k}{2}, m_{i_1}\Big) \gamma^\mu S_F\Big(p - \frac{k}{2}, m_{i_2}\Big) C \Big]_{\alpha_1\alpha_2}$$

(no summation over i_1, i_2) and

$$S_F\Big(p, m\Big) := \frac{\gamma^\mu p_\mu + m}{p^2 - m^2 + i\epsilon} \quad . \tag{4.9}$$

A detailed calculation with respect to this proposition and the subsequent discussion of eigenvalue equations is contained in [3] and [4].

The above given representation of the boson wave function holds for all k, as the calculation was performed in a strictly relativistic invariant way. The integral in (4.8) can be evaluated by standard methods and leads, of course, to a singular behavior on the light cone. Performing the equal–time limit one explicitly sees that $\varphi^{(2)}$ vanishes for large relative distances $\mathbf{r}_1 - \mathbf{r}_2$, so (4.7), (4.8) really describe bound states. Furthermore, one can verify the antisymmetry of (4.7) in the indices I_1,I_2.

The amplitude A_μ characterizes the global behavior of the bosons and has to interpreted as the corresponding vector potential. A_μ is not completely arbitrary but has to satisfy the equations

$$\begin{aligned} A^\mu &= 2i(2\pi)^4 g \int d^4q \Big\{ \big[S(k-q)S(q) + R^\rho(k-q)R_\rho(q) \big] A^\mu \\ &\qquad -2R^\mu(k-q)R_\rho(q)A^\rho \Big\} \end{aligned} \quad (4.10)$$

From equations (4.10) a secular equation for vector bosons can be derived. But where are the field strengths $F^{\mu\nu}$? As far as A^μ and $F^{\mu\nu}$ are concerned, equations (4.1) show that the 'field strengths' $F^{\mu\nu}$ are determined by the 'vector potentials' A^μ, whilst the A^μ themselves are eigenvectors of the homogeneous eigenvalue equations (4.10) for the corresponding eigenvalue. Furthermore, it can be shown that equations (4.10) are compatible with the transversality condition $k^\rho A_\rho = 0$, and it follows that $F^{\mu\nu}$ must have the form $c(k)k^{[\mu}A^{\nu]}$. But attention must be paid to the fact that these relations only hold for $\chi^\mu(0|k)$. In the full exact solutions (4.7) only the vector potentials A^μ occur.

We now discuss the superspin-isospin quantum numbers of (4.8) which are connected with the matrices $T^a_{\kappa_1\kappa_2}$. For vector boson states these matrices must be antisymmetric, and as four-dimensional matrices they can be represented by the antisymmetric elements of the Dirac γ-algebra. From the general field theoretic formalism quantum conditions for the corresponding quantum numbers $f \equiv$ fermion number, $t \equiv$ isospin, $t_3 \equiv z$-component of isospin can be derived. By explicit evaluation of these equations the eigenvectors can be determined to give [3]

	$\hat{T}^1$	$\hat{T}^2$	$\hat{T}^3$	$\hat{T}^4$	$\hat{T}^5$	$\hat{T}^6$
t	0	0	0	1	1	1
t_3	0	0	0	1	−1	0
f	2	−2	0	0	0	0

The bosons attached to the Standard model are characterized by

$$\hat{T}_3 \equiv T_{U(1)} := -i\gamma^5\gamma^2 C \tag{4.11}$$

which constitutes a $U(1)$ singlet state and by the $SU(2)$ triplet

$$\begin{aligned} \hat{T}_4 + \hat{T}_5 \equiv T^1_{\mathrm{SU}(2)} &:= \gamma^5\gamma^3 C \\ i^{-1}\left(\hat{T}_4 - \hat{T}_5\right) \equiv T^2_{\mathrm{SU}(2)} &:= -iC \\ \hat{T}_6 \equiv T^3_{\mathrm{SU}(2)} &:= -\gamma^5\gamma^1 C \end{aligned} \tag{4.12}$$

where these matrices satisfy the corresponding Lie algebra relations. According to the table all these states are eigenstates with the fermion number $f = 0$.

We now turn to the set of vector bosons with the isospin/superspin matrices $T^a \in \{\gamma^5 C,\ \gamma^5\gamma^0 C\}$, which are not contained in the set (4.11) and (4.12) and which according to the table are states with the fermion number $f = \pm 2$. These states are unphysical states. The reason for their dropping out is the superselection rule for fermion numbers.

Finally it is well known in the context of the Standard model that the original input of the U(1) gauge group does not immediatley lead to the photon field. Rather gauge fixing with symmetry breaking and subsequent application of the Glashow-Weinberg transformation is necessary in order to obtain the electromagnetic field from a mixture of the broken SU(2) states and the U(1) states. This phenomenological treatment has its counterpart in the microscopic theory of composite gauge bosons. This theory was evaluated in [3]. It would exceed the scope of this paper to repeat these investigations. So we consider the wave functions (4.7) as representatives of the electro weak gauge bosons although they should undergo some mixture in order to describe the phenomenological fields.

5 Regularization and probability interpretation

So far we have nothing said about the role of the auxiliary field (indices) which appear in the photon equation (4.1) and in its solution (4.7). As we stated already in Section 4, the photon equations arise by means of a new quantum field theoretic nonperturbative calculation method, which allows to use nonrenormalizable field models, like the NJL model for the formulation of basic field laws. The precondition for the use of such models is the introduction of a self consistent regularization scheme which is expressed by the presence of auxiliary fields and the simultaneous emergence of indefinite state spaces. The success of this approach thus depends on our ability to demonstrate that this regularization does not disturb the physical interpretation. In this context the energy equation plays a crucial role. The energy equation can be directly derived from the set of covariant equations (4.1). We resolve the general index I into $I = (Z, x)$ and deduce for eigenstates (4.8) from (4.1) the following exact energy equation

$$\begin{aligned} k_0 \varphi^{(2)}_{Z_1 Z_2}(x_1, x_2) = {} & iD^0_{Z_1 X_1}\Big[D^k_{X_1 X_2}\partial_k(x_1) - m_{X_1 X_2}\Big]\varphi^{(2)}_{X_2 Z_2}(x_1, x_2) \qquad (5.1) \\ & + iD^0_{Z_2 X_1}\Big[D^k_{X_1 X_2}\partial_k(x_2) - m_{X_1 X_2}\Big]\varphi^{(2)}_{Z_1 X_2}(x_1, x_2) \\ & - 3i\Big[D^0_{Z_1 X_1}\hat{U}_{X_1 X_2 X_3 X_4} F_{X_4 Z_2}(x_1 - x_2)\varphi^{(2)}_{X_2 X_3}(x_1, x_1) \\ & - D^0_{Z_2 X_1}\hat{U}_{X_1 X_2 X_3 X_4} F_{X_4 Z_1}(x_2 - x_1)\varphi^{(2)}_{X_2 X_3}(x_2, x_2)\Big] \,. \end{aligned}$$

Now we turn to the auxiliary fields. In the course of regularization of the NJL model and its subsequent evaluation the auxiliary field formalism was developed by use of a strictly canonical quantization. Although the auxiliary fields are deprived of any physical interpretation they are formally treated in the way of ordinary, physical quantum fields and this means that in this formulation each auxiliary field can be separately considered and prepared for 'measurement'. The result is that the GNS matrix elements are singular and the state space metric becomes indefinite.

Obviously these difficulties can only be removed if the regularization of the original 'classical' spinorfield equation can be transferred into its quantum field theoretic formulation. Considering perturbative Pauli–Villars regularization we remember that in the process of calculating vacuum expectation values the singular propagators are replaced by additive regularized expressions. Can one find a nonperturbative counterpart in the Algebraic Schrödinger Representation? This problem and the related problem of probability interpretation was treated by Stumpf [25]. We give a simplified version and refer to the original paper for details.

In the absence of condensation phenomena we assumed the free auxiliary field propagator (4.4) to be a reasonable approximation of the self consistency calculation. If we define

$$\hat{F} \equiv \hat{F}_{\substack{\alpha_1\alpha_2\\ \kappa_1\kappa_2}}(x_1,x_2) := \sum_{i_1,i_2} F_{\substack{\alpha_1\alpha_2\\ \kappa_1\kappa_2\\ i_1 i_2}}(x_1,x_2) \tag{5.2}$$

we obtain a regularized fermion propagator

$$\hat{F} := -i(\gamma^5)_{\kappa_1\kappa_2}\frac{1}{(2\pi)^4}\int d^4p e^{-ip(x_1-x_2)}\left(\prod_{i=1}^{3}\frac{1}{\gamma^\mu p_\mu - m_{i_1}}C\right)_{\alpha_1\alpha_2} \tag{5.3}$$

The essential point of this definition is that one achieves regularization without destroying the relativistic transformation properties of the corresponding expressions, i.e., the regularization does not interfere with the requirements of a relativistic quantum field theory. Although it was exemplified for the special case of a free auxiliary field propagator , in the general theory one has no other means to perform a regularization than to apply the above definition, if a correct quantum field theoretic interpretation of the regularized theory has to be maintained.

Therefore we define the physical, i.e., regularized normal ordered state amplitudes $\hat{\varphi}^{(n)}$ by the single-time expressions

$$\hat{\varphi}^{(n)}_{\substack{\alpha_1\alpha_n\\ \kappa_1\kappa_n}}(\mathbf{r}_1\ldots\mathbf{r}_n|a) := \sum_{i_1\ldots i_n}\varphi^{(n)}_{\substack{\alpha_1\alpha_n\\ \kappa_1\kappa_n\\ i_1 i_n}}(\mathbf{r}_1\ldots\mathbf{r}_n|a)\ , \tag{5.4}$$

and of course apply this definition also to the covariant amplitudes. One immediately realizes that $\hat{\varphi}^{(n)}$ has the same transformation properties as the original $\varphi^{(n)}\ \forall\ n$.

Can one derive an associated dynamical law for (5.4), the physical single-time amplitudes? To answer this question we consider the hard core equation (5.1) in the two–fermion sector in order to avoid lengthy calculations and to argue as transparent as possible. We decompose the index $Z := (A,\alpha,\Lambda,i)$ into $Z = (z,i)$ with $z := (A,\alpha,\Lambda)$ and sum over i_1, i_2 in (5.1). Afterwards we perform the limit to equal times, for instance $t_1 = t_2 = 0$. Due to $\sum_{i_1,i_2} D^0_{x_1x_2}\delta_{i_1i_2}\lambda_{i_2} = 0$ the last bracket in (5.1) vanishes and the equation

$$\begin{aligned} k_0\hat{\varphi}^{(2)}_{z_1z_2}(\mathbf{r}_1,\mathbf{r}_2) &= iD^0_{z_1x_1}D^k_{x_1x_2}\partial_k(\mathbf{r}_1)\hat{\varphi}^{(2)}_{x_2z_2}(\mathbf{r}_1,\mathbf{r}_2) \\ &+iD^0_{z_2x_1}D^k_{x_1x_2}\partial_k(\mathbf{r}_2)\hat{\varphi}^{(2)}_{z_1x_2}(\mathbf{r}_1,\mathbf{r}_2) \\ &-i\sum_{i_1,i_2}\left[D^0_{Z_1X_1}m_{X_1X_2}\varphi^{(2)}_{X_2Z_2}(\mathbf{r}_1,\mathbf{r}_2)+D^0_{Z_2X_1}m_{X_1X_2}\varphi^{(2)}_{Z_1X_2}(\mathbf{r}_1,\mathbf{r}_2)\right] \end{aligned} \tag{5.5}$$

results. Evidently (5.5) is no self consistent equation for the calculation of $\hat{\varphi}^{(2)}$. Rather (5.5) brings about a connection between regularized physical amplitudes and auxiliary amplitudes. This holds a fortiori for the full formalism too, as the operator of equation (5.10) is part of the full formalism.

Hence the auxiliary canonical field formulation in Algebraic Schrödinger Representation is indispensible for the calculation of the dynamical evaluation of the spinorfield. But how can one work with the latter representation if it produces singular functions?

As for a closed system without external forces the inner product structure has to be self consistently defined in accordance with the system dynamics, the only way to proceed is the use of the dynamical equations. Again we demonstrate this for the two–fermion sector. With

$$\hat{\varphi}^{(2)}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2, t) = e^{-iEt} \hat{\varphi}^{(2)}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2) \tag{5.6}$$

in equation (5.5) $k_0 \hat{\varphi}^{(2)}$ can be replaced by $i\partial_t \hat{\varphi}^{(2)}$. From this equation one can derive a current conservation law. For abbreviation we suppress all indices and coordinates aside from the auxiliary field indices. With $\alpha^k(1) := \alpha^k \otimes \mathbf{1}$ and $\alpha^k(2) := \mathbf{1} \otimes \alpha^{\mathbf{k}}$ and with $\hat{\varphi} \equiv \hat{\varphi}^{(2)}$ we obtain

$$\begin{aligned} &\partial_t(\hat{\varphi}^+ \hat{\varphi}) + \partial^1_k \left[\hat{\varphi}^+ \alpha^k(1) \hat{\varphi}\right] + \partial^2_k \left[\hat{\varphi}^+ \alpha^k(2) \hat{\varphi}\right] \\ &\quad + i\hat{\varphi}^+ \beta(1) \sum_{i_1, i_2} \varphi_{i_1 i_2} m_{i_1} - i \sum_{i_1, i_2} \varphi^+_{i_1 i_2} m_{i_1} \beta(1) \hat{\varphi} \\ &\quad + i\hat{\varphi}^+ \beta(2) \sum_{i_1, i_2} \varphi_{i_1 i_2} m_{i_2} - i \sum_{i_1, i_2} \varphi^+_{i_1 i_2} m_{i_2} \beta(2) \hat{\varphi} = 0 \ , \end{aligned} \tag{5.7}$$

with

$$\hat{\varphi}^+ \hat{\varphi} \equiv \sum_{z_1 z_2} \hat{\varphi}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2)^* \hat{\varphi}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2) \tag{5.8}$$

We remember that (5.7) is no self consistent equation for $\hat{\varphi}^{(2)}$ but only an identity following from the time–dependent version of (5.1). Therefore the most general functions which identically satisfy (5.7) are derived from linear combinations of time–dependent energy eigensolutions of (5.1), i.e.,

$$\hat{\varphi}^{(2)}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2, t) = \sum_k c_k e^{-iE_k t} \hat{\varphi}^{(2)}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2) \ . \tag{5.9}$$

We substitute the corresponding functions into (5.7), then with $m_i = m + \delta m_i$ the identity (5.7) reads

$$\partial_t(\hat{\varphi}^+\hat{\varphi}) + \partial_k^1\left[\hat{\varphi}^+\alpha^k(1)\hat{\varphi}\right] + \partial_k^2\left[\hat{\varphi}^+\alpha^k(2)\hat{\varphi}\right] + \sum_{k,k'} c_k^* c_{k'} e^{i(E_k - E_{k'})t} \times \tag{5.10}$$

$$\left\{\sum_{\rho=1}^{2}\left[\hat{\varphi}^+(k)\beta(\rho)\sum_{i_1,i_2}\varphi_{i_1i_2}(k')\delta m_{i_\rho} - \sum_{i_1,i_2}\varphi^+_{i_1i_2}(k)\delta m_{i_\rho}\beta(\rho)\hat{\varphi}(k')\right]\right\} = 0\ ,$$

and for vanishing last term we have current conservation.

Without any assumption about $\varphi^{(2)}$ the last term vanishes in the limit of vanishing mass differences δm_i. This situation resembles Heisenberg's introduction of a dipole ghost with the essential difference that the transition to a dipole ghost and a subfermion for $\delta m_i \to 0$ is performed only in the regularized $\hat{\varphi}^{(n)}$ after all calculations were done.

Then from (5.10) it follows that $\hat{\varphi}^+\hat{\varphi}$ is a conserved positive density. This means: the physical state amplitudes $\hat{\varphi}^{(2)}$ describe stable bound states and are elements of a corresponding Hilbert space.

Let us summarize the result of our discussion:
we have shown that it is possible to derive a bound state wave function for a composite photon within a rigorous field theoretic formalism. The wave function is covariant and its regularized single-time counterpart is element of a corresponding Hilbert space. This allows a probability interpretation of the (sub)structure of the photon and the corresponding wave functions can be used to perform calculations of cross-sections in deep inelastic scattering processes and opens a new way for the understanding of these high energy phenomena.

From a quantum field theoretic point of view the boson wave functions are the ingredients for the construction of many boson states which serve as a starting point for the derivation of effective quantum field theories (in algebraic Schrödinger representation) and which lead in the case of light quanta states to the Maxwell equations as effective dynamical laws, see [3] and [4]. But as the above formalism shows, one is now in a position to analyse the microscopic processes underlying the effective dynamical behavior of these particles or quanta, respectively, with the information gained by these considerations.

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