Comment on the paper "Slightly generalized Maxwell Classical Electrodynamics Can be Applied to Inneratomic Phenomena"*

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In their paper "Slightly generalized Maxwell Classical Electrodynamics Can be Applied to Interatomic Phenomena" Simulik and Krivsky found an interesting relation between the Dirac and Maxwell equations. They found that their "slightly generalized Maxwell equations" have both spin 1 and spin 1/2 symmetries.

Although the mathematical derivations seems to be faultless and nice (except of using the constant $\hbar$, as will be explained latter on), I have serious doubts about their physical interpretation.

First, there is no known transition from quantum equations to classical (non-quantum) equations. All quantum equations and theories must depend on the one mysterious constant, the Planck constant $h = 2\pi\hbar$. Only in quantum theories Planck's constant appears naturally from first principles. Classical (non-quantum) theories can not incorporate the Planck's constant from first principles. The authors use units which hide the quantum effects. They write: "we use the units: $\hbar = c = 1$, transition to standard system is fulfilled by the substitution $\omega \longrightarrow \hbar\omega$, $\mathbf{m}_0 \longrightarrow \mathbf{m}_0 c^2$". The above transition is not complete, one should add also $i\partial_\mu \to i\hbar\partial_\mu$, and hence also (in appropriate places) $curl \to \hbar \cdot curl$, $grad \to \hbar \cdot grad$, and $div \to \hbar \cdot div$. Writing or not writing explicitly the Planck's constant is not a problem of convenience, but an explicit indication of quantum effects.

The authors start from the Dirac equation (in the corrected form):

*NDLR: The author's reply to this comment can be found on p. 523 of this same issue.

$$\left(ic\hbar\gamma^{\mu}\partial_{\mu} - \mathbf{m}_0 c^2 + \gamma^0 \Phi\right)\Psi = 0, \quad \Psi \equiv (\Psi^{\alpha}), \tag{1}$$

using the standard Pauli - Dirac representation for the γ matrices, with $\mathbf{m}_0 \neq 0$ and the interaction potential $\Phi \neq 0$. They look for stationary solutions and assume

$$\Psi(x) = \Psi(\overrightarrow{x})e^{-i\omega t} \Longrightarrow i\partial_0\Psi(x) = \omega\Psi(x). \tag{2}$$

Next they introduce the field

$$\mathcal{E}^{\mu} = E^{\mu} - iH^{\mu}, \tag{3}$$

or

$$\mathcal{E} \equiv \begin{pmatrix} \overrightarrow{\mathcal{E}} \\ \mathcal{E}^0 \end{pmatrix} = \begin{pmatrix} \overrightarrow{E} - i\overrightarrow{H} \\ E^0 - iH^0 \end{pmatrix}, \tag{4}$$

where $\overrightarrow{E}$ and $\overrightarrow{H}$ are denoted as the electric and magnetic fields respectively, and $\mathcal{E}^0 = E^0 - iH^0$ as the complex scalar field. The field $\mathcal{E}$ is related to the Dirac equation wave function Ψ via the unitary transformation

$$\mathcal{E} = W\Psi, \quad \Psi = W^{\dagger}\mathcal{E}, \tag{5}$$

where

$$W = \begin{pmatrix} 0 & iC_- & 0 & C_- \\ 0 & -C_+ & 0 & iC_+ \\ iC_- & 0 & C_- & 0 \\ iC_+ & 0 & C_+ & 0 \end{pmatrix}; \quad C_{\pm} \equiv \frac{1}{2}(C \pm 1), \quad C\Psi \equiv \Psi^*, \quad C\mathcal{E} \equiv \mathcal{E}^*, \tag{6}$$

therefore $\mathcal{E}$ is also a Dirac equation wave function. The stationary Dirac equation now takes the form

$$-ic\hbar\mathrm{curl}\,\overrightarrow{\mathcal{E}} + \left[(\omega\hbar - \Phi)\,C - \mathbf{m}_0 c^2\right]\overrightarrow{\mathcal{E}} = -c\hbar\mathrm{grad}\mathcal{E}^0,$$
$$c\hbar\mathrm{div}\,\overrightarrow{\mathcal{E}} = \left[(\omega\hbar - \Phi)\,C + \mathbf{m}_0 c^2\right]\overrightarrow{\mathcal{E}}. \tag{7}$$

Assuming that $\overrightarrow{E}$, $\overrightarrow{H}$, E^0 and H^0 are real fields, after separating real and imaginary parts, one obtains

$$\mathrm{curl}\,\overrightarrow{H} - \frac{\omega\epsilon}{c}\overrightarrow{E} = \mathrm{grad}E^0, \quad \mathrm{curl}\,\overrightarrow{E} - \frac{\omega\mu}{c}\overrightarrow{H} = -\mathrm{grad}H^0, \tag{8}$$

$$\operatorname{div}\overrightarrow{E} = \frac{\omega\mu}{c}E^0, \quad \operatorname{div}\overrightarrow{H} = -\frac{\omega\epsilon}{c}H^0, \tag{9}$$

$$\epsilon\left(\overrightarrow{x}\right) = 1 - \frac{\Phi\left(\overrightarrow{x}\right) + \mathbf{m}_0 c^2}{\omega\hbar}, \quad \mu\left(\overrightarrow{x}\right) = 1 - \frac{\Phi\left(\overrightarrow{x}\right) - \mathbf{m}_0 c^2}{\omega\hbar}. \tag{10}$$

Equations (8-10) look very similar to Maxwell equations, therefore they have been called "slightly generalized Maxwell equations". But they are still *a different form of the Dirac equation.* They differ from Maxwell equations by the presence of the strange permeabilities of Eq. (10), and by the "complex scalar field" $\mathcal{E}^0 = E^0 - iH^0$. One should remember that this complex scalar field and the "electric and magnetic fields" are part of the Dirac equation wave function, as given by Eq. (5). For $\Phi\left(\overrightarrow{x}\right) = -Ze^2/r$ the solutions of Eqs. (8-10) coincide with the Sommerfeld - Dirac formula

$$\hbar\omega_{nj} = \frac{\mathbf{m}_0 c^2}{\hbar\sqrt{1 + \frac{\alpha^2}{\left(n_r + \sqrt{k^2 - \alpha^2}\right)^2}}}, \quad n_r = n - k, \quad k = j + 1/2, \quad \alpha = e^2/\hbar c. \tag{11}$$

This is a solution of the Dirac equation (even though it resembles the Maxwell equations). Therefore the authors are wrong when they claim that " the result (19) is obtained here not from the Dirac equation, but from the Maxwell equations (1) with sources (3) in the medium (2)." Even for $\Phi\left(\overrightarrow{x}\right) = 0$, many paradoxes will be encountered if the Dirac equation as given by Eqs. (8-10) will be reinterpreted as a new form of Maxwell Equations.

In summary there was here an attempt to reinterpret the Dirac quantum equation in terms of classical Maxwell equations, an impossible task from physical principles. Nevertheless there are in the paper some puzzling relations between these two fields.

The authors should be thanked for presenting us these interesting puzzles.

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