

# Propositional bases for the physics of the Bernoulli oscillators (A theory of the hidden degree of freedom)

## I - Thermodynamic framework

G. MASTROCINQUE

Dipartimento di Scienze Fisiche dell'Università di Napoli  
"Federico II" - Facoltà di Ingegneria - P.le Tecchio - 80125 Napoli

**ABSTRACT.** In a few previous papers, we developed a so-called classical fluctuation model providing (generalized) "symmetrization rules" - these make the classical expressions for energy transfer probabilities compliant with the detailed balance principle. For various physical processes, the symmetrized expressions of the transfer probabilities were shown to be remarkably improved - with respect to the performances of standard semiclassical models - in view of approaching quantum-mechanical results. Therefore, the possibility that a still undiscovered classical physics potential to describe quantum effects may be revealed by the model must be investigated. In this and a few next papers, we introduce some conceptual developments of the model and discuss the fundamental properties of the so-called Bernoulli oscillators (in the present paper I, the thermodynamic properties), whose behavior is analyzed in the light of the assumed existence of a "hidden" degree of freedom. The displayed properties are taken as a basis for a (proposed) classical interpretation of some quantum effects. As a final result, to be deployed in a last paper, a Newtonian-like equation of motion for "quantum" particles (uni-dimensional case) will be introduced, seeming to us the good candidate to set a bridge between classical and quantum physics. By these means - although we remain sometimes within the boundary of a conjectural framework, and limited to the case of translational motion - the possibility to approach a solution to the old problem of inconsistency between classical and quantum mechanics is displayed, and discussed as a proposal.

### 1 Introduction

In a few previous papers [1 – 3] we developed a so-called classical fluctuation model, able to account for the inelastic effects occurring in vibro-translational molecular energy transfers induced by collisions. The

model resulted in some (improved) "symmetrization rules" [4 – 5] which have to be applied to the classical expressions for energy transfer probabilities, so that the detailed balance principle is satisfied. Operating with these rules brought the classical expressions to perform similarly to their quantum-mechanical correspondents; thus we developed, in a few interesting cases, detailed comparisons between the former and the latter ones. By this means, we showed that ameliorations are achieved in view of approaching quantum-mechanical results with a classical model. Since the model revealed some generality, we extended the application domain of the rules to different other topics in physics [6–8], and showed that they have a generalized capability to make the classical expressions considerably approach the corresponding quantum-mechanical results.

The quoted analyses appeared to enlighten a till now undiscovered path towards the development of a theoretical framework able to provide a classical interpretation of quantum-mechanical effects, so that further investigation will be carried on throughout the present work.

In order to enlighten a few key concepts, we make here a specific point out and report a couple of statements describing a few important results we were able to obtain :

i) the application of our "symmetrization rules" to the classical expression of the thermodynamic entropy (or, equivalently, to the "sum-over-states ") corresponding to the case of a harmonic oscillators assembly produces exactly the quantum-mechanical result [6]. This fact has been shown to be consistent with a standard classical thermodynamics apparatus where energy fluctuations are invoked in order to overcome the so-called ultraviolet catastrophe problem. The overall framework in reference [6] results indeed in the correct (uni-modal) Planck/Bose-Einstein distribution for the thermodynamic oscillators energy. The energy fluctuations are interpreted to originate from the interaction of the oscillators with the so-called fluctuation field via the action of a "hidden degree of freedom" (HDF), whose nature and properties will be better explained in this work.

ii) a classical model of tunnelling phenomena has been set up and interpreted, as well, in the light of the assumption of existence of the fluctuation field [8]. This last is able to provide - within the limits allowed by the Heisenberg indetermination principle - to the physical system the extra-energy which it needs to perform the "classical" barrier jump.

These results can be considered representatives of our previous inves-

tigations as concerns a few basic concepts, and are important because they open the way to a well-defined research strategy to the purpose of developing classical models and interpretation for quantum effects. We are especially concerned with the possibility to set up competitive calculus and physical models based on classical mechanics. It is obvious that solving the old problem of inconsistency between the Newtonian concept of particle motion and modern probabilistic/dualistic physics originated by the dominant wave-mechanics is a very big challenge to pursue. Having available even a partial - provided consistent - solution to this problem might give strong impulse to scientific developments with associated consequences.

Our overall work is constituted by four papers where a few developments of our model for physical interactions will be discussed, up to the end where a possible (propositional) solution to the mentioned problem is given.

In the present paper (denoted I), we will first discuss generalities, and will point out the limits of our proposed framework. We will, afterwards, deploy our developments for the thermodynamic model of the oscillators (called the Bernoulli oscillators) interacting with the fluctuation field, and will be introduced to the existence and basic (thermodynamic) properties of the hidden degree of freedom HDF.

In a forthcoming publication (denoted paper II) we will investigate the mechanical counterpart of the thermodynamic model, displaying features of the interaction and giving physical identity to HDF and to its effect on the classical motion equation and energy theorem. A classical-like, propositional interpretation of Heisenberg's incertitude relations will be given and discussed within the same framework.

In another publication (denoted paper III) we will use a simple classical, statistical ensemble model as a reference to understand the correlations between the mechanical and thermodynamic properties. We will find out the state equations governing this (microcanonical) system statistics, both for the reference system and - using some induction - for the generalized case. We will finally write down the appropriate expressions of the sum-over-states and mass-flow-theorem for the Bernoulli oscillators.

Finally in a last publication (denoted paper IV), we will use the previous results for the sake of comparison with the fundamental wave-mechanical equation (taken in the hydrodynamic form). By this means,

we will be able to excerpt an expression for the potential effective on a single particle within the frame of the corresponding classical energy theorem. In this way, a Newtonian-like motion equation (limited to the uni-dimensional case) for the quantum particles in a pure quantum state will be established and discussed.

Conclusive remarks and demonstrations will also be deferred to this final paper.

Here we are first brought to the following considerations.

## 2 Generalities

First of all, we remark that our demonstrations in this domain are affected by (assessed) approximations. In practice, while developing our calculations in the previously quoted references, we generally work within the limits of first order perturbation theory and saddle-point-approximation for integrals evaluation. The WKB or quasi-classical case is generally taken as the reference to discuss the results (as is, for instance, the case for point (ii)). Full coincidence with the exact quantum-mechanical result (which we find indeed in the reported case (i)) must be considered at the present investigation stage only a rather fortunate case, but the important matter is that our results strongly indicate an improvement of the classical equations to the purpose of describing quantum-mechanical reality. An effort must therefore be done to overcome the approximations indicated in our theoretical frame.

This target can be approached in different ways. The most rational investigation strategy is likely to consist in operating to give our theoretical model and procedures higher descriptive and/or computative capability. For instance, we note that the symmetrization rules which we found in previous work can be considered as the specification of generalized rules of the form

$$P_{if}^{CC}(E_i, E_f) = \exp \int_{E_i}^{E_f} [\ln P_C(E)] P(E) dE \quad (1)$$

Here  $P_{if}^{CC}(E_i, E_f)$  must be intended as an improved expression of the fluctuation probability for an energy transfer process, holding in the framework of a generalized, upgraded " Classical Continuum fluctuation model " which might be developed. In equation (1)  $P_C(E)$  is the purely

classical expression corresponding to the investigated process and  $P(E)$  is some (normalized) energy distribution describing the effective energy level density to be accounted for within the framework. Taking  $P(E)$  as a constant over the integration domain gives back our previously quoted symmetrization rules. Equation (1) clearly shows that improvements of the fluctuation model can be obtained by investigating the effect of generalized forms for  $P(E)$ , and is a good starting point to exercise our conjectural attitudes towards the model development. In this work, however, we will follow a different investigation strategy, so that equation (1) has actually been quoted here for later reference.

A conjectural attitude, indeed, can also be exercised straight towards the development of a comprehensive interpretation of quantum mechanics in the light of the assumed existence of energy fluctuations as originated by the hidden degree of freedom. Taking this way may introduce us to a challenging and useful insight into new interpretative ideas in physics. In our papers, we are going indeed to propose and discuss conceptual developments of our model for physical interactions, with specific emphasis to the consequences stemming from the assumption of a classical point of view. Following this path, we will become able to introduce an interpretation of quantum mechanics as the fluid-dynamic appearance of a Newtonian substrate for particle motion, in the known expression of a Bernoulli or mass-flow theorem - although the standard classical expression will be found modified to account for the effect of the hidden degree of freedom. As a result of this analysis, a single-particle motion equation for the uni-dimensional case will be finally identified. It seems to us worthy to be examined as the candidate equation able to fill - although partially, because it is limited to the description of uni-dimensional, translational motion - the gap existing between the classical description of particle motion and the quantum-mechanical description.

Although we cannot afford a description of rotational motion - which would be a major point in the debate - at all in this work, this last will possibly be recognized to display a potential for a discussion about a couple of the most subtle and controversial points of modern physics. The first one concerns the old controversy between the wave or particle nature and behavior of matter, thus involving discussion of the complementarity principle. The second one concerns the two main different interpretations that Heisenberg's indetermination principle has, since long time already, originated amongst the physicists community - whether indetermination

only represents the practical boundary our measurements capability has encountered by the effect of limited experimental tools available, or is an absolute, intrinsic property of the physical reality [9]. As it will be clear in the following, some definite interpretation about both the mentioned points will be essentially promoted by our investigation. The discussion being of some relevance, the present work will - of course - be found unable to give exhaustive answers to the previous questions, or even to provide meaningful forecasts to all the consequences stemming from our model and equations. It is obvious that every new, proposed theoretical frame or interpretation of physical reality must wait for validation checks, all along its development. At present, therefore, our model will only be discussed with reference to the primary properties we have to show being satisfied - in order that it is not (not immediately, at least) found contradictory of consolidated aspects pertaining to the quantum-mechanical description of reality. Necessarily a number of problems, either on the interpretative level or on the practical descriptive level, will be left open or even untouched throughout this investigation.

However, a framework with some logical consistency seems to us being introduced in these papers as a contribute to the many efforts (see f.i. [10 – 22]) aimed to restore a causal interpretation of particle motion and physical reality. The work being only a propositional one with respect to the indicated challenges, it is obvious that further analyses will be required before our arguments either consolidate and call for developments, or decay. We are conscious that external criticism might be found disruptive to this effect. We believe, notwithstanding, that even this negative case - by its consequences and assessments - can be useful to our epistemological advancement.

### 3 Theoretical background

We start our present considerations by resuming the fundamental topic exposed in reference [6] and mentioned already in the previous section (i). This is necessary because we base our argument about the appearance of the hidden degree of freedom on such a discussion. The argument is essentially as is found in the next two subsections. Subsections are introduced in our papers according to the general strategy to discuss briefly a number of different topics deploying the analysis step-by-step.

3.1 *The classical thermodynamic potential*

To be useful as a first reference, let us consider the expression for the thermodynamic potential  $\Psi_C^*$  corresponding to an assembly of harmonic oscillators with the same oscillation frequency  $\nu_c = \omega_c/2\pi$  as is given in classical thermodynamics:

$$\Psi_C^* = -\ln Z_C^* = \frac{F_C^*}{T} = \frac{E_C^*}{T} - S_C^*(T) = -\ln T = 1 - S_C^*(T) \quad (2)$$

$$Z_C^* = \int_0^\infty \int_0^\infty \exp(-H_C(p,x)/T) dp dx \equiv \int_0^\infty \exp(-E'/T) dE' = T = E_C^* \quad (3)$$

$$H_C(p,x) = \frac{1}{2m}p^2 + \frac{1}{2}K^*x^2 \quad (4)$$

In these equations,  $Z_C^*$  is the classical "sum-over-continuum-states",  $E_C^*$  is the thermodynamic energy,  $F_C^*$  is the Helmholtz function and  $S_C^*$  is the entropy of our system.  $H_C(p,x)$  is the classical Hamiltonian for a harmonic oscillator, taken as a function of momentum  $p = mv$  and space-coordinate  $x$  of a particle with mass  $m$ , velocity  $v$ . The elastic constant  $K^*$  is equal to  $m\omega_c^2$ . Idler constants are dropped off in equations (2), (3). As is clear, in these equations we used a unit value for the Boltzmann constant and for the constant-volume specific heat of the system. With these conventions, the thermodynamic energy and absolute temperature become coincident quantities for the two degrees of freedom system. We find this choice comfortable to speed up mathematical handling of our equations.

The relevant expression of the entropy function  $S_C^*(T)$  is therefore :

$$S_C^*(T) \equiv S_C^*(E_C^*) = 1 + \ln(T) = 1 + \ln(E_C^*) \quad (5)$$

### 3.2 *The fluctuation entropy and the quantum-mechanical entropy*

In reference [6] we introduced a thermodynamic model for an ensemble of classical (harmonic) oscillators exchanging energy with a classical electromagnetic field mode at the same oscillator frequency  $\nu_c$ . Within the proposed framework is shown that if we assume that the oscillators thermodynamic energy fluctuates at constant entropy then, in the average, the Bose-Einstein (BE)/ Planck distribution is displayed by the system. Then we advanced the hypothesis that, in a quite similar way, the same oscillators can undergo stationary energy exchanges with an external fluctuation field. This last is invoked in order to explain the deviations of the oscillators physical behavior from the classical expectation, and the appearance of quantum effects. By investigating the fluctuation field effect on our oscillators we found indeed, again, a Bose-Einstein energy distribution and could recover the Bohr-Sommerfeld rule within the same framework. Details of this investigation are not to be discussed here again but a few key arguments, necessary to the further developments, will be technically reported in the sequel. We called the fluctuating-energy oscillators the "Bernoulli" oscillators. These last are classical oscillators but the way they exchange energy with the fluctuation field implies now recognizing the intermediation of a "hidden" degree of freedom HDF - this will be shown here with more details than displayed in [6]. The properties of our oscillators will be seen to evolve towards a more precise definition as far as our understanding of some relevant physical effects will be brought to refinement. We will refer to the model introduced in [6] as to the fluctuating-energy oscillators model (FEOM) all throughout this work.

Let us now report from reference [6] the interesting, key remark, here introduced as follows.

If we perform the integral mean of  $S_C^*(E_C^*)$  throughout an energy variation interval  $E_i^* \leq E_C^* \leq E_f^*$  ( $E_f^* - E_i^* = \Delta E$ ), this is found to result into the quantum-mechanical expectation  $S_{QM}$  :

$$\langle S_C^*(E_C^*) \rangle = \frac{1}{\Delta E} \int_{E_i^*}^{E_f^*} S_C^*(E_C^*) dE_C^* = \frac{1}{\Delta E} \int_{E_i^*}^{E_f^*} \left\{ 1 + \ln(E_C^*) \right\} dE_C^* \quad (6)$$



$$\langle S_C^*(E_C^*) \rangle = \frac{E_f^* \ln E_f^* - E_i^* \ln E_i^*}{\Delta E} = S_{QM}(T) \quad (7)$$

Basing on a principle stated in [6] we have to take indeed, in equation (7) :

$$\frac{E_f^*}{E_i^*} = \exp(\Delta E/T) \quad (8)$$

$$E_i^* = \frac{\Delta E}{\exp(\Delta E/T) - 1} = E_{BE} \quad (9)$$

Here we assume, to be definite,  $E_f^* \geq E_i^*$  so that  $\Delta E$  has to be taken equal to  $\hbar\omega_c$  and  $E_{BE}$  is the Bose–Einstein distribution expression. Further discussion and interpretation of these equations is given in the quoted reference, but in this paper we have to understand more about the correlation between equations (6),(7) and the thermodynamic behavior described by the Bernoulli potential  $\Psi^*$  in equation (11). Here we have to note that introducing the integral mean of the classical entropy expression as shown in equation (6) is equivalent to assume a "symmetrized" expression both for the entropy and sum-over-states as is indicated in the previous paper. We also note that equation (9) determines a Bose-Einstein distribution for the system energy  $\langle E_C^* \rangle$ :

$$\langle E_C^* \rangle = \frac{1}{\Delta E} \int_{E_i^*}^{E_f^*} E_C^* dE_C^* = \frac{E_f^* + E_i^*}{2} = E_{BE} + \frac{\Delta E}{2} \quad (10)$$

Equations (6)÷(10) are obtained in reference [6] by the means of variational procedures applied to a modified classical potential  $\Psi^*$  :

$$\Psi^* = -\ln Z^* = -\ln \frac{Z_C^*(T)}{V^*} = -\ln \frac{T}{V^*} = -\ln P^* \quad (11)$$

This last has been called the Bernoulli thermodynamic potential, or the "hydrodynamic" potential. It differs from the standard classical one (2) by the factor  $\ln V^*$  where  $V^*$  is the oscillators assembly volume.

The origin of this potential will be discussed with a few details in the next section. Here note that it takes an expression  $-\ln P^*$  ( $P^*$  = pressure modulus). As discussed in [6], the pressure  $P$  resulting from the Bernoulli potential is found negative; but, in that same reference, the constitutive equation for the pressure was taken opposite by sign to the standard convention, because we found there more comfortable to operate with a positive-definite pressure  $P^*$  (this is because the expressions of entropy and sum-over-states, as is clear in equations (11) and (30), can be more easily written as functions of  $P^*$ ). Then with equation (11) we find a perfect-gas-like state equation for the pressure modulus  $P^*$  but we have to take care - in [6] as well - that this is because of the convention we used. However, unless we specifically refer to the case of reference [6] which can be easily identified here by the use of asterisks affecting the thermodynamic variables (  $P^*, E^*, S^*$  etc. ), in this work we find more comfortable to come back to the standard convention, and will use the orthodox definition of the pressure  $P$  as is clear, for instance, in equation (29).

Here as a further essential comment to equations (6)÷(11) we remark that they indicate the possibility to set up improved classical models, to the purpose of approaching quantum mechanical results, in the domain of thermodynamics first. However, the big challenge to be pursued in this work is finding the mechanical oscillator model corresponding to the FEOM model, because this mechanical oscillator will exhibit quantum properties while basing its behavior within a classical physics domain.

Moreover, on a more technical ground, the important remark is as follows. Basing on the result (7) one is brought to the conclusion that taking into account the effectiveness of an energy spectrum with values included in a defined interval is the statistical basis to understand the transition from classical to quantum mechanics. In simple words, we have to think that a purely classical system will start to exhibit a quantum–mechanical behavior if energy fluctuations are superimposed to it. The source of these fluctuations has been called in previous work the fluctuation field, and the distinguished parts of the oscillator space and momentum co-ordinates, able to accommodate the energy transfer from it to the oscillator itself, will be identified in this paper as the (so-called) hidden degree of freedom (HDF).

*3.3 The Bernoulli thermodynamic potential*

We have to discuss the origin of the Bernoulli thermodynamic potential (11) with more details, both because we want to give some definite interpretation about it, and because it will be taken as a basis to introduce some generalized expressions at the end of this paper.

It is well known that finding a proper expression for the classical oscillators sum-over-states or thermodynamic potential, able to result in a correct description of effects in modern physics and to overcome the Rayleigh-Jeans law, is a very hard matter to accomplish. The apparent impossibility to avoid the so-called ultraviolet catastrophe by the means of standard classical thermodynamics brought Planck to introduce the revolutionary idea of the existence of energy quanta in physics. A classical-like description of phenomena is, however, promoted by the present and correlated papers. The thermodynamic potential (11) we have assumed in [6] is, in our opinion, an interesting prototype to be investigated to the purpose of assessing alternative procedures and interpretation with respect to Planck's discovery and associated quantum physics framework. We will therefore discuss this subject here.

The potential (11) is representative of a peculiar property of linked systems. If we look at them from the classical mechanics point of view, we find that the oscillator maximum elongation  $x_0$  increases with the total mechanical energy  $E_m$ . For instance, in the harmonic oscillator case we mainly refer to throughout this paper, we have

$$\frac{1}{2}mv^2 + \frac{1}{2}K*x^2 = E_m = \frac{1}{2}K*x_0^2 \tag{12}$$

If we take now a thermodynamic point of view, we can translate - although somewhat "abruptly" - this property into a thermodynamic relation between temperature and volume, provided we assume to stay in a situation as near as possible to mechanics. In order to satisfy this last requirement, we note that the appearance of thermal effects is the distinguishing aspect of thermodynamics compared to mechanics. Then it is clear that importing mechanical properties into a thermodynamic framework only might be allowed within the recognized limit where entropy effects are neglected. The best way to introduce the next equations from (17) to (24) is therefore advertising that they are written, to investigative purposes, neglecting the entropy - i.e. in some way assuming

that the entropy is zero or a constant at least. To show our reasoning, we basically start by using the equipartition principle and we find

$$\left\langle \frac{1}{2}mv^2 \right\rangle = \left\langle \frac{1}{2}K^*x^2 \right\rangle = \frac{T}{2} \quad (13)$$

Here the averages are clearly defined as thermodynamic ensemble ones. Note that the classical potentials pertaining to each of the quadratic degrees of freedom appearing in this equation can be written

$$\frac{\Psi_c^*}{2} = -\frac{1}{2} \ln Z_c^* = -\frac{1}{2} \ln T \quad (14)$$

$$\frac{S_c^*}{2} = \frac{1}{2}(1 + \ln T) \quad (15)$$

We have also

$$\left\langle \frac{1}{2}mv^2 \right\rangle + \left\langle \frac{1}{2}K^*x^2 \right\rangle = \langle E_m \rangle = E^*(T) = T = \frac{1}{2}K^* \langle x_0^2 \rangle \quad (16)$$

Here  $E^*(T)$  is the thermodynamic energy. Now we set

$$\frac{1}{2}K^* \langle x_0^2 \rangle \approx \frac{1}{8}K^*V^{*2} \quad (17)$$

We assume here the (squared) volume  $V^*$  representative of the (squared) mean value of twice the maximum elongation. As advertised, this can be thought true only in the "mechanical" limit, because the volume  $V^*$  is practically equal to  $\langle |2x_0| \rangle$  and in equation (17) we are neglecting the variance due to the energy spread. In this context, the previous equations suggest to us that if we neglect the entropy (then, in a sense, an "adiabatic transformation" along,  $S_m = \text{const}$ ), the volume  $V^*$  will be found to increase with the thermodynamic energy :

$$\frac{1}{8}K^*V^{*2} \Big|_{S_m=\text{const}} \approx T \quad (18)$$

$$\frac{T}{V^{*2}} \Big|_{S_m=\text{const}} \approx \text{const} = \frac{1}{8}K^* \Rightarrow \frac{T}{V^{*2}} = K(S_m) \quad (19)$$

Equations (18) and (19) show that, within the assessed limit and interpretation, the quantity  $T/V^{*2}$  is essentially a function of the entropy which can be written  $K(S_m)$ . We call  $S_m$  (m=mechanical) the entropy resulting from this intermediate analysis step, see equation (24). If this entropy is taken constant, indeed, then the behavior is dominated by mechanics and  $T/V^{*2}$  (for a harmonic oscillator) will take its "mechanical", constant value. The function  $K(S^*)$ , where  $S^*$  is now a generalized thermodynamic entropy, will be called the elastic thermodynamic function. At this stage, we recognize that neglecting entropy would also bring us to the following rough expression for the system sum-over-states :

$$Z_{sl} \approx \exp\left(-\frac{\langle E_m \rangle(V^*)}{T}\right) = \exp\left(-\frac{K^*V^{*2}}{8T}\right) = \exp\left(-\frac{E^*(V^*)}{T}\right) \tag{20}$$

This is just the same form we would have for a single mechanical energy level (sl). From this equation we would find the system state equation

$$P_m V^* = TV^* \frac{\partial}{\partial V^*} \ln Z_{sl} = -TV^* \frac{\partial}{\partial V^*} \frac{\langle E_m \rangle(V^*)}{T} = -\frac{1}{4}K^*V^{*2} = -2T \tag{21}$$

With this procedure, we have deployed the negative pressure property we attributed in [6] to our Bernoulli oscillators. In equation (21) as well as in previous ones, the index m has been given to the concerned variables, because taking the energy  $\langle E_m \rangle$  proportional to the squared volume essentially reproduces the purely mechanical property (18). To go further, however, we have to save the thermal equipartition of energy; so we are interested to an expression for Z where  $E^*$  is taken as a function of the temperature only, while the state equation (21) is maintained. Therefore the more refined forms we will write down - at this investigation stage - to represent our (harmonic) oscillator potentials are as follows :

$$Z_m = \frac{T}{V^{*2}} \tag{22}$$

$$\Psi_m = -\ln Z_m = -\ln \frac{T}{V^{*2}} \tag{23}$$

$$S_m = 1 + \ln \frac{T}{V^{*2}} \quad (24)$$

These equations provide us indeed with the right constant-volume specific heat and state equation. We note that they can be obtained dividing by a term  $V^{*2}$  the argument of the classical entropy  $S_c^*(T)$  ln-part as given by equation (5). They can be considered as the brute, precursory equations for the forms we actually used in reference [6]. The potential  $T/V^*$  appearing in the FEOM model is indeed just something more than the prototype, simple form (22) used to accommodate the fundamental concepts here introduced (elastic function and negative pressure). It stems from further analysis applied - starting from (15) and (19). This is as follows.

When we take the volume  $V^*$  as an independent thermodynamic variable in our potentials, we want to save the equipartition principle, so that the thermodynamic energy  $E^*$  expression always must be kept equal to the temperature  $T$ . Then we have to recognize that introducing the volume  $V^*$  should only be done by perturbing the entropy expression - more specifically, just that part of entropy which depends on the degree of freedom associated with the mechanical potential  $\frac{1}{2}K^*x^2$ . Now that part of the entropy has the standard expression  $S_c^*/2$  given in equation (15), where the term  $\frac{1}{2}$  comes from the thermal equipartition of energy and the logarithm argument  $T$  is the only part of the expression which can be correlated with the elastic function. Then we identify as a precursory (p) expression of the entropy  $S_c^*/2$  (15) the following

$$S_p = \frac{1}{2} + \frac{1}{2} \ln \frac{K^* \langle x_0^2 \rangle}{2} \quad (25)$$

Introducing now the volume  $V^*$  action into this expression as previously understood would bring us to write (idler constants dropped off):

$$S_p \rightarrow \frac{1}{2} + \frac{1}{2} \ln \frac{K^* \langle x_0^2 \rangle}{2V^{*2}} \equiv S_m^* = \frac{1}{2} + \frac{1}{2} \ln \frac{T}{V^{*2}} \equiv \frac{1}{2} + \frac{1}{2} \ln K(S_m^*) \quad (26)$$

As discussed already, this is done in such a way that if a zero variance was attributed to the distribution of elongation values, then our "mechanical" equation (18) would hold, and no (variable) entropy would be generated

at all (ln part of it, as obvious). Now it is clear that the final expression for the oscillators entropy becomes

$$S^* = \frac{1}{2}(1 + \ln T) + S_m^* = 1 + \ln \frac{T}{V^*} \quad (27)$$

and the thermodynamic potential, finally, comes out to be our FEOM expression

$$\Psi^* = -\ln Z^* = -\ln \frac{T}{V^*} = -\ln \frac{Z_c^*}{V^*} \quad (28)$$

From this expression, we find the state equation :

$$P = T \frac{\partial \ln Z^*}{\partial V^*} = T \frac{\partial \ln T/V^*}{\partial V^*} = -\frac{T}{V^*} = -P^* = \frac{P_m}{2} \quad (29)$$

By this procedure, we see that the resulting system pressure is reduced by a half with respect to the "brute" mechanical model, and this is because of a thermodynamic disorder effect. The kinetic degree of freedom entropy is indeed left unperturbed by equation (26). As pointed out earlier in this paper, the pressure P resulting from the Bernoulli potential is negative, and the orthodox convention will be used throughout this work as is clear from the same equation (29).

The entropy full expression in the FEOM model will then be given by

$$S^*(T, V^*) = \frac{E^*}{T} + \ln Z^* = 1 + \ln P^* \quad (30)$$

consistent with equation (11).

#### **4 The FEOM model and the hidden degree of freedom**

Investigations about a HDF possible existence are largely present in the literature and were first developed by de Broglie and Bohm. By our investigations, we are also brought to consider the possibility that HDF and its excitation source (the fluctuation field) are responsible for the deviations that physical systems exhibit from classical behavior. This hypothesis is originated by reference [6] and enforced by the second (ii) result we quoted in the previous section : assuming that the fluctuation field is able to provide extra-energy to alpha particles leads to a very simple explanation of tunnelling effects, while staying in a classical physics frame. Now we want to take advantage of these considerations in further analysis. To this end, the following arguments are introduced.

#### 4.1 The isobaric-isentropic transformation and the Bernoulli oscillators Gibbs energy

Let us now analyze with some details the way thermodynamics works to describe transformations for the system characterized by the potential (11).

For this system, isobaric transformations are coincident with adiabatic ones. Along the adiabatics the First Law will then be written :

$$P^*dV^* = dE^* - TdS^* = dE^* \quad (31)$$

$$S^* = 1 + \ln \frac{T}{V^*} = 1 + \ln P^* = \text{const} \quad (32)$$

In our FEOM model, the system interacts with a classical electromagnetic field mode (CEF) at pulsation  $\omega_c$ . The CEF mode exchanges some heat  $\int TdS_{CEF}$  with the system – within our framework this heat is (reversibly) transformed into work, according to the equation

$$- \int_i^f TdS_{CEF} = \int_i^f P^*dV^* = \pm \Delta E = \pm \hbar\omega_c \quad (33)$$

Equations (31)÷ (33) describe our fluctuation thermodynamics as reported already in reference [6]. Equation (33), however, is a key equation and has to be discussed more in the next section. Here we have to say - as a comment not displayed in [6] - that the intermediation of HDF has to be invoked to explain how this heat/work transformation occurs. If, indeed, appropriate thermodynamic parameters  $f$  and  $\Delta x$  associated to HDF are introduced, the same equation can be written

$$- \int_i^f TdS_{CEF} = \int_i^f TdS(\Delta x) = \int_i^f f d\Delta x = \int_i^f P^*dV^* = \pm \Delta E \quad (34)$$

In this equation,  $S(\Delta x)$  and  $f$  are an entropy and a pressure associated with the thermodynamic HDF "volume" co-ordinate  $\Delta x$ , respectively. The equation shows that HDF is responsible for the heat absorption and work transfer (and vice-versa) to the oscillator. Technically, this is the first appearance of HDF in our framework.



In order to describe the overall interaction process, we also used in [6] an expression for the oscillators Gibbs free energy

$$G^* = E^* + P^*V^* - TS^* \tag{35}$$

This expression is not the one we would find using an orthodox definition. It would be easy to show that in a standard framework where the total volume available for N particles is equal to the volume available for each of them - because they are assumed "not to interact" - then the system Gibbs free energy would be found equal to the Helmholtz free energy and we would have :

$$G^* = F^* = E^* - TS^* \tag{36}$$

Now the form we used

$$G^* = E^* + P^*V^* - TS^* = E^* - PV^* - TS^* \equiv E^* - TS^* + G(f,g,T) \tag{37}$$

is seen to display an extra term  $P^*V^* = -PV^* = T$ . We attribute the appearance of this term, too, to the action of HDF so that - as is clear already from equation (37) - it takes an expression  $G(f,g,T)$ , where a second "pressure" parameter  $g$  is introduced. Further discussion of these equations will follow in the next section.

In order to make clear the sense of the equations used in reference [6], therefore, we conclude resuming here the important remarks. Equation (33) is meaningful only if we can identify a physical mechanism responsible for the invoked transformation of heat into work. On the other hand, the quoted expression for  $G^*$  will be found congruent with a standard model and definition only if we assume that a complementary potential, equal to  $-PV^*$ , is added to it. These remarks mean that our overall framework is congruent with the assumption of existence of a (hidden) degree of freedom HDF, up to now introducing the parameters  $\Delta x$ ,  $f$  and  $g$ . In the next section, this will be shown with greater detail. In the following paper II, the HDF mechanical properties will be deployed and discussed.

#### 4.2 The isobaric-isentropic transformation and the hidden degree of freedom

Here we start again with the equations established in the previous section :

$$- \int_i^f \text{TdS}_{CEF} = \int_i^f \text{TdS}(\Delta x) = \int_i^f f d\Delta x = \int_i^f P^* dV^* = \pm \Delta E \quad (38)$$

$$G^* = E^* - PV^* - TS^* = E^* - TS^* + G(f,g,T) \quad (39)$$

These equations are interpreted in the light of existence of the degree of freedom HDF. By inspection, we come to the conclusion that this last is represented by two (generalized) thermodynamic co-ordinates, which we will call  $\Delta x$  and  $k$ , and their conjugate variables  $f$  and  $g$ , respectively. These co-ordinates are submitted to the First Law as is clear from the following equation:

$$- \int_i^f g dk - \int_i^f f d\Delta x = \Delta U(T, \Delta x, k) - \int_i^f \text{TdS}(\Delta x) - \int_i^f \text{TdS}(k) \quad (40)$$

Here  $U(T, \Delta x, k)$  is the thermodynamic energy pertaining to HDF, which is an unknown function at present - but we do not need to define it here, because in the present framework HDF transforms heat into work totally so that  $\Delta U = 0$ . In the following, by the sake of simplicity, we will drop off idler constants and will assume just the value zero for  $U(T, \Delta x, k_0)$ .

In order to display clearly the effect of these parameters and potentials along the relevant transformation for our oscillators ensemble, we can write again equation (38) as follows :

$$\begin{aligned} - \int_i^f \text{TdS}_{CEF} &= \int_i^f \text{TdS}^* + \int_i^f \text{TdS}(\Delta x) + \int_i^f \text{TdS}(k) = \int_i^f \text{TdS}(\Delta x) + \\ &+ \int_i^f \text{TdS}(k) = \int_i^f g dk + \int_i^f f d\Delta x = \int_i^f P^* dV^* = \pm \Delta E \quad (41) \end{aligned}$$

This equation gives the First Law effect on our system, all the involved parameters taken into account. By comparison of this equation with equations (33), (40) we confirm that the following constraints have been assumed to hold along the transformation:

$$U(T, \Delta x, k_0) = 0 \tag{42}$$

$$\int_i^f TdS(k) = \int_i^f gdk = 0 \Leftrightarrow k = k_0 = \text{const} \Leftrightarrow S(k) = \text{const} = S(k_0) \tag{43}$$

$$\int_i^f fd\Delta x = \int_i^f P^*dV^* \Rightarrow [P^* = \gamma f] \wedge [\Delta x = \gamma V^*] \tag{44}$$

The previous equations describe clearly the energy transfer mechanism between the different degrees of freedom involved. For the sake of consistency of the present model with quantum physics, in the last equation we have to take a (constant) coefficient  $\gamma$  in such a way that the HDF characteristic action associated to  $\Delta x$ ,  $A = m\omega\Delta x^2$ , is of the order of the quantum action  $h$  (we associate a pulsation  $\omega$  to  $\Delta x$  - this will be made clear in the following paper II. In the present context,  $\omega$  is coincident with  $\omega_c$ ). The coefficient  $\gamma$  is able to describe the following limit cases : if  $\gamma \rightarrow 0$  the incertitude  $\Delta x$  and the pressure  $P^*$  will be found negligible and the standard classical case is attained - conversely, if  $\gamma \approx 1$  the particle position incertitude is of the order of the volume  $V^*$  so that a deep quantum limit (small quantum numbers represented) applies. Conjectural expressions for  $\gamma$  can easily be found, but they turn out not to be relevant in the present context. By equation (44) it is seen that the dimension of a [length] is attributed to  $\Delta x$  so that this last is the thermodynamic appearance of the HDF space co-ordinate, whose mechanical counterpart is a variable we will call  $x_z$  (see paper II). To be practical,  $\Delta x$  is the thermodynamic measure of the extension available for  $x_z$  just alike the volume  $V^*$  ( remember this last is a [length] too, because the model is uni-dimensional) is the measure of the space available for the oscillator. The mechanical correspondent of  $\Delta x$  will be interpreted (paper II) as the quantum incertitude on the particle position.

Now we find, using equation (39), that the Gibbs potential associated with HDF can be written as follows

$$G(f,g,T) = G(f,T) + G(g,T) = -PV^* \tag{45}$$

This result can be obtained in the easiest way if we assume

$$TS(k) = [-PV^* - G(f,T)] \ln k/k_0 \implies S(k_0) = 0 \quad (46)$$

$$gk = -PV^* - G(f,T) \quad (47)$$

Equations (38)-(47) make clear that we correlate the isobaric-isentropic transformation to the fact that HDF is perturbed - via its co-ordinate elongation  $\Delta x$  - by the interaction process. HDF is able (reversibly) to absorb heat from the electromagnetic field, transform this energy into work and inject it into the system. It is easily seen, by inspection of our equations, that in order to match the requirements stemming from our FEOM model, we asked to the parameters and potentials some definite behavior and properties. These last can be resumed saying that the parameter  $\Delta x$  is used to transform heat into work, while the parameter  $k$  is taken constant and (concurrently) provides the extra Gibbs potential  $-PV^*$  as required.

#### 4.3 Generalized formalism for the FEOM model and the isobaric-isoelastic transformation

If we recognize that extra parameters have to be considered within the description of the physical processes, then our thermodynamic framework has to evolve towards a generalized formalism. The following equations are therefore established in order to introduce a compact description of the fluctuation process at hand in reference [6]. We have :

$$Z^*(T, V^*) \rightarrow Z^*(T, V^*, \Delta x, k) = Z^*(T, V^*) Z(\Delta x, k) \quad (48)$$

$$Z(\Delta x, k) = Z(\Delta x) Z(k) = \frac{\Delta x}{\gamma} \left[ \frac{k}{k_0} \right]^{-\frac{PV^* + G(f, T)}{T}} \quad (49)$$

$$Z(\Delta x) = \frac{\Delta x}{\gamma} \quad (50)$$

$$\ln Z(k) = -\frac{PV^* + G(f, T)}{T} \ln \left[ \frac{k}{k_0} \right] \quad (51)$$

$$\begin{aligned}
 G^*(P,T) &\rightarrow G^*(T,P,f,g) = G^*(P,T) + G(f,g,T) \equiv \\
 &\equiv F^*(T,V^*) + G(f,T) + G(g,T) = E^* - TS^* - PV^* \quad (52)
 \end{aligned}$$

$$G(f,T) = f\Delta x - TS(\Delta x) = f\Delta x - T \ln \frac{\Delta x}{\gamma} \quad (53)$$

$$G(g,T) = gk - TS(k) = gk - T \ln Z(k) \quad (54)$$

These equations show that the variables  $\Delta x$ ,  $f$ ,  $k$  and  $g$  are introduced into the thermodynamic equations by the means of appropriate additive potentials.

Now, by congruence, we find that the thermodynamic framework corresponding to our FEOM model in reference [6] is expressed by the following equations :

$$Z^*(T,V^*,\Delta x,k) = \frac{T}{V^*} \frac{\Delta x}{\gamma} \left[ \frac{k}{k_0} \right]^{-\frac{PV^*+G(f,T)}{T}} \quad (55)$$

$$E^*(T,V^*,\Delta x,k) = T^2 \frac{\partial \ln Z^*(T,V^*,\Delta x,k)}{\partial T} = E^*(T) + U(T,\Delta x,k) \quad (56)$$

$$U(T,\Delta x,k) = T^2 \frac{\partial \ln Z(\Delta x,k)}{\partial T} \quad (57)$$

$$S^*(T,V^*,\Delta x,k) = \frac{E^*}{T} + \frac{U(T,\Delta x,k)}{T} + \ln Z^*(T,V^*,\Delta x,k) =$$

$$= 1 + \ln Z^*(T,V^*) + S(\Delta x) + S(k) =$$

$$= 1 + \ln \frac{T}{V^*} + \ln \frac{\Delta x}{\gamma} - [PV^* + G(f,T)] \ln \frac{k}{k_0} \quad (58)$$

$$f = T \frac{\partial}{\partial \Delta x} \ln Z^*(T,V^*,\Delta x,k) \quad (59)$$

$$g = T \frac{\partial}{\partial k} \ln Z^*(T, V^*, \Delta x, k) \quad (60)$$

Note here that  $k$  is just the homologous quantity of the function  $K(S_m^*)$  displayed in equation (26), i.e. is an elastic thermodynamic function (pertaining to HDF), able to generate a further (log-part of) entropy in our potential. Since  $k$  is kept constant in our framework, the transformation we are investigating is isoelastic as concerns HDF. On the other hand, from equations (32) and (44) we see that the pressure  $f$  on the co-ordinate  $\Delta x$  is equal to  $P^*/\gamma = \text{const} = f_0$  so that the transformation is isobaric for HDF as well. The interpretation we give to this setting is that we can look at HDF as to a harmonic oscillation superimposed to the classical motion, as is suggested by the FEOM model. Its elongation  $\Delta x$  has been said to be the thermodynamic appearance of the quantum incertitude on the particle position. The previous equation (44) shows that, throughout this transformation, the space extension  $\Delta x$  available for HDF behaves proportionally to the oscillator volume  $V^*$  so that we will also call this transformation "perfect". The elastic function  $k$  - just alike the elongation  $\Delta x$  - may in further developments depend on the volume or other parameters, but is taken independent of the temperature in order to let the constant-volume specific heat of our system unperturbed. This means that the role of HDF is essentially as a physical interface between the classical oscillator and the fluctuation field and - within the present framework - it is not able to store thermodynamic energy. If we keep in mind that "isoelastic" and "perfect" are referred to HDF and that the overall transformation investigated in reference [6], the HDF entropy taken into account, is not isentropic, we may refer to the ensemble of the classical oscillator + HDF as a physical system undergoing a perfect-isobaric-isoelastic transformation. As a further remark, we see that the potential  $G(f, T)$  taken along the isobaric  $f_0$  turns out to be a function of the temperature only. Now equation (47) is a form we will extrapolate down to the case when  $f$  becomes zero, so that by simplicity we will write  $G(f, T) \equiv G_f(T)$ . Then taking into account the equality  $-PV^* = T$  we see that the HDF elastic function is an entropy source (log part)  $2(1 - G_f(T)/T)$  times stronger than its classical counterpart  $K(S_m^*)$  (see eq.(26)), and a chemical potential source able to introduce the thrust potential  $gk_0 = -PV^* - G_f(T)$  into the classical oscillators thermodynamic ensemble.

Along our perfect-isobaric-isoelastic transformation we will find

$$E^*(T) = T \tag{61}$$

$$f = \frac{P^*}{\gamma} = \frac{-P}{\gamma} = \frac{-P_0}{\gamma} = f_0 = \text{const} \tag{62}$$

$$\Delta x = \gamma V^* = \frac{\gamma T}{P_0^*} \tag{63}$$

$$k = k_0 = \text{const} \tag{64}$$

$$Z^*(T, V^*, \gamma V^*, k_0) \equiv P_0^* \frac{\Delta x}{\gamma} \equiv Z_c^* \equiv E_c^* \equiv E^* = T \tag{65}$$

$$U(T, V^*, k_0) = T^2 \frac{\partial \ln Z(\Delta x, k)}{\partial T} \Big|_{V^*, k_0} = 0 \tag{66}$$

$$S^*(T, V^*, \gamma V^*, k_0) = \frac{E^*}{T} + \ln Z^*(T, V^*, \gamma V^*, k_0) =$$

$$= 1 + \ln Z^*(T, V^*) + S(\Delta x) \Big|_{V^*} + S(k_0) =$$

$$= 1 + \ln \left[ P_0^* \frac{\Delta x}{\gamma} \right] \equiv 1 + \ln T \equiv 1 + \ln E^* \equiv S_c^*(E^*) \equiv S_c^*(E_c^*) \tag{67}$$

$$gk \Big|_{k_0} \equiv gk_0 = -PV^* - G_f(T) \tag{68}$$

We have also

$$f \Delta x = T \equiv E^* \tag{69}$$

#### 4.4 The perfect-isobaric-isoelastic transformation and the fluctuation entropy

Now from equation (41), always referring to the FEOM model, we see that the system total entropy ( given by  $S^* + S(\Delta x) + S(k)$  ) fluctuates with the energy  $E^*$ . Its mean value  $\langle S^*(T, V^*, \gamma V^*, k_0) \rangle$ , given by the integral performed over the variational domain with extension  $\Delta E$  and boundaries  $E_i^*, E_f^*$ , is seen by the effect of equation (67) to be coincident with an expression we have reported in a previous section already:

$$\begin{aligned} \langle S^*(T, V^*, \gamma V^*, k_0) \rangle &\equiv \langle S_C^*(E_C^*) \rangle = \frac{1}{\Delta E} \int_{E_i^*}^{E_f^*} S_C(E_C^*) dE_C^* = \\ &= \frac{1}{\Delta E} \int_{E_i^*}^{E_f^*} \left[ 1 + \ln(E_C^*) \right] dE_C^* = \frac{E_f^* \ln E_f^* - E_i^* \ln E_i^*}{\Delta E} = S_{QM}(T) \quad (70) \end{aligned}$$

As remarked, due to the correlation between  $E_i^*$  and  $E_f^*$ , this mean value is also found coincident with the quantum-mechanical expectation  $S_{QM}$ . We recall that the formalism expounded in reference [6] provides, at the same time, the value of the mean thermodynamic energy  $\langle E_C^* \rangle$  over the fluctuation (given in equation (10)), and the mean Helmholtz energy of the system. The last quantity turns out to be expressed by the arithmetic mean over the extreme values attained in the fluctuation. All of these quantities are found to be coincident with the quantum-mechanical expectations, in agreement with the following equation :

$$\begin{aligned} -T \langle \ln Z^*(T, V^*, \gamma V^*, k_0) \rangle &= -T \frac{1}{2} \ln (E_i^* E_f^*) \equiv -T \ln Z_{QM}(T) = \\ &= \langle E_C^* \rangle - T \langle S_C^*(E_C^*) \rangle = \\ &= \frac{E_f^* + E_i^*}{2} - T \frac{E_f^* \ln E_f^* - E_i^* \ln E_i^*}{\Delta E} \equiv E_{BE} + \frac{\Delta E}{2} - TS_{QM}(T) \quad (71) \end{aligned}$$

Both the Helmholtz energy and the entropy shown in these equations result from the application of symmetrizing equations of the form (1)



taken with an energy distribution function  $P(E_C^*) = 1/[E_f^* - E_i^*]$ . If we set, indeed

$$P_C(E_C^*) \equiv \exp \left[ S_C^*(E_C^*) - \frac{E_C^*}{T} \right] = Z_C^* \quad (72)$$

then we have :

$$\begin{aligned} P_{if}^{CC}(E_i^*, E_f^*) &\equiv \exp \int_{E_i^*}^{E_f^*} [\ln P_C(E_C^*)] P(E_C^*) dE_C^* = \\ &= \exp \frac{1}{\Delta E} \int_{E_i^*}^{E_f^*} \left[ S_C^*(E_C^*) - \frac{E_C^*}{T} \right] dE_C^* = \exp \left[ \langle S_C^*(E_C^*) \rangle - \frac{E_f^* + E_i^*}{2T} \right] \end{aligned} \quad (73)$$

so that

$$\begin{aligned} &\left\{ P_C(E_C^*) \rightarrow P_{if}^{CC}(E_i^*, E_f^*) \right\} \Leftrightarrow \\ &\Leftrightarrow \left\{ \left[ S_C^*(E_C^*) \rightarrow \langle S_C^*(E_C^*) \rangle \equiv S_{QM}(T) \right] \wedge \left[ E_C^* \rightarrow \frac{E_f^* + E_i^*}{2T} \right] \right\} \end{aligned} \quad (74)$$

i.e.

$$Z_C^*(E_C^*) \rightarrow Z_{QM}(T) \quad (75)$$

and

$$-T \ln Z_C^*(E_C^*) \rightarrow -T \ln Z_{QM}(T) \quad (76)$$

These equations show that CC-symmetrization of the classical quantities  $S_C^*(E_C^*)$ ,  $E_C^*$ ,  $Z_C^*$  results into their quantum-mechanical correspondents. On the other hand, we might also set

$$P_C(E_C^*) \equiv Z^*(T, V^*, \gamma V^*, k_0) \equiv Z_C^* \equiv E_C^* \quad (77)$$

and use in this case the classical-discrete (CD) fluctuation model [2]. We would obtain:

$$P_{if}^{CD}(E_i^*, E_f^*) = \sqrt{P_C(E_i^*)P_C(E_f^*)} = \sqrt{E_i^* E_f^*} \quad (78)$$

Then we would find, as pointed out in [6] already,

$$\left\{ P_C(E_C^*) \rightarrow P_{if}^{CD}(E_i^*, E_f^*) \right\} \Leftrightarrow \left\{ Z_C^* \rightarrow \sqrt{E_i^* E_f^*} \equiv Z_{QM}(T) \right\} \quad (79)$$

These last relationships have been found in the previous, quoted reference - they are reported here by the sake of completeness and clarity of the overall framework proposed. In equations (72)÷(79) idler normalization constants for the probability distributions have been omitted.

## 5 Generalized framework

The previous equations (12)÷(79) give deeper congruence and interpretative level to the equations used in reference [6]. As equations (65) and (67) are concerned, it is interesting to note that the investigated perfect isobaric-isoelastic transformation equations (61)÷(64) characterizing the fluctuation make the expression of  $Z^*(T, V^*, \Delta x, k)|_{P, k_0}$  coincident with the purely classical expression  $Z_C^*$  :

$$Z^*(T, V^*, \Delta x, k)|_{P, k_0} = Z^*(T, V^*, \gamma V^*, k_0)|_{P= T} = Z_C^* \quad (80)$$

This is because the functions  $\Delta x / (\gamma V^*)$  - which last can be named  $z(\Delta x, V^*)$  - and  $Z(k)$  are equal to 1 across our perfect-isoelastic transformation. We may say that these functions are "silent" or "hidden" into the classical sum-over-states expression  $Z_C^*$ . We will use again this concept later on in this section.

Also interesting, we see that the form

$$Z^*(T, V^*, \Delta x) = \frac{T}{V^*} \frac{\Delta x}{\gamma} \quad (81)$$

can be obtained using the classical sum-over-continuum-states method when the very simple variations one can appreciate from the following equations are applied (two degrees of freedom reference case) :

$$Z_C^* = \int_0^\infty \int_0^\infty \exp(-H_C(p, x)/T) dp dx \equiv T \quad (82)$$

$$\begin{aligned}
 Z^*(T, V^*, \Delta x) &= Z^*(T, V^*)Z(\Delta x) = Z_c^* \frac{\Delta x}{\gamma V^*} \equiv \\
 &\equiv \int_0^\infty \int_0^\infty \exp(-H_c(p, x)/T) \frac{dp dx}{\Delta p V^*} \quad (83)
 \end{aligned}$$

(idler constants  $h$  and  $\gamma$  now dropped off). This equation shows clearly that our thermodynamic framework is correlated to a standard classical-continuum formalism for the sum-over-states, from which it can be obtained by normalizing the integral with a (variable) action  $\Delta p V^*$ .  $\Delta p$  in equation (83) is equal to  $h/\Delta x$  as given by Heisenberg's indetermination principle (note yet that this last will take in our frame an interpretation different from the orthodox one, and the incertitudes  $\Delta x$  and  $\Delta p$  will assume peculiar definitions to be given in the following paper II). However,  $\Delta p$  is only an auxiliary, dependent (on  $\Delta x$ ) variable in the present frame. For further investigation we will instead base our generalized expression, displayed in the next equation (85), on extrapolating the constitutive function  $\Delta x/V^*$  in equation (81) into the following form

$$\frac{\Delta x}{V^*} \rightarrow \sqrt{\frac{\Delta x \Delta p^*(\Delta x)}{m \omega V^2}} \quad (84)$$

In this expression, the quantity  $\Delta p^*$  is a function which must be considered coincident with the quantum momentum incertitude only when the Heisenberg's principle is assumed as an effective, strict constraint - i.e. in the full quantum limit. It is clear that in the quasi-classical limit, instead, this constraint is relaxed, and we will take  $\Delta p^*$  just proportional to  $\Delta x$  when we want to approach the FEOM framework. In this way, we see that  $\Delta x \Delta p^*(\Delta x)$  is an action varying between the values  $h$  and  $A = m \omega \Delta x^2$  peculiar of the FEOM model ( $A$  being of the order of  $h$  too, the asterisk in  $\Delta p^*$  just indicates that in the FEOM case  $\Delta p^*$  takes indeed the functional expression  $\omega \Delta x$ ).

As a consequence of equation (84), in the full quantum limit the sum-over-states expression will be found independent of  $\Delta x$  so that (as remarked already) the corresponding pressure  $f$  will be found de-coupled from the pressure  $P^*$  and takes a zero value. This setting is also due to the fact that, in our papers, the full quantum case analysis is limited to the case of stationary pure quantum  $n$ -states, with density  $\rho_n$ . The specific volume  $V \equiv \rho_n^{-1}$  then in the full quantum case will not be a

fluctuating quantity (as it was in the FEOM model) any more, and there is no work done by the pressure  $f$  to account for. It is therefore understood, in our framework, that the pressure  $f$  originates from the mixing of pure states.

### 5.1 The generalized thermodynamic potential and sum-over-states expressions

The generalized expression we are going to introduce by the means of equation (85) is a silent expression (in the quantum limit where  $\Delta p^* \equiv \Delta p \approx h/\Delta x$ ; note also that  $k$  will just take the determination  $k_0$  along the relevant transformation) in the HDF variables. This is noted in view of further analysis to be developed in paper III. This analysis will concern microcanonical ensembles of particles, so that the thermodynamic variables will be referred to such cases. We advance therefore that a "microcanonical temperature"  $T_m$  will be defined, and the relevant volume will be set equal to the inverse of density  $\rho$  and named  $V$ . We have first :

$$\begin{aligned} \psi &= -\ln z(T_m, V, \Delta x, k) = \\ &= -\ln \sqrt{\frac{\Delta x \Delta p^*}{\{m\omega V^2\}}} + h(T_m) - \frac{\int P(V)dV}{T_m} - \frac{\{PV\} + G_f(T_m)}{T_m} \ln \frac{k}{k_0} \equiv \\ &\equiv -\ln z(V) \equiv \ln V \equiv -\ln \rho \end{aligned} \quad (85)$$

$$h(T_m) = \frac{\int P(V)dV}{T_m} + \frac{PV + G_f(T_m)}{T_m} \ln \frac{k}{k_0} + \frac{1}{2} \ln \frac{\Delta x \Delta p^*}{m\omega} \quad (86)$$

The function  $z(T_m, V, \Delta x, k)$  is a peculiar constitutive factor of the sum-over-states we will introduce in the following. Its utility will indeed be clear from the following equations (87)÷(90). It is expressed in equation (85) as a function of the appropriate (thermodynamic) independent variables  $T_m, V, \Delta x, k$ ; but note that the potential  $PV$  and the action  $m\omega V^2$  must be considered only parametric functions in that expression (the brackets indicate that we have not to take derivatives of  $\{PV\}$ ,  $\{m\omega V^2\}$  while setting up the constitutive equation for the pressure  $P$ ).

The function  $h(T_m)$  is a normalizing factor which can be calculated by the same equation (85) taking care that - given a transformation of interest - all the parameters have been expressed as a function of  $T_m$ . By the effect of such normalization (i.e. by our definition), the sum-over-states  $z$  is always equal to the particle density  $1/V$ . The variable  $z$  displays its utility when we express the statistical sum over (energy-bounded) micro-canonical ensembles to set up the full system thermodynamic potential. To see this, let us distinguish here the two relevant cases of the FEOM quasi-classical limit and the full quantum limit. We have indeed easily

$$\begin{aligned}
 Z_m^*(H_C, T_m, T, V, \Delta x, k) &= z(T_m, V, \Delta x, k) \exp(-H_C(p, x)/T) \equiv Z_m^*(H_C, T, V) \equiv \\
 &\equiv \frac{1}{V} \exp(-H_C(p, x)/T) \tag{87}
 \end{aligned}$$

Here the index  $m$  means "microcanonical", and idler constants are dropped off. Then we have

$$\begin{aligned}
 Z^*(T, V^*, \Delta x, k_0) &= \int_0^\infty Z_m^*(H_C, T_m, T, V, \Delta x, k_0) dH_C = \\
 &= \int_0^\infty z(T_m, V, \Delta x, k) \exp(-H_C(p, x)/T) dH_C \equiv Tz(T_m, V^*, \Delta x, k_0) \equiv |_{\Delta x=V^*} \\
 &\equiv Z^*(T, V^*) \equiv \frac{T}{V^*} \qquad \{ \text{FEOM case} \} \tag{88}
 \end{aligned}$$

$$Z_n^{QM}(E_n, T, V_n) = z(T_m, V_n, \Delta x_n, k_n) \exp(-E_n/T) = \frac{\exp(-E_n/T)}{V_n} \tag{89}$$

$$Z_{QM}(T, V^*) = \sum_n Z_n^{QM}(E_n, T, V_n) = \sum_n \frac{\exp(-E_n/T)}{V_n} =$$

$$= \frac{1}{V^*} \sum_n \exp(-E_n/T) \quad \{\text{Full quantum case}\} \quad (90)$$

Here and in the previous equation (88) the volume  $V^*$  appears, as is obvious, as a mean value calculated over the energy states ensemble. By these equations, we see that the FEOM case is the quasi-classical limit of the full quantum case. The last equation can also be integrated in the  $x$ -co-ordinate domain because we take in this case the volume  $V_n$  as a function of  $x$  and equal to the inverse power of the (normalized) quantum-mechanical particle density  $\rho_n(x)$ , so that

$$\begin{aligned} Z_{QM}(T) &= \int Z_{QM}(T, V_n) dx = \sum_n \int \frac{\exp(-E_n/T)}{V_n(x)} dx = \\ &= \sum_n \int \rho_n(x) \exp(-E_n/T) dx = \sum_n \exp(-E_n/T) \\ &\quad \{\text{Full quantum case}\} \end{aligned} \quad (91)$$

Integration of equation (88) over the  $x$ -co-ordinate domain also makes the volume  $V^*$  to disappear, and plainly brings to the standard classical result.

These equations and remarks complete - as concerns the present context - the correlation frame between our proposed theory and consolidated known results. They will also turn out useful for applications to be shown in the following papers II and III. Note yet that both the FEOM and the full quantum case as described by equations (87) and (89) appear consistent with defining a Gibbs distribution [23]  $w_n$  in the form

$$w_n = \rho_n(x) \exp(F - E_n)/T \quad (92)$$

This distribution displays extra entropy terms  $\ln \rho_n$  within it when compared with the standard definition. We can interpret this difference as a peculiar property expressed by the microcanonical ensembles pertaining to the energy values  $E_n$ , meaning that some extra degeneration of states with respect to the standard case should be accounted for when

forming the statistical mixtures with particles submitted to the HDF action. We are going indeed, in the following paper III, to set up a statistical model able to account for this degeneration - as well as to serve as a basis to characterize better the physical meaning of the HDF parameters  $\Delta x$ ,  $\Delta p$ ,  $f$ ,  $g$ ,  $k$  and their influence on the overall physical behavior of our system. Here, as a conclusive statement, we will give the expression of the stationary mass-flow theorem consistent with the thermodynamic framework we exposed in this paper, inclusive of the effect of HDF, for a microcanonical particles ensemble characterized by the partition function  $z(T_m, V, \Delta x, k) \exp(-E/T)$ .

*5.2 The mass-flow and energy theorem expressions*

To introduce this subject, let us first consider the following. The FEOM model is set up with a thermodynamic assembly of classical oscillators whose attractive centers are at rest in the same position of the space. Their volume potential, defined as a function of the (linear) co-ordinate  $x$ , is harmonic and, in the classical thermodynamic average, is responsible for half the constant volume specific heat of the system ( $c_v = 1$ ). The volume  $V^*$  of the system represents the total extension of space occupied by the oscillating particles. The FEOM model, therefore, describes the behavior of a thermalized system on a "macroscopic" space scale. In the following we want, instead, to have deeper insight into the system physical properties which are observable within a smaller scale region of space. This means that we are interested into the local properties displayed by the statistical ensemble of particles which can be found into every region of space enclosed between the current co-ordinate values  $x$  and  $x+dx$ . To this end, we have to introduce some "local" thermodynamics of the system, and this will be based on the expressions of the distributions (87) or (89) (depending on the case) and on the flow-of-mass theorem.

In the region of space enclosed between  $x$  and  $x+dx$  we will find  $\rho(x)dx = (1/V)dx$  oscillating particles, i.e.  $\rho(x)$  is the (numerical) particle density at the space position  $x$  and  $V$  is the corresponding specific volume. A drift velocity  $v_D(x)$  of the ensemble has also to be considered - it will depend on the mass-flow rate we define for the particles ensemble. The  $x$ -dependent classical volume potential is centered in  $x = 0$ , is equal for all the particles and will be called from now on  $\Phi(x)$ . It will be taken into account as a mechanical quantity in our following equations, so that the constant volume specific heat of the system will now be only due to the kinetic degree of freedom ( $c_v = \kappa/2$ ). As we will see in paper

III, indeed, our microcanonical energy distributions will also be characterized by some degree of thermalization  $\kappa$  concerning the kinetic degree of freedom and by their own "microcanonical temperature"  $T_m$ , to be introduced. Clearly, our FEOM model and associated considerations on HDF in this paper will sometimes become obsolete because of the thermalization of the potential degree of freedom having been removed; but - the thermodynamic potentials pertaining to different degrees of freedom being additive - we are allowed to use the previous results on HDF as a strong reference. Moreover, we want since now evolve towards a generalized analysis where our previous results will be - as is obvious - used critically. Within this context, the peculiar entropy term  $-\ln V$  introduced in the FEOM model will be maintained within the generalized framework - we have seen already that it remains attached to the microcanonical distribution as is clear from the expression of the statistical sum  $z(T_m, V, \Delta x, k) \equiv 1/V$  introduced previously. At the same time, taking the derivative with respect to  $V$  in the generalized equation for  $z$ , we see easily that the pressure definition of the microcanonical ensemble has been de-coupled from the entropy expression, so that the possibility to have different state equations is provided. In these conditions, we can even remove the constraint that the volume potential is harmonic and evolve towards the general case - where  $\Phi(x)$  is whatever and, in every position  $x$ , HDF is still acting on the particles while they travel.

Within this framework, we are brought to write down the mass-flow theorem for our ensemble of particles. The HDF action must be accounted for into it and we write:

$$\frac{1}{2}mv_D^2 + I_D(\nu(x), \kappa) + \Phi(x) + \langle \Phi_{\text{HDF}}(x, \xi_i(x)) \rangle |_x + \int \Delta x df + \int k dg + \int V dP = \text{Inv} \quad (93)$$

Here  $v_D$  is the ensemble drift velocity,  $\langle \Phi_{\text{HDF}}(x, \xi_i(x)) \rangle |_x$  is the (statistical ensemble mean) mechanical potential energy supplied by HDF ( $\Phi_{\text{HDF}}$  will be supposed, as shown in the next papers, to depend on a statistical parametric function  $\xi(x)$  assuming a value  $\xi_i(x)$  for the  $i$ th particle). The quantities  $\int \Delta x df$  and  $\int k dg$  are the constant -temperature parts of the HDF-dependent thermodynamic chemical potentials, while  $\int V dP$



is the standard classical, relevant (for mass-flow) part of the system chemical potential or indicated work function. The term  $I_D(\nu(x), \kappa)$  is a mechanical-statistical potential, determined by the microcanonical distribution broadening ( $\nu(x)$  is the corresponding volume-flow of particles) and "thermalization level"  $\kappa$ . This potential is assumed not to depend explicitly on the pressures  $f$  and  $g$ , so that it has not to be discussed within the present framework and will be introduced with details in paper III. Note here finally that if we take the limit of equation (93) when  $P \rightarrow 0$ , then we have a microcanonic-statistical expression for the energy theorem in the form

$$\left[ \frac{1}{2} m v_D^2 + \langle \Phi_{\text{HDF}}(x, \xi_i(x)) \rangle_x + \int \Delta x df + \int k dg \right] |_{P=0} + I_D(\nu(x), 0) + \Phi(x) =$$

$$= \text{Inv} |_{P=0} \tag{94}$$

This equation will be exploited in the following papers.

According to a previous remark, when considering pure quantum n-states the quantity  $\int \Delta x df$  will be taken equal to zero.

## 6 Conclusion

In this paper, we analyzed the thermodynamic behavior of a classical oscillators ensemble in the light of the assumed existence of the so-called hidden degree of freedom. HDF behaves as an interface between the system and the external environment, and is assumed responsible for the appearance of quantum-like effects. A few thermodynamic properties of this physical actor have been brought to evidence, using essentially the requirement that the overall framework describing the relevant interactions and behaviors is able to match standard thermodynamics. A simple generalized apparatus including an expression of the mass-flow theorem able to describe the evidenced properties has been also set up for later use, to be displayed in the following papers II and III.

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