THE ELECTROMAGNETIC ORIGIN OF QUANTIZATION AND THE ENSUING CHANGES IN COPENHAGEN INTERPRETATION

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ABSTRACT. The pre-1925 quantum prescriptions of Planck, Einstein, Bohr, Sommerfeld and recently Aharonov-Bohm permit a recasting as part of a complete set of electromagnetic residue integrals such as used in a mathematical discipline known as de Rham cohomology. The ensuing spacetime topological reorganization of early quantum aspects seems well supported by Josephson- and quantum Hall effects. This reversal of priorities demands a physical readjustment of standard nonclassical Copenhagen pronouncements. The Schroedinger equation becomes a tool solely applicable to ensembles consisting of single systems of random phase and -orientation. This reorganization is a return to the ensemble initiatives of the Thirties by Slater, Popper, Kemble and others, which now can be given a compelling form by identifying long standing classical counter-examples to Copenhagen's nonclassical propositions. Heisenberg uncertainty and zero-point energy have to yield their pedestal of universal absolute status. They now become manifestations governing order-disorder transitions in ensembles.

key words: quantum topology, Copenhagen, ensemble, single system

1   Introduction

This essay aims at new basic perspectives for electromagnetic theory that are contingent on an incisive revision of the Copenhagen interpretation of quantum mechanics. This revision does not affect established mathematical procedures of quantum mechanics. Instead, it addresses a more precise de-

lineation of its objects of description and how they relate to reality. The interdisciplinary nature of this presentation has to call on explanatory compromise. Major parts pertaining to period integrals as well as the ensuing changes in Copenhagen interpretation have been discussed earlier in some detail in book form [21]. The justification for this overview in the form of an article is really based on a need to bring out more salient aspects why one has to call on different disciplines to strengthen the motivation to accept the need for here suggested incisive changes.

A vast majority of physicists views the Copenhagen interpretation with some reservation, say as something in lieu of a more realistic and precise view yet to come. By the same token, suggested alternatives have been viewed with equal distrust, because there is reluctance to change unless there is substantial epistemic gain. As a result, physics is now confronted with a status quo of three quarter century during which undefined nonclassical recipes have been ruling supreme.

Since recipes beget other recipes, it is not surprising if a state of fatigue is apparent, due to too many new recipes that have been too limited in scope. Quantum theory, as other theories, started out with recipes, in fact, Schroedinger's equation is still a recipe today. Over the past three quarter century, this equation has worked so well so as to entice physics to remain in the recipe stage. This essay is almost an act of despair to prevent physics from being spoiled by a too exclusively ontic modus operandi. Knowing **how** does not obviate a need later for also knowing **why**.

The major point of this essay claims that most nonclassical metaphors are due to the insidious single system choice of the Ψ function interpretation presented in the standard Copenhagen views**.** Acknowledging this error of decision does not detract from what we know already. It mostly makes life simpler and more realistic by getting away from a contemporary overdose of nonclassical hype. For those beholden to Copenhagen traditions, the process must be anticlimactic. **Since metrology provides here a major substantiation for the initiative supporting a more epistemic oriented view,** an historical perspective may be helpful on how our thinking has been molded by a succession of discoveries, combined with at times blinding success.

2   Chronology of Events leading up to Quanta of charge e, action h, flux h/e.

The following chronology of quantum experimentation has been selected to focus on its implications that point at a discrete nature of matter and asso-

ciated dynamic processes. This overview can be later used for referral in the process of developing new value judgments. By taking advantage of now available more encompassing information, a reexamination of currently accepted foundations seems in order. There is evidence that in the past, physics has forced itself into premature decisions by focusing too exclusively on the quantum of action, perhaps neglecting the quanta of charge and flux. Here is a combined overview of basic experimentation that had a major role in the conceptualization surrounding the three major quanta of nature:

Chronology of Events

1803
Dalton's atomic view of chemistry and Avogadro's number are first systematic and persistent reminders pertaining to a discreteness of matter.

1836
Faraday's electrolytic experiments, in conjunction with Dalton's concept of atoms, give decisive signs of the discreteness of electric charge. In conjunction with Avogadro's number, it is indicative of a smallest unit of electric charge.

1897
The discovery of the electron by J J Thomson puts many earlier speculations in a more concrete and realistic light. The existence of a quantum of electricity e is now an established fact.

1900
Planck extends the quantum notion to an abstract quantity known as action, its unit is h. He felt compelled into making this decision, because it seemed a sine qua non for arriving at an experimentally correct spectral radiation law of heat. He uses the frequency ν to define energy quanta $h\nu$.

1905
Einstein uses Planck's energy quantum $h\nu$ to account for a very spectacular and sharp frequency cut-off of the photo-electric effect. The h values obtained from the photo-electric effect appear to gibe with the h values obtained from Planck's radiation law. This second application strengthened the epistemic nature of Planck's initially ontic proposition of quantum h.

1910
Millikan succeeds in measuring, in a direct way, the quantum e of electric charge. The values gibes with earlier estimates of Faraday and data obtained from deflecting electron beams.

1912

Planck [1], in his book THEORY OF HEAT RADIATION introduces the concept of zero-point energy as an average energy residual of hν/2 per oscillator. It is necessary to keep an ensemble of harmonic oscillators in a phase random state. He views quantization as a cellular nature of phase space, say as symbolically expressed by the two-dimensional integral:

$$\iint_{cell} dpdq = h .$$

1913

Bohr gives Planck's abstract quantum of action a physically tangible form by taking angular momentum to be a multiple of h. In doing so he could derive Balmer's spectral formula for hydrogen and hydrogen-like ions. This boosted the nature of Planck's quantum from ontic to epistemic.

1915

Sommerfeld [2] succeeded in converting Bohr's condition in a more useful tool as a cyclic integral of momentum, which he equated to multiples of the quantum of action:

$$\oint pdq = nh \qquad\qquad \text{with} \quad nh; n = 1, 2,....,$$

known as the Bohr-Sommerfeld quantization integral. This integral was also pioneered by others. Sommerfeld's name stuck, because his famous fine structure calculations using relativity mechanics were experimentally confirmed.

1916

Einstein's [3] very elegant derivation of Planck's law using spontaneous- and induced emission.

1923

Using the Bohr-Sommerfeld relation, William Duane [4] gives a particle-based theory of X-ray diffraction.

Compton's electron-photon scattering proves the validity of energy-momentum conservation under quantized conditions.

1924

Louis de Broglie [5] calls attention to a proportionality of the frequency-wave- and energy-momentum- vectors with Planck's h as factor of propor-

tionality. This formulation reveals Planck's quantum as a relativistic invariant. This is so to say a *material* parallel of Duane's X-ray work.
1925
Uhlenbeck and Goudsmit [6] propose spin and magnetic moment properties for the electron.
1926
Schroedinger [7] uses de Broglie's "matter" wave metaphor to obtain a wave equation, which was then (prematurely) hailed as superseding all previous quantization procedures pertaining to action.
1927
Davisson-Germer [8] confirm de Broglie's diffraction of electrons as a rest-mass counter part of Duane '23 X-ray proposition.

Heisenberg [9] launches his uncertainty relation as a universal, always present limitation on physical perception. This universality proposition is here restricted by changes to be proposed later.

Dirac [10] creates a Lorentz invariant counter part of Schroedinger's wave equation, surprisingly yielding quantitative evidence of the Uhlenbeck-Goudsmit electron spin proposition.
1932
In a footnote remark in his book on Superfluids, Fritz London [11] introduces for the first time a fundamental quantum of flux of magnitude h/e.
1961
Doll [12] and Fairbanks [13] experimentally establish the existence of flux quanta of magnitude $h/2e$. Onsager [13] relates the factor 2 difference to Cooper's electron pairing in the superconducting ring samples.
1962
Brian Josephson [14] discovers thin film tunneling of super currents. The associated ac effect yields extremely precise data for h/e.
1980
Working with a constant magnetic field and varying electron density in their MOSFET samples, Klaus von Klitzing *et al* [15] discover the integer quantum Hall effect. The Hall impedance (transverse Hall voltage over sample current) is found to equal h/e^2 divided by an integer. Josephson ac effect, measuring h/e, and quantum Hall effect, measuring h/e^2, yield a metrology milestone in precision data for the quanta h and e.
1982
Working with a variable magnetic field and unchanged electron density in their sample, physicists at Bell Telephone Labs [16] discover, in addition to

the 1980 integer effects, a fractional quantum Hall effect. Their Hall impedance equals h/e^2 divided by a rational fraction.

3   A Comparative Evaluation of Three Fundamental Quanta

The given chronology covers a time-span of nearly two century in dealing with discreteness related observations in nature. From a preliminary inspection, it is quite apparent that some quanta have received more attention than others. The quantum of action h has received by far the bulk of attention. At the time of its discovery at the beginning of the twentieth century, the quantum of electric charge e was almost well established, whereas the quantum of flux had not yet made its debut. The flux quantum h/e was first suggested by London [11] in 1932 and it would take nearly three decades before its existence would be verified by the experiments of Doll [12] and Fairbanks [13]. It thus seems attention and interest in the existence of quanta has been somewhat unevenly distributed. The quantum of action has received a disproportionate attention. In the course of time, it became a key to a new so-called nonclassical era in physics, while the much earlier quantum of charge had been accepted with less fanfare as compared to the upheavals associated with action. All this happened at a time when a quantum of flux was still perfectly unknown.

Taking a look at all three quanta, flux h/e, charge e and action h, only two of them are independent. The product of flux and charge gives action or if you will the ratio of action and charge yields flux. Such close relations ought to indicate a family membership. Let us now inquire whether, according to their origin, these quanta are distinguished by conspicuous mathematical features.

All quanta have in common that they have lately been determined with a numerical precision covering nine decimal places. These values don't depend on location or time of observation. In fact, spectroscopy yields ample evidence that **the quanta are the same throughout the visible universe.** This observation suggests a global nature, which indicate that this intrinsic nature should have far-reaching invariance connotations.

There is no evidence indicating that e and h values are affected by strong gravitational fields, even black holes if you will. Since gravitational fields reflect on a spacetime metric structure, it would indicate that **the quanta are metric-independent entities.**

Spectroscopic observation of very distant stellar objects have indicated that the fine structure "constant" measured for quasars is the same as what is

spectroscopically observed on earth. Since the fine structure constant is a ratio of an E-M free-space impedance and the quantum Hall impedance h/e^2, it follows: **The free-space impedance ought to be a universal constant.**[1]

On the basis of these very compelling facts, the quanta here identified are universal constants throughout our spacetime domain of experience. So, one has to conclude that quanta are spacetime scalars independent of position and instant of time. **Hence h and e, as well as combinations h/e and h/e^2 occur as scalars of constant absolute value throughout our spacetime domain of observation.**

The just presented observations call for mathematical specifications in basic physics, capable of accommodating these quanta features as here identified. Gauss' integral law of electrostatics, which dates back to the early nineteenth century, can guide the way as to how quanta are identifiable. Gauss' law of electrostatics is a two-dimensional closed surface integral of the dielectric displacement. It identifies the charge or charges inside its enclosure. *Mathematically this integral law belongs in a category now referred to as Stokes' generalized law. The latter is valid in general differential manifolds regardless of whether they have a defined metric.*[2]

Note how this observation in italics verifies the earlier bold face remark on the metric- and gravity-independence of quanta. This recurring feature in which quanta are pre-metric entities identifiable by pre-metric (Stokes) integral laws calls for an adjustment of loosely defined notions in a realm that has been denoted as quantum gravity.

This intermezzo of abstract analysis is essential, because a closed loop one-dimensional integral of the vector potential similarly identifies units of flux linked in its integration loop. It was used by Fritz London [11]. He published his prediction of a flux quantum unit in the form of a footnote during the Thirties. Spacetime applications by Aharonov and Bohm [17] have made this integral a principal tool in quantum interferometry. Yet notwithstanding this outstanding record of achievement, the integral still retains a position as a secondary tool with respect to Copenhagen's anointed instrument of presumed primary quantization: Schroedinger's equation.

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[1] A point of view supported on the grounds of universe transparency by Hermann Poeverlein; emeritus professor of geophysics at the Darmstadt Institute of Technology in Germany.

[2] For elementary proofs of metric-independent invariance of curl and divergence operations see ref.27.

As it now stands the quanta of charge and flux exhibit a purely electromagnetic nature. Since the product of flux and charge yields action quanta, one may wonder: can the latter also be reduced to a purely electromagnetic origin as a residue of an integral?

Planck's initial application as an action unit for the harmonic oscillators inside a black body points at an initial electromagnetic origin. It was Bohr who converted this abstract Planck assumption into the more tangible assumption pertaining to the mechanical angular momentum of the orbiting electron in an hydrogen atom. Ever since, it has remained that way, because quantum theory became quantum mechanics not quantum electromagnetism.

The question whether or not action or angular momentum can be recast in the form of a residue-type integral has been answered by Robert M Kiehn [18]. The integral in question is indeed electromagnetic in nature. Its cyclic integration domain has to be extended over a closed three-dimensional domain of integration. Hence the conceptual reality of this theoretic artifact is contingent on whether or not this closed three-dimensional cycle of integration can be imbedded in a spacetime manifold. As a possible form of the integrand, Kiehn suggests the exterior product of the differential one-form defining the Aharonov-Bohm(AB) integral and the two-form of the Ampère-Gauss(AG) integral; the latter [19] being the spacetime generalization of the Gauss integral of electrostatic theory. Kiehn shows how for a single charge this three-dimensional integral reduces to the familiar Bohr-Sommerfeld condition of quantization. The reader be aware that the reduction is contingent on equating field- and particle-based quantities; *e.g.,* **eA-->p**.

In the perspective of the action integral's reducibility to Bohr-Sommerfeld, one may wonder whether the hydrogen spectrum can be obtained by using solely electromagnetic quantization criteria. A sample calculation to this effect appears in the appendix. The spectrum is the same, yet their are subtle differences when relativity is taken into account.

This preliminary overview of trying to learn from a different angle at presenting the milestones of the past invokes mathematical demands that need to be met if hitherto unfamiliar perspectives are to be substantiated. Standard physical description based on the groups R(3) and L(1,3), defined by metric invariance may leave metric invisible, but the metric is still hidden. True metric-independence can only be made **visibly explicit** by imposing general invariance Diffeo(4). There is no other way to reliably check whether the metric remains absent. **Hence experience with wider groups of description is a sine qua non for becoming discerning about these features of quantum topology**.

4   New Cyclic Integration Perspectives in Mathematical Physics

The contemporary traditions of quantum mechanics have placed the exactness of Schroedinger results well above results obtainable by the pre-1925 methods of quantization. The two have been known to be subject to asymptotic relations such as expressed by the WBK relations. Euphoria following the discovery and successes of the new 1925 quantum mechanics, rather than logical evidence, have led to convictions that the Schroedinger result had to be exact, by contrast the pre-1925 results were to be relegated to a realm of approximations.

The just cited traditional view is a superficial one, because it silently assumes the pre-1925 methods and the Schroedinger process as addressing the same category of physical situations. Many macroscopic quantum interferometer experiments and corresponding low temperature work do give an impression that Aharonov-Bohm type arguments not only give adequate answers, the results have shown precision and reproducibility that raise serious doubts about their officially presumed approximate status.

The most reliable h and e values presently come from the Josephson and quantum Hall effects. While these effects were originally argued from a Schroedinger angle, closer inspection really shows the Aharonov-Bohm integral at work. Josephson's phase single valuedness is for all practical purposes an Aharonov-Bohm argument. **For the quantum Hall effect, zero-point energy has prevented Schroedinger's process from yielding a tight and truly honest account of reality**. In fact the Schroedinger approach has led to all sorts of fudging such as artifact distinctions between integer and fractional quantum Hall effects. However, an Aharonov-Bohm type argument [21], by comtrast, provides a direct and tight relation between experiment and description, without any need whatsoever for taking recourse to fudging or unproven assumptions concerning fractional, elementary, electric charge quanta or compound fermions.

Summarizing, these arguments indicate the existence of a category of quantum effects for which the cyclic integrals AB and AG make a much stronger showing than Schroedinger's process. Existing traditions in quantum instruction have so far not been favorable to giving this conceptual alternative a constructive role in contemporary physics.

Acting on this suggestion for change, it will be necessary to report here about some mathematical progress related to cyclic integrals. Gauss' integral of electrostatics may well be regarded as the early prototype of this sort of

integrals in the mathematical and physical literature. Let us recall how this integral is invariant under deformations of its closed integration surface, as long as the deformations remain in the domain where the divergence of its integrand is zero. Integrals obeying this type of properties are called period-integrals or residue integrals. This operation of deforming the surface permits in principle **an exploration of the shape** of the object enclosed. We have here the beginning of a topological probing of an enclosed object of interest.

Conceivably inspired by the appearance of the topological concepts of linking and enclosing in Maxwell theory, George de Rham [20], a Swiss mathematician, has made the methods of Maxwell field theory into a sophisticated general tool for the exploration of the global structure of manifolds and the objects imbedded therein. This discipline, called de Rham cohomology, has become accepted in the realm of mathematics as a viable tool in modern topology.

Conversely, de Rham's investigation should be expected to have a bearing on physics, because its first input came from there. Yet, inadequacies of the mathematical vehicles used in contemporary physics have been a stumbling block that has prevented reaping the gains from improved mathematical insight. Since topologicsl features need to remain invariant under a wide class of transformation changes, standard vector and tensor methods as used in contemporary physics are inadequate. Recalling the apparent gravity independence of the flux, action and charge quanta, a metric-independent realm of physics has to be singled out for exploring the realm of quanta.

Traditional mathematical tooling in physics, restricted to R(3) or Lorentz group, is defined through metric invariance, not metric-absence. Topology is homeomorphic invariant, requiring Diffeo(4) in the differential realm. Gravity independence of quanta invites a separating out of metric-independent features in physical description. Ironically, such metric-independent elements of physics had surfaced in the Twenties. Yet at the time those studies could not be directly related to urgent physical question and the results were ignored as not relevant. Since details of this tedious story have been elaborately discussed in refs.[21] and [27] and references therein, the quanta counting integrals are here listed in Diffeo-4 form language. Once we see these integrals as in part already familiar structures in physics, we can look at them in an additional perspective of being quanta counters. In retrospect, quanta counting should be metric-independent:

The London-Aharonov-Bohm or (L)AB integral; c_1 is a one-dimensional cycle (loop):

$$\oint_{c1} A = nh/e$$

n = half integer in self-field, integer in external field.

The Ampère-Gauss- or AG integral; c_2 is a two-dimensional cycle:

$$\iint_{c2} \widetilde{G} = se \qquad \text{for boson counting s is even.}$$

The R Kiehn or RK integral, c_3 is a three-dimensional cycle

$$\oiiint_{c3} A \wedge \widetilde{G} = nsh \qquad \text{s and n defined as above.}$$

For a single charge RK reduces to the Bohr-Sommerfeld integral.

At least two of these three integrals have already had a prominent role in physics, albeit not always in a connotation of exact laws. After the metrology strides unleashed by Josephson ac and quantum Hall effects, the following extension of their validity domain can be given a good measure of justification.

The AB, AG and RK integrals can assume a status of exact physical law, iff their integration cycles everywhere reside in domains where the exterior derivatives of their integrands vanish. The latter condition indicates they have assumed a period status, meaning the quanta of action flux and charge are spacetime topological invariants. The latter conclusion takes issue with past speculations of a quantum of action depending on time. Note also, how all primary quantization is electromagnetic, not mechanical in nature.

It is good to keep in mind that the first intriguing applications of the AB integral dealt with macroscopic application for which conditions of integration in a field-free environment could only be approximately met. Anybody who knows how difficult it is to realize perfectly field-free environments would be reluctant to ascribe exactness to those AB configurations. While the AG integral is easier associated with exactness, due to the seemingly confined nature of electric charge, no such measure could be extended to the AB integral. The existence of remaining stray field was regarded as too much of an obstacle.

Only in superconductivity is it possible to create perfectly field-free regions by virtue of the Meissner effect. In fact, that is exactly how flux quantization was discovered in 1961 by Doll *et al* and Fairbanks *at al*. Therefore, some conditions are realizable in which the AB integral indeed assumes the same status of exactness as the AG integral. It is now necessary to check whether or not those conditions are met in the two experiments that have given us those high precision h and e values.

In the Josephson ac effect the time integration loop of one ac period resides inside the sandwich domain with its E field, obviously violating the residue integral condition. Similarly, for the quantum Hall effect, electrons orbit reside in a strong magnetic field, so how can there be an integration loop in a field-free region? Both cases, it seems, present major obstacles against elevating the AB integral to a stature of exactness. Yet, the nine decimal places precision of the end result for h and e continues to tell us something.

The only way of rescuing an apparent exact applicability of the AB integral in external fields is by endowing charge itself with a natural field-free interior. View this as a n.a.s. electrodynamics condition for charge to exist. If this proposition is true, as it seems supported by the metrology revolution, then a model, say of the electron itself, would have to be envisioned as having a nontrivial one- and two-connectedness. Elsewhere reported experimental calculations [21] with trefoil tube models simultaneously account for electron-positron pairing, a one half (differential) spin, magnetic moment and measures of anomaly.

If these perspectives can be further verified as germane to the basic laws of quantum physics, questions need to be confronted as to how this relates to traditional Copenhagen views. Model-based attempts at particle description seem at variance with the spirit of Heisenberg's uncertainty relation. The latter, as a presumed universal, always valid, restriction seems to cast a cloud over such particle specifics. The mostly successful applications of these single system integral tools to Josephson ac effect and quantum Hall effect raise a fundamental question: Can the Schroedinger equation and this residue method both be legitimate tools applicable to single systems?

Since the Schroedinger equation is a statistical tool whereas the residue integrals are not, they could not possibly apply to the same physical realm. It seems difficult to associate classical statistical features with Josephson ac effect and quantum Hall plateaus. Schroedinger applications compound the situation by having to separate integer and fractional quantum Hall effects. Retaining zero-point energy in this ordered realm moreover necessitates

measures of fudging to obtain observed results. It is not that surprising, when applying an inherently statistical tool as the Schroedinger equation to an inherently nonstatistical situation, one may end up calling on nonclassical miracles to make it work.

If the residue approach is, by far, the more natural procedure for single system quantum phenomena distinguished by a very pronounced order, then it follows that the only way of accommodating the statistical implications of standard quantum mechanics is by assuming Schroedinger's equation to be a tool solely describing real physical statistical ensembles.

By taking recourse to nonclassical options, the Copenhageners silently assumed their Schroedinger ensemble to be an abstract Gibbs ensemble of perhaps conceivable states pertaining to a single system. In a belief of having found the golden grail of single system quantum mechanics, the Copenhageners did not consider an option of real physical ensembles. It has been a saving grace for Copenhagen that a choice between a Gibbs abstract ensemble and a real physical ensemble is inconsequential for most quantum mechanical calculations of weakly interacting systems. Yet, an interpretation of zero-point energy is found to be critically dependent on this distinction.

For the abstract Gibbs ensemble, Copenhageners drew a fateful conclusion that no harmonic oscillator could be in a zero energy state. This assumption led to the notorious vacuum infinities, which during all those years have really subverted Planck's original spectral radiation law. Boersma[26] has shown a relevant classical angle of the Casimir effect dispensing with those zero-point infinities.

Before world war I and the birth of the Schroedinger equation, Planck (Theory of Heat Radiation p.141) [1] had introduced the concept of zero-point energy as an ensemble average, necessary to keep an ensemble in a state of phase disorder. Planck's concept of zero-point energy did not require harmonic oscillators to retain a lowest energy $h\nu/2$. Hence the need for vacuum infinities did not arise. By the same token, Planck's calculation invalidated the concept of a nonclassical statistics by counter example.

The Feynman Lectures [22] twice (in Vol. II, and appendix Vol. III) give a *perhaps unintended* support for the real physical ensemble with a calculation of the mechanical angular momentum average in a perfectly classical orientation averaging procedure. Feynman *et al* do not disclose why they did not call on this counter example to invalidate the legitimacy of Copenhagen's call for a nonclassical statistics. All of this shows the tremendous inertia of the heavy nonclassical investments made over such an extended time.

Man is sometimes forced to develop a measure of comfort with questionable virtues of his past.

If the residue integrals are really primary laws let us investigate whether the Schroedinger equation can be regarded as a statistical consequence thereof.

5   Schroedinger's Recipe as Derivation from first Principles

The quantum revolution of the mid-twenties had hailed the Schroedinger-Dirac procedures as a break-through, because its results corresponded tightly with experimental spectral observation, which at that time were prevailingly of an ensemble origin. All this gibes well with the present findings that the Schroedinger-Dirac process relates to ensembles.

At the time, there still was a virtual absence of real single system observations. This apparently created the false impression that physics had obtained with Schroedinger-Dirac a final exact formulation of quantum mechanics. The early pre-1925 recipes were accordingly relegated to a realm of approximations. There were as yet no dramatic quantum Hall experiences to question the latter decision as premature.

Hence, in the light of knowledge available at the time, Schroedinger's procedure for obtaining his wave equation had to be qualified as a recipe, because those approximate pre-1925 quantum manipulations had somehow brought to life a result that was then believed to be exact. Later, Lamb shift and the electron's magnetic moment anomaly began to detract a bit from that absolute exact status assigned to Schroedinger-Dirac. However, it was not yet enough to call for abdicating the status of exactness.

The era of the quantum Hall (QH) effect can now help us to clarify the difference between exact and a little bit exact. The metrology success of this QH effect holds powerful evidence to restore an exact status of the pre-1925 methods, provided they can be seen in the context of the residue integrals of section IV as having exactness for single systems. This should include ordered arrays of single systems such as occurring in the plateau states of the quantum Hall effect. Outside plateau states Schroedinger-Dirac prevails. The ensuing situation invites us to qualify the residue integrals as exact for single systems and the Schroediger-Dirac process as a near-exact tool for appropriately randomized ensembles.

Now having a new look at Schroedinger's procedure for obtaining his wave equation, we see a process that now seems to be moving from an exact single system input towards a near-exact ensemble description of Schroed-

inger-Dirac. **The recipe of the past now assumes a potential for being a derivation from an exact primary residue integral to a near-exact derived ensemble tool, thus injecting the implication that single systems and ensembles are now to be treated by different tools.** Let us hereto briefly summarize Schroedinger's steps that led him to this most frequently used equation of modern physics.

Schroedinger starts from the Hamilton-Jacobi (HJ) equation, say for stationary energy. He transcribes the action variable S with the substitution S=h ln Ψ, in which h is Planck's constant and Ψ is a dimensionless quantity called the wave function. Single valuedness of Ψ now assumes the role of a pre 1925 quantization. Schroedinger then uses this HJ equation as the functional of a variational integral over physical space say for one particle, or a configuration space for more than one particle. The Euler-Lagrangean derivative of the integrand then yields the second order eigenvalue differential equation which is, by definition, the Schroedinger equation with its extrema for the energy eigenvalue parameter. The discrete eigenvalues come about by imposing Ψ single valuedness as primary law aspect and square integrability of Ψ accounting for the ensemble aspect.

Let us now translate into more tangible physical language these abstract mathematical operations, which have produced this magic equation. Extremizing the HJ equation yields an optimization average for the totality of manifold solutions each corresponding to energy eigenvalues E.

The just cited operations are comparable to a maximum probability operation in the sense of statistical mechanics. A fixed set of integration constants defines a particular HJ solution. Yet in the variational process an infinity of solutions is involved, they can be defined as points of the solution manifold spanned by the integration constants. *The HJ solution manifold represents an ensemble of identical systems.* Hence Schroedinger's recipe becomes, in essence, a universal continuum version of earlier cited calculations by Planck, Feynman and Kompaneyets rolled into one. **All this places Schroedinger's recipe in the category of derivations, which is appropriate and well deserved for an equation that has rendered so many services.**

6   Conclusion: Textbook QM obstructs the Quantum Topology of EM theory

So, what really is the central error in Copenhagen's interpreting of the results of the Schroedinger-Dirac procedures? The multifaceted answer centers

around Copenhagen's creation of a totally undefined nonclassical statistics that was assumed to apply to single systems without needing a so-called **universe of discourse for that statistics**. Somebody among the Copenhageners, or a dominant group among them, single-handedly decided to wave the *universe of discourse* condition. Their unproven decision stuck for no good reason and has been accepted ever since by the physics community They exempted a Schroedinger-based statistics from having to prove that a single system could accommodate the statistics with the required *universe of discourse.* When, at the time, they could not put a finger on a legitimate candidate they opted for a nonclassical statistics, rather than seeking an object of description that could accommodate a proper universe of discourse. So, the Copenhageners sort of passed the bug to nature, which Einstein in retrospect so quaintly characterized by saying that they had the good Lord playing dice.

Let us make up for the neglect of a universe of discourse by saying that one single system can be thought of assuming an ensemble of conceivable states. In this context Copenhagen's neglect can be brought back into semi-legal territory by the proposition that **Copenhageners unfortunately made a wrong choice of this so-called Gibbs ensemble rather than considering the possibility of an ensemble of real systems.**

In the rush of new revelations, perhaps restrained by Heisenberg's unduly extrapolated universal uncertainty, Copenhageners held back from inquiring further into an explicit statistics pertaining to a Gibbs ensemble. During the Thirties the subsumed nonclassical statistics strangely dominated the single system and ensemble supporters both. The Copenhagen option really coexisted with a real ensemble option as advocated by Popper [23], Kemble [24] and many others, perhaps favored by Slater and quite a few others. As we now know from a letter to Popper in 1935, Einstein favored such an ensemble view. At the time, Einstein was involved with the EPR paper attempting to carry Bohr's single system position ad absurdum.

The two options of interpretation remained perfectly equivalent for most practical data in a vast number of applications. Yet, Copenhagen's Gibbs choice precipitated interpretation predicaments for quantum uncertainty, duality, all the way to readings of Bell's theorem [28, 29].

After the conceptual turmoil of the mid-Twenties, Heisenberg uncertainty had been welcomed as some sort of relief from earlier science arrogance. One now had a seemingly scientific reason for soothing the conscience about not answering certain classical questions. It absolved physics from probing further into a territory that was uprooting the peace of mind. Physics

had already received its unexpected share of fortune's favor with the event of the Schroedinger equation. It would be bad manners and pushing one's luck too far by asking for more. Physics would pay a price for this silent compromise. It entered an era of over-reliance on recipes. Some of the ensuing ontic outrages were frequently defended with the arrogant fervor typical of a theocratic debate.

Yet, prior to that era, unhampered by Heisenberg uncertainty, Planck already had transgressed into a territory where modern physicists are dissuaded from going. He introduced his ensemble-based concept of zero-point energy in 1912. Later in the Fifties and Sixties, Feynman and Kompaneyets (perhaps others earlier) also made excursions in this now forbidden territory, without really admitting that by doing so, **they had inadvertently disproven Copenhagen's myth of a nonclassical statistics.**

The tragic result of the nonclassical myth was a failing to recognize the two-tier nature of the quantum world. There was a totally unjustified illusion that single systems and ensembles belonged under the same mathematical and physical heading. Adding insult to injury, Copenhageners then proceeded with a fateful contingency error of relegating whatever at the time was available in single system tools to the realm of approximations. **Let the precision of quantum Hall data serve as reminder that those old "approximate" tools held perspectives that cannot be washed away by conceptions of fractional charge or compound fermion.**

Now looking back in retrospect to the mid-Twenties, the present population of physicists has matured in an atmosphere of nonclassical mystique over a time span of three generations. It was an unavoidable and essential part of their reality. Any initiative of seriously seeking to replace their foregone convictions unerringly met with fierce opposition. The contemporary younger generation of peer reviewers take attempts at undoing the nonclassical realm as an attack on sacrosanct values. They either censor such contributions or have them intercepted at editorial levels.

Mara Beller [25} has made an incisive study of that era in her recent book: **Quantum Dialogue.** She approaches the subject matter from the realms of philosophy, psychology and history. It brings out how physics' disconnect with philosophy is now catching up with its procedures. It shows how its insular attitude is now visible to those without detailed familiarity with the subject. Such disconnect is no longer a token of justified self-confidence. Mara's plea may help in opening up this critical Copenhagen era to new scrutiny. In case the present document gets out in the open in physics media, let us summarize for emphasis major changes are involved:

Copenhagen's Change of Venue:

I: *The Schroedinger and Dirac equations need to step back from the pedestal of single system tools. They, instead, describe behavior of phase- and orientation randomized ensembles.*

II: *Zero-point energy and Heisenberg uncertainty are not universal, but pertain to ensembles in a state of positional and phase randomness. As a single system feature zero-point energy leads to vacuum infinities. The latter have no role in the Casimir effect. Compare hereto S L Boersma's maritime variant of this effect* **[26].**

III: *The cyclic integrals of Aharonov-Bohm (AB), Ampère-Gauss (AG) and Kiehn (RK) are single system tools that assume exactness, if and only if so-called period conditions can be met.*

IV: *If a single system assessment of Schroedinger-Dirac is in error, so is the concept of wave-particle duality and complementarity. There is at best a wave- many particle duality.*

V: *Classical statistical calculations duplicating Schroedinger results and derivation aspects acquired by Schroedinger's recipe invalidate continued use of undefined notions of nonclassical statistics.*

General Aspects:

The three period integrals are spacetime topological invariants in full compliance with the principle of general covariance of the general theory of relativity. Their metric-independent nature is a sine qua non for being exact quanta counters.

Since topological specification is a prerequisite to metric exploration, mathematical descriptions of physics ought to start from pre-metric topological via metric general-invariant, to invariances under conformal, Lorentz- and rotation subgroups.[*]

Yet, false pedagogy suggests an opposite course. For those who care, dimensional analysis based on pure- quanta and -metric units helps in illustrating structural perspectives of physical descriptions that separate metric-independent aspects of physics [27].

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[*] The group sequence is: Homeomorphism>Diffeomorphism> Conformal>Lorentz> Rotations. The Galilei group has except the unit operation no elements in common with the Conformal or Lorents groups. **Hence, contrary to textbook assertions, the Diffeo(4) invariant Maxwell equations are invariant under Galilei- as well as under Lorentz-groups.(a result due to Kottler and Poincaré in the early Twenties)**

While many of these points are discussed in ref. [21], so far reviewers have remained strangely silent about the compelling nature of its Copenhagen related message. This reassessment in different setting is meant to invite more of an open forum about Copenhagen revision. Such revision clearly favors particles over waves, as convincingly demonstrated in the striking experiments by Tonomura *et al* [28]. All of which homes in on an unavoidable reality that without a Copenhagen revision, physics will fail to recognize the independent physical significance of the period integral structure of electromagnetism as an example of primary topology-based quantization.

One reviewer has pointed out the work of Canals-Frau and Kracklauer [29. 30] in dismantling the saga surrounding the Bell theorem and the Aspect experiments. It is all evidence converging on Beller's request for Copenhagen views to come down from their high horse of nonclassical hype. A descending from hype to reality is bound to be an anticlimactic experience. Another reviewer has expressed concern whether these changes would affect the observed Casimir effect of attracting parallel plates. S L Boersma's [26] perfectly classical variation of this effect should dispel lingering doubts that vacuum infinities have any role in this effect, they simply cancel. The Casimir effect is a force differential at the low frequency end of the spectrum, without any infinites whatsoever.

With Robert Kiehn [18] uncovering a cohomological superstructure of electromagnetism, Mara Beller [25] delineating the inadequacies of traditional Copenhagen arguments, Canals-Frau [29], Kracklauer [30] and Collins [31] reducing the Saga of Bell's theorem to normal statistics, a two-tier Quantum Reprogramming [21] of ensembles versus single systems seems unavoidable!

John Slater's [32] early opinions to this effect brought him a fall-out with Bohr. Yet, Bohr's great legacy to physics would not be served if the theocratic angle of Copenhagen attitudes keeps lingering on. Retaining the current Copenhagen status quo is becoming logically too disruptive.

Appendix:
Hydrogen Spectrum from Electro-Flux Quantization

Among the early quantum conditions Bohr's angular momentum condition seemed to be of an explicit mechanical origin. In order to prove that the hydrogen spectrum is derivable from a purely electromagnetic condition, let us apply here the Aharonov-Bohm integral to the Rutherford model of Hy-

drogen. The electron orbits in an **external** electric field **E** of the proton nucleus, thus calling for quanta h/e not h/2e. The Aharonov-Bohm quantum condition in an external field becomes

$$\int_0^T \frac{e}{4\pi\varepsilon_0 r}dt = n\frac{h}{e} \tag{1}$$

in which the integrand is the potential at the orbital position of the electron, integrated over one period of circulation T. Using the orbital equation the following change of variables is indicated

$$\int_0^{2\pi} \frac{e}{4\pi\varepsilon_0 r\dot\varphi}d\varphi = n\frac{h}{e} \tag{2}$$

The angular momentum theorem yields an expression in which L is a constant of the motion

$$mr^2\dot\varphi = L \quad \text{or} \quad \dot\varphi = \frac{L}{mr^2} \tag{3}$$

Substitution in Eq.2 yields

$$\int_0^{2\pi} \frac{em}{4\pi\varepsilon_0 L}rd\varphi = \frac{em}{4\pi\varepsilon_0 L}\int_0^{2\pi} rd\varphi = n\frac{h}{e} \tag{4}$$

For A > B, the orbital equation is given by an ellipse

$$r = \frac{1}{A + B\cos\varphi} \tag{5}$$

The integral Eq.4 becomes after complex integration:

$$\int_0^{2\pi} r d\varphi = \int_0^{2\pi} \frac{d\varphi}{A + B\cos\varphi} = \frac{2\pi}{\sqrt{A^2 + B^2}} \tag{6}$$

Substitution of Eq.6 in Eq.4 gives

$$\frac{em}{2\varepsilon_0 L\sqrt{A^2+B^2}} = n\frac{h}{e} \tag{7}$$

Standard text book literature gives A and B expressed in terms of the integration constants of angular momentum L and energy E, one finds

$$A^2 - B^2 = -\frac{2mE}{L^2} \tag{8}$$

Substituting Eq.8 into Eq.7 one finds that L drops out of the end result

$$\frac{em}{2\varepsilon_0\sqrt{-mE}} = n\frac{h}{e} \tag{9}$$

Solving for E and using $\varepsilon_0\mu_0 = c^{-2}$ gives the familiar Bohr result

$$E = -\frac{mc^2}{2}\frac{\alpha^2}{n^2} \tag{10}$$

In Eq.10 α is the fine structure constant, which in Eq.11 is represented as a ratio of free-space impedance and quantum Hall impedance

$$\alpha = \frac{e^2}{2h}\sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{11}$$

Since L drops out of Eq.9, the n of Eq,1 appears right away as a principal quantum number in Eq.10. It seems that relativity corrections don't give "uninvited" spin terms as in the Sommerfeld and Dirac procedures. From

this one might infer that the Pauli treatment of spin is really more fundamnetal than the Dirac treatment; for other arguments to this effect see [21].

A brief note about subtle differences seems in order. While the relativity approaches of Bohr-Sommerfeld and Dirac yield electron spin and magnetic moment, seemingly without injecting those features beforehand. The relativity variant of the here given Aharonov-Bohm based approach does not yield an automatic fine structure contribution. Hence spin and magnetic moment are no longer magically created but require instead separate and independent propositions.

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