The invariant formulation of special relativity, or the "true transformations relativity", and electrodynamics

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ABSTRACT. In the invariant approach to special relativity (SR), which we call the "true transformations (TT) relativity," a physical quantity in the four-dimensional spacetime is mathematically represented either by a true tensor or equivalently by a coordinate-based geometric quantity comprising both components and a basis. This invariant approach differs both from the usual covariant approach, which mainly deals with the basis components of tensors in a specific, i.e., Einstein's coordinatization of the chosen inertial frame of reference,.and the usual noncovariant approach to SR in which some quantities are not tensor quantities, but rather quantities from "3+1" space and time, e.g., the synchronously determined spatial length. This noncovariant formulation of SR is called the "apparent transformations (AT) relativity." The principal difference between the "TT relativity" and the "AT relativity" arises from the difference in the concept of sameness of a physical quantity for different observers. In the second part of this paper we present the invariant formulation of electrodynamics with the electromagnetic field tensor F^{ab}, and also the equivalent formulation in terms of the four-vectors of the electric E^a and magnetic B^a fields.

1. INTRODUCTION

At present there are two main formulations of the classical electrodynamics. The first one is the manifestly covariant formulation, which deals with the component form, in Einstein's coordinatization, of tensor quantities and tensor equations in the four-dimensional (4D) spacetime, and where the electromagnetic field tensor $F^{\alpha\beta}$ (the component form; for the notation see the next section) contains all the information about the electromagnetic field. (In the Einstein ("e") [1] coordinatization

Einstein's synchronization [1] of distant clocks and cartesian space coordinates x^i are used in the chosen inertial frame of reference (IFR).) The second one is the noncovariant formulation dealing with the three-vectors (3-vectors), the electric field $\mathbf{E}$ and the magnetic field $\mathbf{B}$, and with equations containing them. The whole latter formulation is given in "3+1" space and time and was constructed by Maxwell before the appearance of Einstein's theory of relativity [1]. In [2, 3] I have presented an alternative covariant formulation of vacuum electrodynamics with the electric and magnetic 4-vectors E^α and B^α (also the component form), which is equivalent to the usual covariant formulation with $F^{\alpha\beta}$. Recently [4] the invariant formulation of vacuum electrodynamics is presented with the electric and magnetic 4-vectors E^a and B^a (true tensors), which is equivalent to the invariant formulation with F^{ab} (also true tensor). For the covariant formulation of electrodynamics with E^α and B^α (the component form of tensors in the "e" coordinatization) see also [5] and [6]. The covariant formulation with $F^{\alpha\beta}$ and the usual formulation with the electric and magnetic 3-vectors $\mathbf{E}$ and $\mathbf{B}$ are generally considered to be equivalent. It is shown in [2, 3] ([4]) that, contrary to the general opinion, there is not the equivalence between covariant (invariant) formulations, either the usual one with $F^{\alpha\beta}$ (F^{ab}), or equivalently the alternative one with E^α and B^α (E^a and B^a), and the usual noncovariant formulation.

It seems that the work on the foundations of electromagnetic theory is again in a continuos progress and I only quote two recent references, [7] and [8], which present an interesting part of this work.

In the first part of this paper a general discussion on the "TT relativity" will be presented, and in the second part we consider the invariant formulation of electrodynamics with F^{ab}, and E^a and B^a.

2. A GENERAL DISCUSSION ON THE "TT RELATIVITY

In [9] Rohrlich introduced the notions of the true transformations (TT) and the apparent transformations (AT) of physical quantities and emphasized the role of *sameness* of a physical quantity for different observers. This concept of sameness is also considered in the same sense by Gamba [10].

The principal difference between the "TT relativity," the usual covariant formulation and the "AT relativity" stems from the difference in the concept of *sameness* of a physical system, i.e., of a physical quantity,

for different observers. This concept actually determines the difference in what is to be understood as a relativistic theory.

In this paper we explore a formulation of special relativity (SR) that is borrowed from general relativity. This is the formulation in which all physical quantities (in the case when no basis has been introduced) are described by *true tensor fields*, that are defined on the 4D spacetime, and that satisfy *true tensor equations* representing physical laws. The true tensors and true tensor equations are defined without any reference frame. For the formulation of spacetime theories without reference frames see, e.g., [11]. When the coordinate system is introduced the physical quantities are mathematically represented by the coordinate-based geometric quantities (CBGQs) that satisfy the coordinate-based geometric equations (CBGEs). *The CBGQs contain both the components and the basis one-forms and vectors* of the chosen IFR. Speaking in mathematical language a tensor of type (k,l) is defined as a linear function of k one-forms and l vectors (in old names, k covariant vectors and l contravariant vectors) into the real numbers, see, e.g., [12, 13, 14]. If a coordinate system is chosen in some IFR then, in general, any tensor quantity can be reconstructed from its components and from the basis vectors and basis 1-forms of that frame, i.e., it can be written in a coordinate-based geometric language, see, e.g., [14]. The symmetry transformations for the metric g_{ab}, i.e., the isometries, leave the pseudo-Euclidean geometry of 4D spacetime of SR unchanged; if we denote an isometry as Φ^* then $(\Phi^* g)_{ab} = g_{ab}$. At the same time they do not change the true tensor quantities, or equivalently the CBGQs, in physical equations. Thus *isometries* are what Rohrlich [9] calls *the TT*. The formulation of SR that deals with true tensor quantities and the TT is called the "TT relativity." In the "TT relativity" different coordinatizations of an IFR are allowed and they are all equivalent in the description of physical phenomena. (An example of a nonstandard synchronization, and thus nonstandard coordinatization as well, that drastically differs from the Einstein synchronization is considered in detail in [4].) In the "TT relativity" the concept of *sameness* of a physical quantity is very clear. Namely *the CBGQs representing some 4D physical quantity in different relatively moving IFRs, or in different coordinatizations of the chosen IFR, are all mathematically equal.* Thus they are really *the same quantity* for different observers, or in different coordinatizations. We suppose that in the "TT relativity" such 4D tensor quantities are well defined not only mathematically but also *experimentally*, as measurable

quantities with real physical meaning. *The complete and well defined measurement from the "TT relativity" viewpoint is such measurement in which all parts of some 4D quantity are measured.* Different experiments that test SR are discussed in [15] and it is shown that all experiments, which are complete from the "TT relativity" viewpoint, can be qualitatively and quantitatively explained by the "TT relativity," while some experiments cannot be adequately explained by the "AT relativity."

In this paper I use the same convention with regard to indices as in [4, 15]. Repeated indices imply summation. Latin indices $a, b, c, d, ...$ are to be read according to *the abstract index notation*, see [12], Sec.2.4.; they "...should be viewed as reminders of the number and type of variables the tensor acts on, *not* as basis components." They designate geometric objects in 4D spacetime. Thus, e.g., l_{AB}^a (a distance 4-vector $l_{AB}^a = x_B^a - x_A^a$ between two events A and B with the position 4-vectors x_A^a and x_B^a) and $x_{A,B}^a$ are (1,0) tensors and they are defined independently of any coordinate system. Greek indices run from 0 to 3, while latin indices $i, j, k, l, ...$ run from 1 to 3, and they both designate the components of some geometric object in some coordinate system, e.g., $x^\mu(x^0, x^i)$ and $x^{\mu'}(x^{0'}, x^{i'})$ are two coordinate representations of the position 4-vector x^a in two different inertial coordinate systems S and S'. The true tensor x^a is then represented as the CBGQs in different bases $\{e_\mu\}$ in an IFR S and $\{e_{\mu'}\}$ in a relatively moving IFR S' as $x^a = x^\mu e_\mu = x^{\mu'} e_{\mu'}$, where, e.g., e_μ are the basis 4-vectors, $e_0 = (1, 0, 0, 0)$ and so on, and x^μ are the basis components when the "e" coordinatization is chosen in some IFR S. Similarly the metric tensor g_{ab} denotes a tensor of type (0,2). The geometry of the spacetime is generally defined by this metric tensor g_{ab}, which can be expanded in a coordinate basis in terms of its components as $g_{ab} = g_{\mu\nu} dx^\mu \otimes dx^\nu$, and where $dx^\mu \otimes dx^\nu$ is an outer product of the basis 1-forms. Thus the geometric object g_{ab} is represented in the component form in an IFR S, and in the "e" coordinatization, i.e., in the $\{e_\mu\}$ basis, by the 4×4 diagonal matrix of components of g_{ab}, $g_{\mu\nu} = diag(-1, 1, 1, 1)$, and this is usually called the Minkowski metric tensor.

It has to be noted that the "TT relativity" approach to SR differs not only from the "AT relativity" approach but also from the usual covariant approach. The difference lies in the fact that *the usual covariant approach* mainly deals with *the basis components of tensors* (representing physical quantities) and the equations of physics are written out *in the component form.* Mathematically speaking the concept of a tensor in the usual covariant approach is defined entirely in terms of *the transfor-*

mation properties of its components relative to some coordinate system.
Obviously in the usual covariant approach (including [9] and [10]) *the basis components of a true tensor,* or equivalently of a CBGQ, that are determined in different IFRs (or in different coordinatizations), are considered to be *the same quantity* for different observers. Although the basis components of a true tensor refer to the same tensor quantity they, in fact, are not the same quantity. They depend on the chosen reference frame and the chosen coordinatization of that reference frame. Thus the basis components are the coordinate quantities.

In contrast to the TT *the AT* are not the transformations of spacetime tensors and they do not refer to the same 4D quantity. Thus they are not isometries and they *refer exclusively to the component form of tensor quantities and in that form they transform only some components of the whole tensor quantity.* In fact, depending on the used AT, only a part of a 4D tensor quantity is transformed by the AT. Such a part of a 4D quantity, when considered in different IFRs (or in different coordinatizations of some IFR) corresponds to different quantities in 4D spacetime. An example of the AT is the AT of the synchronously defined spatial length [1], i.e., the Lorentz "contraction." It is shown in [9, 10], and more exactly in [4], that the Lorentz "contraction" is an AT. The spatial or temporal distances taken alone are not well defined quantities in 4D spacetime. Further it is shown in [2, 3], and more exactly in [4], that the conventional transformations of the electric and magnetic 3-vectors **E** and **B** (see, e.g., [16] Sec.11.10) are also the AT. The formulation of SR which uses the AT we call the "AT relativity." An example of such formulation is Einstein's formulation of SR which is based on his two postulates and which deals with different AT. Thus in the "AT relativity" quantities connected by an AT, e.g., two spatial lengths connected by the Lorentz contraction, are considered to be the same quantity for different observers. However, as explicitly shown in [4], the quantities connected by an AT are not well defined quantities in 4D spacetime and, actually, they correspond to different quantities in 4D spacetime.

(In the following we shall also need the expression for the covariant 4D Lorentz transformations $L^a{}_b$, *which is independent of the chosen coordinatization of reference frames* (see [17], [3] and [4]). It is

$$L^a{}_b \equiv L^a{}_b(v) = g^a{}_b - \frac{2u^a v_b}{c^2} + \frac{(u^a + v^a)(u_b + v_b)}{c^2(1 + \gamma)}, \qquad (1)$$

where u^a is the proper velocity 4-vector of a frame S with respect to itself, $u^a = cn^a$, n^a is the unit 4-vector along the x^0 axis of the frame S, and v^a is the proper velocity 4-vector of S' relative to S. Further $u \cdot v = u^a v_a$ and $\gamma = -u \cdot v/c^2$. In the Einstein coordinatization $L^a{}_b$ is represented by $L^\mu{}_\nu$, the usual expression for pure Lorentz transformation which connects two coordinate representations, basis components x^μ, $x^{\mu'}$ of a given event. x^μ, $x^{\mu'}$ refer to two relatively moving IFRs (with the Minkowski metric tensors) S and S',

$$
\begin{aligned}
x^{\mu'} &= L^{\mu'}{}_\nu x^\nu, \qquad L^{0'}{}_0 = \gamma, \quad L^{0'}{}_i = L^{i'}{}_0 = -\gamma v^i/c, \\
L^{i'}{}_j &= \delta^i_j + (\gamma - 1)v^i v_j/v^2,
\end{aligned}
\tag{2}
$$

where $v^\mu \equiv dx^\mu/d\tau = (\gamma c, \gamma v^i)$, $d\tau \equiv dt/\gamma$ and $\gamma \equiv (1 - v^2/c^2)^{1/2}$. Since $g_{\mu\nu}$ is a diagonal matrix the space x^i and time t ($x^0 \equiv ct$) parts of x^μ do have their usual meaning.)

As already mentioned different experiments that test SR are discussed in [15]. In numerous papers and textbooks it is considered that the experiments on the length contraction and the time dilatation test SR, but the discussion from [15] shows that such an interpretation of the experiments refers exclusively to - the "AT relativity," and not to - the "TT relativity." When SR is understood as the theory of 4D spacetime with pseudo-Euclidean geometry then instead of the Lorentz contraction and the dilatation of time one has to consider the 4D tensor quantities, the spacetime length l, $l = (g_{ab}l^a l^b)^{1/2}$, or the distance 4-vector $l^a_{AB} = x^a_B - x^a_A$. Namely in the "TT relativity" the measurements in different IFRs (and different coordinatizations) have to refer to the same 4D tensor quantity, i.e., to a CBGQ. In the chosen IFR and the chosen coordinatization the measurement of some 4D quantity has to contain the measurements of all parts (all the basis components) of such a quantity. However in almost all experiments that refer to SR only the quantities belonging to the "AT relativity" were measured. From the "TT relativity" viewpoint such measurements are incomplete, since only some parts of a 4D quantity, not all, are measured. It is shown in [15] that the "TT relativity" theoretical results agree with all experiments that are complete from the "TT relativity" viewpoint, i.e., in which all parts of the considered tensor quantity are measured in the experiment. However the "AT relativity" results agree only with some of the examined experiments (and this agreement exists only for the specific coordinatization, i.e., the "e" coordinatization. Moreover the agreement of

the "AT relativity" and the experiments is, in fact, an "apparent" agreement, which is usually obtained by means of an incorrect treatment of 4D quantities. This is explicitly shown in [15] for some of the well-known experiments: the "muon" experiment, the Michelson-Morley type experiments, the Kennedy-Thorndike type experiments and the Ives-Stilwell type experiments.

In this paper we only give a short discussion of the Michelson-Morley experiment and for the details see [15]. In the Michelson-Morley experiment two light beams emitted by one source are sent, by half-silvered mirror O, in orthogonal directions. These partial beams of light traverse the two equal (of the length L) and perpendicular arms OM_1 (perpendicular to the motion) and OM_2 (in the line of motion) of Michelson's interferometer and the behavior of the interference fringes produced on bringing together these two beams after reflection on the mirrors M_1 and M_2 is examined. The Earth frame is the rest frame of the interferometer, i.e., it is the S frame, while the S' frame is the (preferred) frame in which the interferometer is moving at velocity $\mathbf{v}$.

In the Michelson-Morley experiment the traditional, "AT relativity," derivation of the fringe shift $\triangle N$ deals only with the calculation, in the "e" coordinatization, of t_1 and t_2 (in S and S'), which are the times required for the complete trips OM_1O and OM_2O along the arms of the Michelson-Morley interferometer; . The null fringe shift obtained with such calculation is only in an "apparent," not true, agreement with the observed null fringe shift, since this agreement was obtained by an incorrect procedure. Namely it is supposed in such derivation that, e.g., t_1 and t_1' refer to the same quantity measured by the observers in relatively moving IFRs S and S' that are connected by the Lorentz transformation. However, as shown in [4, 15], the relation for the time dilatation $t_1' = \gamma t_1$, which is used in the usual explanation of the Michelson-Morley experiment, is not the Lorentz transformation of some 4D quantity, and, see [15], t_1' ($t_1' = 2L/c(1 - v^2/c^2)^{1/2}$) and t_1 ($t_1 = 2L/c$) do not correspond to the same 4D quantity considered in S' and S respectively but to different 4D quantities.

Our "TT relativity" calculation, in contrast to the "AT relativity" calculation, deals always with the true tensor quantities or the CBGQs; in the Michelson-Morley experiment it is the phase of a light wave

$$\phi = k^a g_{ab} l^b, \qquad (3)$$

where k^a is the propagation 4-vector, g_{ab} is the metric tensor and l^b is

the distance 4-vector. All quantities in (3) are true tensor quantities and thus (3) is written without any reference frame. These quantities can be written in the coordinate-based geometric language and, e.g., the decompositions of k^a in S and S' and in the "e" coordinatization are

$$k^a = k^\mu e_\mu = k^{\mu'} e_{\mu'}, \qquad (4)$$

where the basis components k^μ of the CBGQ are transformed by $L^{\mu'}{}_\nu$ (2), while the basis vectors e_μ are transformed by the inverse transformation $(L^{\mu'}{}_\nu)^{-1} = L^\mu{}_{\nu'}$. By the same reasoning the phase ϕ (3) is given in the coordinate-based geometric language as

$$\phi = k^\mu g_{\mu\nu} l^\nu = k^{\mu'} g_{\mu\nu} l^{\nu'}. \qquad (5)$$

As shown in [15] the "TT relativity" calculations yields the observed null fringe shift and that result holds for all IFRs and all coordinatizations.

In addition, it is shown in [15] that the usual "AT relativity" actually deals only with the part $k^0 l_0$ (i.e., ωt) of the whole phase ϕ, (3) or (5). This contribution $k^0 l_0$ is considered in the interferometer rest frame S, while in the S' frame, in which the interferometer is moving, the usual "AT relativity" takes into account only the contribution $k^0 l_{0'}$ (i.e., $\omega t'$); the k^0 (i.e., ω) factor is taken to be the same in S and S' frames. Thus in the usual "AT relativity" two different quantities $k^0 l_0$ and $k^0 l_{0'}$ (only the parts of the phase (3) or (5)) are considered to be the same 4D quantity for observers in S and S' frames, and these quantities are considered to be connected by the Lorentz transformation. Such an incorrect procedure then caused an apparent (not true) agreement of the traditional analysis with the results of the Michelson-Morley experiment. Since only a part of the whole phase ϕ (3) or (5) is considered the traditional result is synchronization, i.e., coordinatization, dependent result.

Driscoll [18] *improved* the traditional "AT relativity" derivation of the fringe shift taking into account the changes in frequencies due to the Doppler effect. (Recall that in the traditional approach ω is the same in S and S'.) The improved "AT relativity" calculation of the fringe shift from [18] finds a "surprising" non-null fringe shift. It is shown in [15] that the non-null theoretical result for the fringe shift from [18] is easily obtained from our "TT relativity" approach taking only the product $k^{0'} l_{0'}$ in the calculation of the increment of phase ϕ' in S' in which the apparatus is moving. Thus again as in the usual "AT relativity" calculation two different quantities $k^0 l_0$ and $k^{0'} l_{0'}$ (only

the parts of the phase (3) or (5)) are considered to be the same 4D quantity for observers in S and S' frames, and consequently that these two quantities are connected by the Lorentz transformation. Since only a part $k^{0'} l_{0'}$ of the whole 4D tensor quantity ϕ (3) or (5) is considered the non-null fringe shift can be shown to be quite different in another coordinatization, see [15].

The same conclusions can be drawn for the Kennedy-Thorndike type experiments, and for the modern laser versions of both, the Michelson-Morley and the Kennedy-Thorndike type experiments, see [15].

This short consideration illustrates the main differences in the interpretation of the well-known experiments from the point of view of the traditional "AT relativity" and from the viewpoint of the "TT relativity."

3. THE INVARIANT FORMULATION OF ELECTRODYNAMICS WITH F^{ab}

Let us now apply the above general consideration of the invariant formulation of SR to the electrodynamics.

The usual covariant Maxwell equations with $F^{\alpha\beta}$ and its dual $^*F^{\alpha\beta}$

$$\partial_\alpha F^{a\beta} = -j^\beta/\varepsilon_0 c, \quad \partial_\alpha \, ^*F^{\alpha\beta} = 0, \qquad (6)$$

where $^*F^{\alpha\beta} = -(1/2)\varepsilon^{\alpha\beta\gamma\delta}F_{\gamma\delta}$ and $\varepsilon^{\alpha\beta\gamma\delta}$ is the totally skew-symmetric Levi-Civita pseudotensor, are actually the equations in the "e" coordinatization for basis components in a chosen IFR. We first show how these equations for the basis components are derived from the true tensor equations (when no basis has been introduced). The true tensor equations can be written in the abstract index notation as

$$\nabla^a F_{ab} = -j_b/\varepsilon_0 c, \quad \varepsilon^{abcd}\nabla_b F_{cd} = 0, \qquad (7)$$

where ∇_b is the derivative operator (sometimes called the covariant derivative), see, e.g., [12]. The tensor equation (7) can be written in the following form

$$(-g)^{-1/2}\partial_a((-g)^{1/2}F^{ab}) = -j^b/\varepsilon_0 c, \quad \varepsilon^{abcd}\partial_b F_{cd} = 0, \qquad (8)$$

where g is the determinant of the metric tensor g_{ab} and ∂_a is an ordinary derivative operator. When some coordinatization is chosen in a

specific IFR S, e.g., the "e" coordinatization, then the relations (8) can be written in the coordinate-based geometric language as the equations that contain the basis vectors as well,

$$\partial_\alpha F^{\alpha\beta} e_\beta = -(1/\varepsilon_0 c)j^\beta e_\beta, \quad \partial_\alpha \, {}^* F^{\alpha\beta} e_\beta = 0. \qquad (9)$$

(We remark that (9) follows from (8) for those coordinatizations for which the basis vectors are constant, e.g., the "e" coordinatization.) From (9), which contain the basis (1,0) tensors (4-vectors), one finds the already written equations for basis components (6); every equation in (9) is the equality of two tensors of the same type, two 4-vectors, and if two 4-vectors are equal then the corresponding components are equal, and that holds in all bases. In many treatments only the covariant Maxwell equations (6) for the basis components are used forgetting that they are obtained from the tensor equations (8) or (9).

Similarly one finds from (8) the equations corresponding to (9) and to (6) but in the $\{e_{\mu'}\}$ basis, i.e., in the S' frame and in the "e" coordinatization, by replacing the unprimed quantities with the primed ones.

¿From this consideration some important conclusions can be derived regarding the mathematical form of the physical laws in the "TT relativity." From the mathematical viewpoint the (1,0) tensor quantity $(-g)^{-1/2}\partial_a((-g)^{1/2}F^{ab})$ can be written in the coordinate-based geometric language in the "e" cordinatization, and in S as $\partial_\alpha F^{\alpha\beta} e_\beta$, while in S' as $\partial_{\alpha'} F^{\alpha'\beta'} e_{\beta'}$, where all primed quantities (including the basis vectors) are obtained by the TT, i.e., by the Lorentz transformation $L^\mu{}_{\nu,e}$ (2) from the corresponding unprimed quantities. Thus

$$(-g)^{-1/2}\partial_a((-g)^{1/2}F^{ab}) = \partial_\alpha F^{\alpha\beta} e_\beta = \partial_{\alpha'} F^{\alpha'\beta'} e_{\beta'}, \qquad (10)$$

which shows that *the equalities in (10) refer to the same quantity in 4-D spacetime.* Analogously, the mathematics yields for the (1,0) tensor (4-vector) $-j^b/\varepsilon_0 c$ the relations

$$-j^b/\varepsilon_0 c = -(1/\varepsilon_0 c)j^\beta e_\beta = -(1/\varepsilon_0 c)j^{\beta'} e_{\beta'}. \qquad (11)$$

A similar analysis can be applied to the second Maxwell equation in (8).

The physical laws expressed as tensor equations, e.g., (8), or equivalently as CBGEs, for example, (9), *set up the connection between two*

geometric quantities, in this case, two 4-vectors, that are given by equations (10) and (11). *The experiments in which all parts of tensor quantities are measured then play the fundamental role in deciding about the validity of some physical law mathematically expressed as tensor equation.* We see from the equations (10) and (11) that *when the physical laws are expressed as tensor, geometric, equations (8) or (9) then these equations are invariant upon the Lorentz transformations.* It is not so for the equations in the component form, e.g., (6). Of course the covariance of physical equations, when they are written in the component form, is a simple consequence of the invariance of tensor quantities, or equivalently, of the CBGQs, upon the mentioned TT, that is upon the isometries. *The invariance of physical laws, that are expressed as tensor equations, or equivalently as the CBGEs, means that all physical phenomena proceed in the same way (taking into account the corresponding initial and boundary conditions) in different IFRs. Thus there is no physical difference between these frames, what automatically embodies the principle of relativity. We remark that in the "TT relativity" there is no need to postulate the principle of relativity as a fundamental law. It is replaced by the requirement that the physical laws must be expressed as tensor equations (or equivalently as the CBGEs) in the 4D spacetime.*

This consideration is used in [4] to derive an important result, i.e., to show that, contrary to the general belief, *the usual Maxwell equations with E_i and B_i, or with the 3-vectors $\mathbf{E}$ and $\mathbf{B}$, are not equivalent to the tensor equations (8), i.e., to the CBGEs (9).* Further it is explicitly shown in [4] that the conventional transformations for $\mathbf{E}$ and $\mathbf{B}$ (see, e.g., [16] Sec.11.10) actually connect different quantities in 4D spacetime, and thus that they are not the TT but the AT.

4. THE INVARIANT FORMULATION OF ELECTRODYNAMICS WITH E^a AND B^a

In this section we present the formulation of electrodynamics introducing the 4-vectors E^a and B^a instead of the usual 3-vectors $\mathbf{E}$ and $\mathbf{B}$. The Maxwell equations are formulated as tensor equations with E^a and B^a, which are equivalent to the tensor Maxwell equations with F^{ab}, (7) or (8). We define the electric and magnetic fields by the relations

$$E_a = (1/c)F_{ab}v^b, \quad B^a = -(1/2c^2)\varepsilon^{abcd}v_b F_{cd}. \tag{12}$$

The E^a and B^a are the electric and magnetic field 4-vectors measured by an observer moving with 4-velocity v^a in an arbitrary reference frame,

$v^a v_a = -c^2$, and ε^{abcd} is the totally skew-symmetric Levi-Civita pseudotensor (density). These fields satisfy the conditions $v_a E^a = v_b B^b = 0$, which follow from the definitions (12) and the antisymmetry of F_{ab}. In the usual treatments (see, e.g., [12], [20], [19]) the tensors E^a and B^a are introduced in the curved spacetimes or noninertial frames, but at the same time the usual Maxwell equations with the 3-vectors **E** and **B** are considered to be valid in the IFRs. One gets the impression that E^a and B^a are considered only as useful mathematical objects, while the real physical meaning is associated with the 3-vectors **E** and **B**. Our results obtained in [4] and in Sec. 2. imply that it is necessary to use the 4-vectors E^a and B^a in IFRs as well. This means that the tensor quantities E^a and B^a do have the real physical meaning and not the 3-vectors **E** and **B**. The inverse relation connecting the E^a, B^a and the tensor F_{ab} is

$$F_{ab} = (1/c)(v_a E_b - v_b E_a) + \varepsilon_{abcd} v^c B^d. \tag{13}$$

The tensor Maxwell equations with E^a, B^a in the curved spacetimes are derived, e.g., in [20]. Here we specify them to the IFRs, but in such a way that they remain valid for different coordinatizations of the chosen IFR. First we write the tensor Maxwell equations (8) with F^{ab} as the CBGEs (9). Then we also write the equation (13) in the coordinate-based geometric language and the obtained equation substitute into (9). This procedure yields

$$\begin{aligned}
\partial_\alpha(\delta^{\alpha\beta}{}_{\mu\nu} v^\mu E^\nu + c\varepsilon^{\alpha\beta\mu\nu} B_\mu v_\nu)e_\beta &= -(j^\beta/\varepsilon_0)e_\beta, \\
\partial_\alpha(\delta^{\alpha\beta}{}_{\mu\nu} v^\mu B^\nu + (1/c)\varepsilon^{\alpha\beta\mu\nu} v_\mu E_\nu)e_\beta &= 0,
\end{aligned} \tag{14}$$

where E^α and B^α are the basis components of the electric and magnetic field 4-vectors E^a and B^a measured by a family of observers moving with 4-velocity v^α, and $\delta^{\alpha\beta}{}_{\mu\nu} = \delta^\alpha_\mu \delta^\beta_\nu - \delta^\alpha_\nu \delta^\beta_\mu$. The equations (14) correspond in the E^a, B^a picture to the equations (9) in the F^{ab} picture. From the relations (14) we again find the covariant Maxwell equations for the basis components (without the basis vectors e_β), which were already presented in [2], [3] and [4].

$$\begin{aligned}
\partial_\alpha(\delta^{\alpha\beta}{}_{\mu\nu} v^\mu E^\nu + c\varepsilon^{\alpha\beta\mu\nu} B_\mu v_\nu) &= -(j^\beta/\varepsilon_0), \\
\partial_\alpha(\delta^{\alpha\beta}{}_{\mu\nu} v^\mu B^\nu + (1/c)\varepsilon^{\alpha\beta\mu\nu} v_\mu E_\nu) &= 0.
\end{aligned} \tag{15}$$

(It has to be mentioned that the component form of Maxwell equations, (15), was also presented in [5], and with $j^\beta = 0$ in [6]. However in [6]

the physical meaning of v^α is unspecified - it is any unitary 4-vector. The reason for such choice of v^α in [6] is that there E^α and B^α are introduced as the "auxiliary fields," while **E** and **B** are considered as the physical fields. In our "invariant" approach with E^a and B^a the situation is just the opposite; E^a and B^a are the real physical fields, which are correctly defined and measured in 4D spacetime, while the 3-vectors **E** and **B** are not correctly defined in 4D spacetime from the "TT viewpoint." The equations (15) for basis components correspond to the covariant Maxwell equations for basis components (6). Instead of to work with F^{ab}- formulation, (9) and (6), one can equivalently use the E^a, B^a formulation with (14) and (15). For the given sources j^a one could solve these equations and find the general solutions for E^a and B^a.

4.1 The comparison of Maxwell's equations with E and B and those with E^a and B^a

The comparison of this invariant approach with E^a and B^a and the usual noncovariant approach with the 3-vectors **E** and **B** is possible in the "e" coordinatization. If one considers the "e" coordinatization and takes that in an IFR S the observers who measure the basis components E^α and B^α are at rest, i.e., $v^\alpha = (c, \mathbf{0})$, then $E^0 = B^0 = 0$, and one can derive from the covariant Maxwell equations (15) for the basis components E^α and B^α the Maxwell equations which contain only the space parts E^i and B^i of E^α and B^α, e.g., from the first covariant Maxwell equation in (15) one easily finds $\partial_i E^i = j^0/\varepsilon_0 c$. We see that the Maxwell equations obtained in such a way from the Maxwell equations (14), or (15), are of the same form as the usual Maxwell equations with **E** and **B**. From the above consideration one concludes that all the results obtained in a given IFR S from the usual Maxwell equations with **E** and **B** remain valid in the formulation with the 4-vectors E^a and B^a *(in the "e" coordinatization), but only for the observers who measure the fields E^a and B^a and are at rest in the considered IFR.* Then for such observers the components of **E** and **B**, which are not well defined quantities in the "TT relativity," can be simply replaced by the space components of the 4-vectors E^a and B^a (in the "e" coordinatization). It has to be noted that just such observers were usually considered in the conventional formulation with the 3-vectors **E** and **B**. However, the observers who are at rest in some IFR S cannot remain at rest in another IFR S' moving with V^α relative to S. Hence in S' this simple replacement does not hold; in S' one cannot

obtain the usual Maxwell equations with the 3-vectors $\mathbf{E}'$ and $\mathbf{B}'$ from the transformed covariant Maxwell equations with $E^{\alpha'}$ and $B^{\alpha'}$.

Some important experimental consequences of the "TT relativity" approach to electrodynamics have been derived in [3]. They are the existence of the spatial components E^i of E^a outside a current-carrying conductor for the observers (who measure E^a) at rest in the rest frame of the wire, and the existence of opposite (invariant) charges on opposite sides of a square loop with current, both when *the loop is at rest* and when it is moving.

The similar external second-order electric fields from steady currents in a conductor at rest are also predicted in, e.g., [21]. But this prediction is made on the basis of Weber's theory and thus the theory from [21] is an action-at-a-distance theory.

5. SUMMARY AND CONCLUSIONS

In this paper we have presented the invariant (true tensor) formulation of SR. This "TT relativity" is compared with the usual covariant approach to SR and with the usual "AT relativity" formulation, i.e., with the original Einstein's formulation.

The principal concept that makes distinction between the "TT relativity" formulation, the usual covariant formulation and the "AT relativity" formulation of SR is the concept of *sameness* of a physical quantity for different observers. In the "TT relativity" the same quantity for different observers is the true tensor quantity, or equivalently the CBGQ, only one quantity in 4D spacetime.

In the usual covariant approach one deals with the basis components of tensors and with the equations of physics written out in the component form, and all is mainly done in the "e" coordinatization. There one considers that the basis components, e.g., l^μ and $l^{\mu'}$, represent the same quantity for different observers. These quantities, in fact, are not equal $l^\mu \neq l^{\mu'}$, but they only refer to the same tensor quantity l^a_{AB}. If only one coordinatization is *always* used, usually the "e" coordinatization, then the conventional covariant approach can be applied. However the physics must not depend on the chosen coordinatization, which means that the theory has to be formulated in the manner that does not depend on the choice of some specific coordinatization. The Einstein coordinatization is nothing more physical but any other permissible coordinatization. This requirement is fulfilled in the "TT relativity."

In the "AT relativity" one does not deal with tensor quantities but with quantities from "3+1" space and time, e.g., the synchronously determined spatial lengths, or the temporal distances taken alone. The AT connect such quantities and thus they refer exclusively to the component form of tensor quantities and in that form they transform only *some* components of the whole tensor quantity. In the "AT relativity" the quantities connected by an AT are considered to be the same quantity, but such quantities are not well defined in 4D spacetime, and actually they correspond to different quantities in 4D spacetime.

The difference between the traditional "AT relativity" and the invariant formulation of SR, i.e., the "TT relativity," is also illustrated by the difference in the interpretation of the Michelson-Morley experiment.

In Sec. 3 we have presented Maxwell equations as the true tensor equations (7) or (8) and as the CBGEs (9). It is discussed how from these equations one finds the usual covariant Maxwell equations (i.e., the component form) (6).

In Sec.4 we have introduced the 4-vectors E^a and B^a instead of the usual 3-vectors $\mathbf{E}$ and $\mathbf{B}$ and we have formulated the Maxwell equations as tensor equations with E^a and B^a, i.e., as the CBGEs (14) and the equations for the basis components E^α and B^α (15) (all in the "e" coordinatization). These equations are completely equivalent to the usual covariant Maxwell equations in the F^{ab}- formulation, (9) and (6). It has been explicitly shown in Sec. 4.1 that all the results obtained in a given IFR S from the usual Maxwell equations with $\mathbf{E}$ and $\mathbf{B}$ remain valid in the formulation with the 4-vectors E^a and B^a (in the "e" coordinatization), but only for the observers who measure the fields E^a and B^a and are at rest in the considered IFR. Thus we conclude that the tensor quantities E^a and B^a do have the real physical meaning and not the 3-vectors $\mathbf{E}$ and $\mathbf{B}$.

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