Dirac monopoles and gravitation

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ABSTRACT. According to Dirac, the definition of a magnetic charge in Electrodynamics is related to a semi-infinite vector-potential singular string. Contrary to Wu and Yang interpretation, recent studies have shown that a Dirac string is a physical observable if the magnetic charge is nonzero in QED. In this work we propose that Dirac strings naturally lead to space-time distortions, which can be interpreted as the gravitational field of a point-mass located in the monopole. Thus a Dirac string is observable: It can be detected as a source of gravitation. The description of QED particles as structured by spinless charges and monopoles leads to the interpretation that any QED particle is a source of gravitation as well.

1 Introduction

The magnetic charge or monopole proposed by Dirac in 1931 [1] is based on Maxwell Electromagnetic theory, so that the monopole magnetic field $\mathbf{B}$ is described by a vector potential $\mathbf{A}$, i.e., $\mathbf{B} = \nabla \times \mathbf{A}$. In order to describe a source of magnetic field, Dirac defined this vector potential as singular at some regions of space-time. The shape of the singularity related to the vector-potential is like a semi-infinite string with its end at the monopole position. The proper definition of the equations of motion for charges and monopoles requires that charges never touch a monopole's string, [2].

In 1974 Wu and Yang [3] proposed that singular strings (Dirac strings) are non-observable physical entities. Instead of deforming the string in order to not cross any charge's path, they introduced the notion of sub-spaces. According to their model, a particular vector-potential is defined for each sub-space (a sub-space is a place where particles can

move freely without crossing with Dirac strings as it is smaller than the
entire space and does not cover the region where a Dirac string is posi-
tioned). The aim is to be able to freely distribute a particle at any point
of space-time around a monopole: if a particle is present at some point,
then the Dirac string is placed in a region not interfering with it. The
multiple sub-spaces overlap (two at least are required to map the entire
space-time). Since a special vector-potential is associated to each sub-
space, a test-particle moving from a region to another (from a sub-space
to another) has an associated quantum phase $\int \mathbf{A} \cdot \mathbf{dl}$ for each sub-space.
For each overlapping region a gauge transformation connects the phases
associated to each sub-spaces. If Dirac quantization condition is valid,
then any closed path performed by a test-particle has no physical impact
due to Dirac strings.

However, twenty years later, He, Qiu and Tze [4] presented a different
result: if a monopole charge is non-zero, then a physical interpretation to
Dirac strings exists. According to them, Stokes theorem was misused in
previous works, treating Dirac strings as gauge artifacts (as in the case
with Wu and Yang's view). More recently, Singleton [5] introduced (be-
yond the ordinary four-vector potential A_μ) a second pseudo-four-vector
while discussing eletrodynamics in the presence of magnetic charges. His
approach avoids the use of singular nonlocal variables, i.e., Dirac strings.
The outcome of the theory is a system in which two types of photon
co-exist (respectively associated to each four-potential). As in fact a sin-
gle photon is observed in Nature and the other one was never observed
(which is massive according to Singleton), the theory lacks confirmation.
According to the results of He *et al*, the important point to emphasize
is that a non-zeroed charge monopole implies in some physical conse-
quences. In Singleton model it is the new massive photon according to
our interpretation.

Therefore according to these works [4, 5] some new physics is involved
when the monopole charge is different from zero. The present work ad-
dresses an interpretation in which Dirac strings are related to gravita-
tion. The origin of this connection is based on the physical notion of
space-time deformation caused by singular strings with nonzero volume
(nonzero cross-section) since no particle or field are allowed to be defined
at the string region. The relation between monopoles or magnetic-flux-
based particles and gravitation also appears in other works: Davidson
and Karasik [6] recently presented the idea of cosmic solenoids where the
inclusion of gravity implies in the inexistence of infinitely small cross-

section magnetic flux tubes or strings. Also Israelit [7], with a modified Weyl-Dirac theory, presented a complete basis for deriving gravitation and electromagnetism from geometry at once with the inclusion of monopoles. Monopoles seem to be essential in this unifying picture.

2   The definition of a Dirac monopole

The definition of a particle in Electrodynamics involves a number of hypothesis concerning electromagnetic fields. In the case of electric charges, there is a minimum radius assigned to the elementary source. Let us consider two electrically charged particles in the classical approach: Using "1" and "2" to label the charges, the scalar potentials are V_1 and V_2. The electric fields are $\mathbf{E}_1 = -\nabla V_1$ and $\mathbf{E}_2 = -\nabla V_2$ respectively. The energy of the system is

$$\frac{\varepsilon_0}{2} \int \left(\mathbf{E}_1 \cdot \mathbf{E}_1 + \mathbf{E}_2 \cdot \mathbf{E}_2 + 2\mathbf{E}_1 \cdot \mathbf{E}_2 \right) dv, \tag{1}$$

where dv is the differential volume of the observable universe. The first two terms are easily calculated as $(e_1^2/a_1 + e_2^2/a_2)/(8\pi\varepsilon_0)$, where $e_{1,2}$ is the charge of the particle 1, 2 and $a_{1,2}$ its radius. Applying Green theorem to the third term and defining a closed surface with an external shell S^e at infinity plus two inner shells $S_{1,2}^i$ around each particle (just over their radii) one gets:

$$\varepsilon_0 \int \nabla V_1 \cdot \nabla V_2 dv \tag{2}$$
$$= \frac{\varepsilon_0}{2} \left(\oint V_1 \nabla V_2 \cdot \mathbf{dS}^e - \oint V_1 \nabla V_2 \cdot \mathbf{dS}_2^i - \int V_1 \nabla^2 V_2 dv \right)$$
$$+ \frac{\varepsilon_0}{2} \left(\oint V_2 \nabla V_1 \cdot \mathbf{dS}^e - \oint V_2 \nabla V_1 \cdot \mathbf{dS}_1^i - \int V_2 \nabla^2 V_1 dv \right).$$

Let us consider the first parentheses on the right hand side of equation (2): the first term vanishes because the external surface is placed at infinity (and consequently the fields tend to zero). The third term also vanishes as there are no source of field inside the region enclosed by the surfaces. The second term is nonzero, and it is possible to approximate the scalar potential of particle 1, V_1, as its value at the second particle position is almost constant given $a_2 \to 0$. Therefore $a_2/(|\mathbf{r_1} - \mathbf{r_2}|) \to \mathbf{0}$.

The same observations are valid for the second parenthesis so that

$$\frac{\varepsilon_0}{2}\left(V_1\left(\mathbf{r}_2\right)\oint E_1\cdot dS_2^i\right)+\frac{\varepsilon_0}{2}\left(V_2\left(\mathbf{r}_1\right)\oint E_2\cdot dS_1^i\right)\qquad(3)$$

$$=\quad\frac{V_1\left(\mathbf{r}_2\right)e_2+V_2\left(\mathbf{r}_1\right)e_1}{2}.$$

The results above are only valid if no fields are prescribed inside the region delimited by the $S_{1,2}^i$ surfaces. In the complete theory of the electron this hypothesis is relevant [8] even if field quantization is assumed both in the self-energy derivation as in the interaction energy among particles.

Let us now consider the monopole proposed by Dirac. The simplest definition of vector potential associated to it is

$$\mathbf{A}=\hat{\phi}g\frac{1+\cos\theta}{r\sin\theta step(\theta-\delta)},\qquad(4)$$

where spherical coordinates were used. The function $step(x-x_0)$ is equal to 1 if $x>x_0$, zero if $x<x_0$ and $1/2$ for $x=x_0$. The parameter δ is an arbitrary angle. The resulting magnetic field is

$$\mathbf{B}=2\pi g(1+\cos\delta)\mathbf{r}/r^3,(\theta>\delta,\forall\phi),\qquad(5)$$

that is, it is not defined in the region of the string. When $\delta=0$ the vector potential is singular on a semi - infinite line, while the magnetic field remains well defined everywhere.

The interaction energy between two monopoles ("i" and "j") is proportional to $\int\mathbf{B}_i\cdot\mathbf{B}_j dv$. The same result for the general case, i.e., when $\delta\neq0$, has no meaning since the monopole magnetic field is not defined in the region of the string according to equation (5). A similar result is also obtained from the momentum integral $\int\mathbf{E}_i\times\mathbf{B}_j dv$, if now one of the particles is an electric charge and the other a monopole. Again, if $\delta\neq0$, no magnetic field is defined in the string region implying in no momentum density definition for this region as well.

In order to have a finite interaction energy among particles no fields should be defined inside the string region, i.e., the Dirac string of the general case $\delta\neq0$ (as in the previous case of the electrically charged particles, equations (1) to (3) where we assume no field is defined inside the particle's radii). On the other hand, a magnetic charge (the magnetic

flux through a closed surface surrounding the monopole) has to be finite. Taking into consideration equations (4) and (5) with $\delta \neq 0$, it is clear that it is not possible to take the integration $\int \mathbf{B} \cdot \mathbf{ds}$ over the entire closed surface. The calculation must exclude the region where the Dirac string crosses the surface (as the string cannot be removed by any gauge transformation, [4]). This assumption implies that, for a given time and distance R from the monopole, the flux should be measure over an area smaller than $4\pi R^2$, i.e., $\sqrt{area/4\pi} < R$, any R. The actual available space was then reduced due to the presence of the string. In other words, we are saying that the string induces a space-time distortion.

It is interesting to find a physical interpretation for the existence of forbidden regions in space-time that serves here to solve both the problem of energy interaction as the monopole charge definition.

The singular string has physical implication if $\delta \neq 0$, i.e., if the Dirac string has a non-null volume: let bus consider an elementary electric charge described by a Schröedinger-type field ψ. If an external vector potential $\mathbf{A}$ is such that it is infinite ($\mathbf{A} \to \infty$) in some region of space, then $\psi = 0$ in this region. Such observation follows as a physical solution of Schröedinger equation, and not as an mere assumption. Now if the external vector field potential is generated by a monopole, the charged particle always interacts with the local finite potential. As a result, the particle electromagnetic field cannot exist inside the Dirac string region as well: as the potential acting upon the electric charge is finite since its wave function ψ is zero in the singular region. If now the momentum is calculated using the momentum density relation, $\int \mathbf{E} \times \mathbf{B} dv$, it is clear that no electric field $\mathbf{E}$ by the charge is allowed inside the singular string region, otherwise the result diverges. The same argument is valid for two monopole particles described by Schröedinger wave functions. In conclusion, if a particle wave function is zero in the singular region, its electromagnetic fields cannot be defined in this region as a result of solving Schroedinger equations and the equivalence between this and the calculation via the electromagnetic fields.

Space-time reduction is a physical consequence of Schröedinger equation solutions in a flat three-dimensional space containing Dirac monopoles since neither particles nor fields are allowed in the string region. Electrodynamics is well defined in flat three-dimensional spaces. In the presence of Dirac monopoles there is a physically induced reduction of space-time where particles and fields are found, that is, particles and fields can exist only outside Dirac strings (in the general case with

non-null volume).

3   Dirac string and space-time distortion

The definition of a volumetric Dirac string is to some extent a matter of choice, i.e., one is free to choose the value of δ in equation (4). For some fixed $\delta \neq 0$, the magnetic charge is $g(1 + \cos \delta)/2$, which is a constant. The magnetic field of the monopole follows the law $1/r^2$. In the next lines of the present section two convergent ideas are presented in order to show the physical effect of volumetric Dirac strings. The first one uses the simple approach of a Dirac string represented by equation (4) and its implication for the behavior of a test-particle with no magnetic or electric charge close to the monopole. The second idea is the identification of forbidden regions as due to a point-like mass as seen from a flat three-dimensional world in Einstein theory, which is a volumetric string-like singularity for particles and fields in the same way as volumetric Dirac strings.

The time-rate of a clock was one of the most immutable aspects of nature before Einstein. It is the simplest and most direct aspect to be observed when one searches for space-time distortions. In order to estimate the physical effects due to Dirac strings, we place a clock close to a monopole and study its behavior. The most suitable clock has a time-rate depending solely upon the fundamental constants of nature. Such a clock was first designed by Marzke and Wheeler [9], the so called *geometrodynamic clock*. It is composed of a pair of parallel mirrors between which a free photon bounces back and forth determining the clock period. Any local time-rate variation can only be due to a local space-time distortion.

Although the image of a Dirac string is limited to a cone region as seen from a flat three dimensional space, it affects the entire space surrounding it. Geometric aspects related to space-time reveals the distortion: as we have seen, the definition of a magnetic charge from a volumetric Dirac string requires that, for a given distance r from the monopole $\sqrt{area/4\pi} < r$ for any r. To get a geometrical image of the space-time distortion caused by a Dirac string from the point of view of a flat three-dimensional space, we imagine the string divided in infinite sub-parts equally spread in space and creating a infinite set of tiny strings emanating from the monopole. As a result, closed spherical surfaces enclosing the monopole have infinite "holes" equally spread as elementary forbidden regions. The distance covered by any particle on

this closed surface as seen from the flat three-dimensional space is then reduced by the amount of holes times their diameters. Such geometrical view helps to understand the working of a geometrodynamic clock placed near a monopole.

Let us consider a geometrodynamic clock near a monopole so that the string is placed between the parallel mirrors (the clock is at distance r from the monopole center). A photon moves in a trajectory perpendicular to the line formed by the string. The distance between mirrors is L. Now let us consider the monopole described by equation (4) with $\delta \neq 0$. The path covered by the photon between the walls is then reduced by $2r\delta$. If the clock natural frequency (without the Dirac string) is $\omega^0 = c/L$ (c the velocity of light) the same frequency becomes

$$\omega = \frac{c}{L(1 - 2\delta r/L)}. \tag{6}$$

Taking $\delta \to 0$ the frequency may be approximated by

$$\omega \simeq \omega^0(1 + 2\delta r/L). \tag{7}$$

In this calculation the mirror walls completely cover the string. The density of elementary strings (with the geometrical view presented above) may be obtained by multiplying $2r\delta$ by $2\pi(1 - \cos\delta)/4\pi$. For the sake of simplicity we omit this factor.

If the clock moves with velocity $\mathbf{v}$, the relativistic correction to the frequency relative to the laboratory (where the monopole is at rest) is

$$\omega = \omega^0\sqrt{1 - v^2/c^2}, \tag{8}$$

with $v = |\mathbf{v}|$. For $v << c$ the last expression is approximated by

$$\omega \simeq \omega^0(1 - v^2/2c^2). \tag{9}$$

If the time-interval of a resting clock is dt (in the absence of monopoles), the same quantity becomes

$$dt\left\{1 + 2\delta r/L - v^2/2c^2\right\} \tag{10}$$

in the presence of the monopole. The total time interval along a trajectory performed by the clock is then

$$\int \left(2\delta r/L - v^2/2c^2\right) dt, \tag{11}$$

which multiplied by the constant $-mc^2$ (where m is the inertial mass of a test particle at rest relative to the clock) results

$$\int \left(-c^2(2\delta/L)mr + \frac{mv^2}{2} \right) dt. \qquad (12)$$

Using the principle of minimum action, the test particle will behave as if under the action of a potential proportional to the distance from the monopole. Therefore, the non-charged test particle performs a parabolic trajectory given an initial impulse as induced by an attractive potential towards the monopole. Also, this result is equivalent to the existence of a gravitational field generated by a point-like particle at the monopole position provided that variations in r are small compared to changes in the gravitational field.

We are now going to study the second approach, i.e., we are looking for string-like forbidden structures in space-time as due to a point-like mass. Let us consider a spherical distribution of mass m at the center of the coordinate system with a given radial distribution function. Then the metric interval for $r > 2m$ is:

$$ds^2 = (1 - \frac{2m}{r})dt^2 - (1 - \frac{2m}{r})^{-1}dr^2 - r^2 d\theta^2 - r^2 \sin^2 \theta d\phi^2 \qquad (13)$$

according to Schwarzschild.

As seen from the three-dimensional flat world the metric can be estimated by the measure of geometric constants like the ratio between the spherical surface area and its corresponding radius. For $r = R$, the physical area Δ is $4\pi R^2$, while the physical radius Λ will be

$$\begin{aligned}
\Lambda &= \int_{R_0}^{R} r/\sqrt{r^2 - 2mr}\, dr \\
&= \left[\sqrt{(r^2 - 2mr)} + m\ln\left(r - m + \sqrt{(r^2 - 2mr)} \right) \right]_{R_0}^{R},
\end{aligned} \qquad (14)$$

where R_0 is an internal radius ($R_0 < R$) in the region of the matter distribution. The expression above is assumed to be vanishingly small for the parameter R_0 (of the order of $2m$) if compared to the same expression for R, so that the physical radius becomes

$$\Lambda = \sqrt{(R^2 - 2mR)} + m\ln\left(R - m + \sqrt{(R^2 - 2mR)} \right), \qquad (15)$$

and for $R >> 2m$,

$$\Lambda = R + m \ln (2R - 2m), \qquad (16)$$

i.e., the actual physical distance $\Lambda > R$.

Therefore, for a given physical distance Λ, the available area to cover the shell of radius R is $4\pi R^2$, that is, it is not possible to fill the entire surface of radius Λ in the flat three-dimensional space. For particles and fields surrounding the mass, there is no discontinuity in space-time, but as seen from flat R^3, a forbidden region appears (with the shape similar to a cone in R^3). On the other hand it is very well known that an average curvature can be assigned to a spherical region of radius r in R^3 with area $4\pi r^2$ by means of a defect. The connection of this idea to the theory of gravitation is done by the conceptual significance of the stress-energy tensor component G_4^4. The average curvature $R_{12}^{12} + R_{23}^{23} + R_{13}^{13}$ of the three-space is perpendicular to time as dictates a well known interpretation of gravitation, [10]. Such result is equivalent to the previous one (conic string measured by geometridynamic clock) for $m \to 0$ and small variations of R (where the mass is the source of gravitation).

In the analysis above using the geometrodynamic clock, the space-time defect has the shape of a cone (equation (4), $\delta \neq 0$) which means that, for a given R, the avaliable physical area is not $4\pi R^2$ but $2\pi R^2(1 + \cos \delta)$ instead, i.e., it is still proportional to R^2 due to to the conic shape of the string. Assuming Einstein theory as seen from the flat three-dimensional world the avaliable physical area will be $4\pi(\Lambda^2 - R^2)$, which is similar to the conic-like string if Λ is proportional to R. Taking equation (15) with small variations of R around km (where k is a positive constant, $k > 2$) we have

$$\Lambda = m\sqrt{k^2 - 2k} + \frac{k-1}{\sqrt{k^2 - 2k}}(R - mk),$$

and for $m \to 0$, $\Lambda \approx ((k-1)/\sqrt{k^2 - 2k})R$, i.e., Λ is proportional to R, showing that under this approximation, the previous results obtained with volumetric Dirac strings agree with the behavior predicted starting from Einstein gravitational theory for point-like masses.

4   Electrons and Photons

The electric charge and spin distribution associated to a particle in Dirac electron theory can be calculated. The approach also allows to find the

energy associated to the magnetic field. By taking averages over all possible spin orientations we find

$$\frac{1}{2\pi} \lim_{a \to 0} \frac{e^2\hbar^2}{3m^2c^2a^3}, \tag{17}$$

in the case of a single electron. This corresponds to the field energy of a magnetic dipole density concentrated in a sphere of radius a.

If the single electron self-energy e^2/a is to be finite and equal to mc^2, we obtain a radius $a = e^2/mc^2$ for a net charge e. The spin is $\hbar/2$ and the magnetic moment $e\hbar/2mc$. The elementary particle (or single electron) in Dirac theory is an object containing such characteristics. The physical image is that of a small sphere of radius a with electric charge, magnetic moment and angular momentum. The minimum physically significant distance from such a particle is defined to be e^2/mc^2, which means that any measure of field flux is made by the use of a surface that completely contains the sphere of radius a. The net electric flux is equal to e and the magnetic equivalent is identically zero since the magnetic poles or electric currents (composing a dipole-like magnetic field) exist only within the particle internal region.

We expose here the idea of representing the electron as composed of a spinless electric charge plus two spinless Dirac monopoles of opposite charges inside the structure (defined by radius a). This representation is possible if the hyper-fine energy structure of the s-states of Hydrogen is contemplated. The origin of this hyper-fine energy structure is due to the association of magnetic moments and electric currents in electrons and not to static magnetic poles in its internal structure, [11]. The model (Dirac monopoles inside the electron internal structure as source of magnetic moments) is still valid since a Dirac monopole may be represented in two ways: a semi-infinite line composed of magnetic dipoles or a semi-infinite solenoid carrying an electric current ([11], Chapter 6). The representation of a single electron with electric charge and magnetic moment is then valid since Dirac monopoles can be viewed as originating from electric currents. In order to give a proper mathematical definition for the proposed structure, it is necessary to write down the general equations for charges and monopoles as single entities as well as the conditions involved in the variational principle from which the field and particle equations of motion are derived.

Let us consider the general case of electric charges and Dirac poles in a system. The $g > 0$ monopole is defined as a source of magnetic field

while for $g < 0$ the monopole is a sink for the field. This assumption allows to construct a magnetic dipole as a combination of a positive and a negative monopole. According to this definition, the generalized Maxwell equations including electric and magnetic currents, are

$$\partial_\mu F^{\mu\nu} = -j_e^\nu,$$
$$\partial_\mu F_*^{\mu\nu} = j_g^\nu, \tag{18}$$

where $F^{\mu\nu}$ is the electromagnetic field tensor connected to the four-vector j_e^ν formed by the electric charge and current densities, $F_*^{\mu\nu}$ is the dual of $F^{\mu\nu}$ (which is connected to the four-vector j_g^ν formed by the magnetic charge and current densities)

$$F_*^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\sigma\delta} F_{\sigma\delta}, \tag{19}$$

and $\varepsilon^{\mu\nu\sigma\delta}$ is the four-dimensional Levi-Civita tensor ($\varepsilon_{0123} = -\varepsilon^{0123} = 1$, completely anti-symmetric).

Each charge and monopole can be described by a corresponding Lorentz equation

$$m_e \frac{d^2 z^\mu}{ds^2} = e \frac{dz_\nu}{ds} F^{\mu\nu}(z),$$
$$m_g \frac{d^2 x^\mu}{ds'^2} = -g \frac{dx_\nu}{ds'} F_*^{\mu\nu}(x), \tag{20}$$

where m_e and m_g are the electric and magnetic particle inertial masses, $z_\mu(s)$ is the charge world-line as a function of the proper time s and $x_\mu(s')$ is a corresponding four-coordinate system defining the monopole position.

The local action principle depends on the fundamental interaction between an electric charge and a magnetic monopole. Considering a system formed by a single charge and monopole, the fields appearing in the equations of motion represent radiative reaction terms and the retarded fields of their mutual interaction.

It is possible to define two four-vectors very similar to those defined by Singleton, [5], W^μ and V^μ such that $F^{\mu\nu} = W^{\mu\nu} - V_*^{\mu\nu}$; $F_*^{\mu\nu} = W_*^{\mu\nu} + V^{\mu\nu}$, with $W^{\mu\nu} = \partial^\mu W^\nu - \partial^\nu W^\mu$, $V^{\mu\nu} = \partial^\mu V^\nu - \partial^\nu V^\mu$. By the use of Lorentz condition, $\partial_\mu W^\mu = 0$ and $\partial_\mu V^\mu = 0$, we get

$$\partial^2 W^\mu = -j_e^\mu; \partial^2 V^\mu = j_g^\mu. \tag{21}$$

In order to obtain the term $-eV_*^{\mu\nu}dz_\nu/ds$ on the right side of the charge equation, the action integral should have a term like

$$-e \int \frac{dz_\mu}{d\tau} V_*^\mu(z) d\tau, \qquad (22)$$

which is only possible if $\partial^\mu V_*^\nu - \partial^\nu V_*^\mu = V_*^{\mu\nu}$. Lets consider the non-local potential as a consequence of $j_g^\mu \neq 0$:

$$V_*^\mu(z) = \int_{-\infty}^0 V_*^{\alpha\beta}(\xi) \frac{\partial \xi_\beta}{\partial z^\mu} \frac{\partial \xi_\alpha}{\partial \tau} d\tau, \qquad (23)$$

where the four-vector $\xi^\mu(z,\tau)$ is defined by

$$\begin{aligned} \xi^\mu(z,0) &= z^\mu, \qquad (24) \\ \lim_{\tau \to -\infty} \xi^\mu(z,\tau) &= space - like - infinity, \end{aligned}$$

where "space-like infinity" means relative to the monopole position, the charge position in the infinite past. The path from the distant point to z is given by ξ as τ varies from $-\infty$ to 0 and then

$$\partial^\mu V_*^\nu - \partial^\nu V_*^\mu = V_*^{\mu\nu} + \int_{-\infty}^0 \frac{\partial \xi_\beta}{\partial z^\nu} \frac{\partial \xi_\gamma}{\partial z^\mu} \frac{\partial \xi_\alpha}{\partial \tau} (\partial^\gamma V_*^{\alpha\beta} + \partial^\alpha V_*^{\beta\gamma} + \partial^\beta V_*^{\gamma\alpha}) d\tau, \qquad (25)$$

with $(\partial^\gamma V_*^{\alpha\beta} + \partial^\alpha V_*^{\beta\gamma} + \partial^\beta V_*^{\gamma\alpha}) = \varepsilon^{\alpha\beta\gamma\sigma} [j_g]_\sigma(\xi)$. In order to keep $\partial^\mu V_*^\nu - \partial^\nu V_*^\mu = V_*^{\mu\nu}$ in the last equation, the action will be non-local provided the charge world-line never cross a monopole world-line at a given space-time point.

Considering now the general case of charge and monopole line crossing at some space-time point. The contribution for $F^{\mu\nu}$ as given by the second term on the right side of equation (26) will be

$$\lim_{\tau \to 0} - \left[\frac{\partial \xi_\alpha}{\partial \tau} \right]_{\tau=0} \varepsilon^{\alpha\mu\nu\sigma} \int_\tau^0 [j_g]_\sigma(\xi) e \frac{dz_\nu}{ds} d\tau. \qquad (26)$$

This contribution will vanish provided ($[j_g]_\sigma$ written in terms of dx_σ/ds)

$$\frac{dz_\nu}{ds} = \frac{dx_\nu}{ds}, \qquad (27)$$

as in this case $\varepsilon^{\alpha\nu\mu\sigma}(dz_\sigma/ds)(dx_\nu/ds) = 0$ with $\nu = \sigma$. Therefore the local character of the action principle, from which the equations of motion can be derived, is guaranteed as long as the world-line paths of charges and monopoles never cross and charges and monopoles have identical four-velocities if they are located at the same point. In addition, in order to have a general variational principle from which both particle and field equations are simultaneously derived in a system of distinct particles, $j_g^\mu = 0$ is valid everywhere [12]. Therefore, there is no new photon predicted by Singleton model [5] as no free monopole is allowed by the present approach.

The base of the model is then established, i.e., spinless particles (electric charge plus a pair of monopoles) can be considered as structuring an electron. Lets define an elementary particle as composed of a point electric charge and a magnetic dipole: in a Cartesian coordinate system, consider a positive monopole g at $(0, 0, z_0)$ with ($z_0 > 0$, z_0 a real number), a negative monopole $-g$ at $(0, 0, -z_0)$ and an electric charge e at the origin. Now z_0 is the particle radius. The angular and magnetic momenta of the system is contained in the electromagnetic field angular momentum, $\mathbf{L}_{em}$, i.e., $\mathbf{L}_{em} = \frac{1}{4\pi c} \int \mathbf{r} \times (\mathbf{E} \times \mathbf{B}) \, dv$ where $\mathbf{r}$ is the distance, $\mathbf{E}$ and $\mathbf{B}$ the electric and magnetic fields produced by the system (at rest in the laboratory frame), and dv the volume integration element. The angular momentum is

$$\mathbf{L}_{em} = \frac{(2g) \, e}{c} \hat{z}, \tag{28}$$

with $\hat{z}$ an unary vector toward the z axis. The result does not dependent on z_0 but only on e and g. Monopoles can then be placed *arbitrarily* close to the charge resulting in systems with invariant angular momentum. The magnetic charge, g, represents here the net magnetic flux of a monopole (which appears in equation (5) multiplied by $(1 + \cos\delta)$).

Since any charge-monopole pair has an associated angular momentum, the simplest momentum quantization (as pointed out by Saha and Wilson [13]) leads to the condition $\mathbf{L}_{em} = n\hbar/2$ or $n\hbar$ with n an integer. Such quantization condition implies $(2g) \, e/c = n\hbar/2$ (or $n\hbar$) for the system. If the magnetic moment of a particle is $e\hbar/(2mc)$, (in terms of the present description, $2gz_0$) we therefore obtain

$$z_0 = \frac{e^2}{mc^2} \left(\frac{1}{n} \right). \tag{29}$$

For an arbitrary large value of angular momentum, the size of a particle composed of an electric charge and two monopoles of opposite charge may be taken as arbitrarily small (for $n \to \infty$, the spin $n\hbar/2 \to \infty$ and $z_0 \to 0$). Let us take $n = 1$: the angular momentum is $\hbar/2$, $z_0 = e^2/mc^2$ and the magnetic moment $e\hbar/2mc$ for a net electric charge e. With $m = 0.511$ MeV/c^2 and $e = 1.602 \cdot 10^{-19}$ C the values for the magnetic moment and particle radius agree with the well known observed values of the electron [8].

Not only fermions can be formed by Dirac monopoles. Bosons are also possible candidates: photons have been described as a pair of fermions according to de Broglie description [14]. If fermions are described in terms of Dirac monopoles, the same space-time structure will be observed as caused by photons.

5   Conclusions

The present contribution is an initial approach to Dirac strings as sources of space-time distortions. The model is still in its initial steps as no clear relation between the string parameters (cross-section, etc) and the value of the gravitational mass was given.The main result is that Dirac monopoles cause gravitation to appear as a consequence of volumetric singular Dirac strings. One question then arises: is there any physical reason for Dirac strings to have non-null volumes? The generalized flux quantization rule (magnetic flux plus dual electric charge equals to an integer) derived by A Davidson and D Karasik [6] suggests that a finite magnetic flux cannot be confined indefinitely. In their approach this is associated to the inclusion of gravity in the theory, which shows a qualitative agreement with the present ideas.

Although the mathematical expression relating the monopole charge to the corresponding gravitational mass is still lacking in the present study, there is clear indication that for small δ (which corresponds to small gravitational fields according to equation (12)) and for small variations of distance of the test particle from the magnetic charge, monopoles may be considered as the source of gravitational effects related to point-like elementary particles.

In other works, [7], monopoles are related to Electromagnetism and Gravitation in unification efforts. In them torsion was included in the Weyl-Dirac theory causing the dual of the field strength tensor to have a nonzero divergence, both for charges and monopoles within the context of Proca equation.

In conclusion, it was possible to show that generalized Dirac monopoles composed of volumetric strings may deform space-time similarly to what happens in the case of a vanishing point-like mass when the distance of the test particle to the monopole does not change much. The generalization of the present results will be addressed in a future work.

6   Acknowledgements

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