Energy-dependent metric for gravitation and the breakdown of local Lorentz invariance

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ABSTRACT. We analyze the data on the comparison of clock rates between a flying clock and a clock at ground, carried out by Alley and coworkers at the end of '70's .The fit to such data is in favour of an energy-dependent metric for gravitation. We discuss also the results of a recently proposed electromagnetic test of breakdown of local Lorentz invariance - based on the detection of a voltage induced by a stationary magnetic field - and show that the obtained positive evidence for such an effect seems to support the derived form of the energy-dependent gravitational metric.

1 Introduction

The geometrical structure of the physical world - both at a large and a small scale -has been debated since a long. After Einstein, the generally accepted view considers the arena of physical phenomena as a four-dimensional space-time, endowed with a *global,* curved, Riemannian structure and a *local,* flat, Minkowskian geometry.

However, a recent analysis of some experimental data concerning physical phenomena ruled by different fundamental interactions seems to provide evidence for a local departure from Minkowski metric[(1-6)]: among them, the lifetime of the (weakly decaying) K_s^0 meson[(7)], the Bose-Einstein correlation in (strong) pion production[(8)] and the superluminal propagation of electromagnetic waves in waveguides[(9)]. These phenomena seemingly show a (local) breakdown of Lorentz invariance, together with a plausible inadequacy of the Minkowski metric; on the other hand, they can be interpreted in terms of a deformed Minkowski space-time, with metric coefficients depending on the energy of the process considered[1−6].

All the above facts suggested to us a (four-dimensional) generalization of the (local) space-time structure based on an energy-dependent "deformation" of the usual Minkowski geometry, whereby the corresponding deformed metrics ensuing from the fit to the experimental data seem to provide an *effective dynamical description of the relevant interactions (at the energy scale and in the energy range considered)*. Moreover, it was also shown[10] that *the four-dimensional energy-dependent space-time is just a manifestation of a larger, five-dimensional space in which energy plays the role of a fifth dimension.* In fact, *all* the phenomenological metrics discussed in refs. [1-6] can be obtained as solutions of the Einstein equations in such a five-dimensional space-time.

An analogous energy-dependent metric seems to hold for the gravitational field (at least locally, i.e. in a neighborhood of Earth) when analyzing some classical experimental data concerning the slowing down of clocks[11]. We have given a preliminary form of such a metric in paper [6]. However, our derivation was based on a theoretically assumed analogy of the gravitational energy-dependent metric with the strong one; moreover, the experimental data were only used as a consistency check.

In the present paper, we want instead to derive the energy-dependent metric for gravitation by fitting them. Moreover, we will put in evidence some interesting connections between such a metric and the electromagnetic one, on account of a recent experimental test apparently providing evidence for an electromagnetic breakdown of local Lorentz invariance[12,13].

The paper is organized as follows. In sect. 2 we review the formalism of the deformed Minkowski space, and the main phenomenological results of interest to us. In sect. 3 we fit the data of the Alley experiment on the comparison of clock rates and derive the form of the energy-dependent metric for gravitation. Sect. 4 describes the proposed test of the local Lorentz invariance, based on the detection of a voltage induced by the stationary magnetic field of a coil, and the preliminary positive evidence for such an effect provided by a first experimental run. In sect. 5 we put forward a possible connection between the results of the two previous sections, and discuss its physical implications.

2 Deformed Minkowski space and metric description of interactions

Let us shortly review the main ideas and results concerning the (four-dimensional) "deformed" Minkowski space-time.

The four-dimensional "deformed" metric scheme introduced in [1-6] is based on the assumption that space-time, in a preferred frame which is *fixed* by the scale of energy E , is endowed with a metric of the form

$$ds^2 = b_0^2(E)c^2dt^2 - b_1^2(E)dx^2 - b_2^2(E)dy^2 - b_3^2(E)dz^2 = \eta_{\mu\nu}(E)dx^\mu dx^\nu;$$
$$\eta_{\mu\nu}(E) = diag\left(b_0^2(E), -b_1^2(E), -b_2^2(E), -b_3^2(E)\right), \tag{1}$$

with $x^\mu = (x^0, x^1, x^2, x^3) = (ct, x, y, z)$, c being the usual speed of light in vacuum. We named "*Deformed Special Relativity*" (DSR) the relativity theory built up on metric (1)[(6)].

Although uncommon, the use of an energy-dependent space-time metric is not new. It can be traced back to Einstein himself, just in connection with the problem of clock behavior in a gravitational field. In fact, in order to account for the modified clock rate due to gravity, Einstein was the first to generalize the usual special-relativistic interval by introducing a "time curvature" as follows[(11)]:

$$ds^2 = \left(1 + \frac{2\phi}{c^2}\right)c^2dt^2 - dx^2 - dy^2 - dz^2, \tag{2}$$

where ϕ is the Newtonian gravitational potential. This expression of the gravitational metric is now known to be valid in the Newtonian (weak field) approximation.

The metric (1) is supposed to hold locally, i.e. in the space-time region where the process occurs. It is supposed moreover to play a *dynamical* role, and to provide a geometric description of the interaction, in the sense that each interaction produces its own metric, through different specializations of the parameters $b_\mu(E)$. We notice explicitly that the space-time described by (1) is flat (it has zero four-dimensional curvature), so that the geometrical description of the fundamental interactions based on it differs from the general relativistic one (whence the name "deformation" used to characterize such a situation). Although for each interaction the corresponding metric reduces to the Minkowskian one for a suitable value of the energy E_0 (which is characteristic of the interaction considered), the energy of the process is fixed and cannot be changed at will. Thus, in spite of the fact that *formally* it would be possible to recover Minkowski space by a suitable change of coordinates (e.g. by a rescaling), this would amount, in such a framework, to be a mere mathematical operation devoid of any physical meaning. Actually, it can be shown that the physics of the interaction lies in the curvature of a five-dimensional space-time, with energy as fifth dimension, in which the four-dimensional, deformed Minkowski space is embedded[(10)]. Let us stress that this result is in agreement with the approach by Jackiw[(14)] to the breakdown of Lorentz invariance, based on the introduction of an additional field in the usual electromagnetic Lagrangian. Indeed, adding new fields amounts, on many respects, to adding new dimensions to space-time.

Among the kinematical implications of metric (1), let us mention the existence, inside the deformed Minkowski space, of an upper limit of propagation of signals, i.e. *a maximal causal speed*, given by

$$u_k = \frac{b_0}{b_k} c \tag{3}$$

which is characteristic of a given interaction (and/or physical process), a priori different for different spatial directions, and different, in general, from the light speed in vacuum[(5)]. It can be regarded as the speed of propagation of the interaction considered (i.e. the speed of the corresponding quanta which mediate it). Let us notice that the possible theoretical existence of upper speeds different from the light one traces back to the early 70's [(15)], and is present in other generalizations of Special Relativity[1] and in recently discussed models of Lorentz-noninvariant effects (the *"maximum attainable speed"* of Coleman and Glashow[(16)]).

Moreover, the following formula for time dilation holds in DSR:

$$d\tau = \sqrt{\eta_{00}}dt = b_0(E)dt, \tag{4}$$

where τ denotes the proper time.

As far as phenomenology is concerned, it is important to recall that a local breakdown of Lorentz invariance may be envisaged for all the four fundamental interactions (electromagnetic, weak, strong and gravitational) whereby *one gets evidence for a departure of the space-time*

[1] A main account can be found in ref.[6].

metric from the Minkowskian one (in the energy range examined). The explicit functional form of the metric (1) for the first three interactions can be found in refs. [1-6]. To our present aims, we recall the following basic features of these energy-dependent phenomenological metrics:

1) Both the electromagnetic and the weak metric show the same functional behavior, namely[(4−6)]

$$\eta(E) = diag\left(1, -b^2(E), -b^2(E), -b^2(E)\right); \tag{5}$$

$$b^2(E) = \begin{cases} (E/E_0)^{1/3}, & 0 \leq E < E_0 \\ 1, & E_0 \leq E \end{cases} \tag{6}$$

with the only difference between them being the threshold energy E_0 , the energy value at which the metric parameters are constant, i.e. the metric becomes Minkowskian ($\eta_{\mu\nu}(E \geq E_0) \equiv g_{\mu\nu} = diag(1, -1, -1, -1)$); the fits to the experimental data yield

$$\begin{aligned} E_{0el} &= 4.5 \pm 0.2 \mu eV \;; \\ E_{0w} &= 80.4 \pm 0.2 GeV; \end{aligned} \tag{7}$$

Notice that for either interaction the metric is isochronous, spatially isotropic and *"sub-Minkowskian"*, i.e. it approaches the Minkowskian limit from below (for $E < E_0$). Both metrics are therefore Minkowskian for $E > E_{0w} \simeq 80 GeV$, and then our formalism is fully consistent with electroweak unification, which occurs at an energy scale $\sim 100 GeV$.

2) For strong interactions, the metric reads[(2,6)]:

$$\eta(E) = diag\left(b_0^2(E), -b_1^2(E), -b_2^2(E), -b_3^2(E)\right); \tag{8}$$

$$b_0^2(E) = b_3^2(E) = \begin{cases} 1, & 0 \leq E \leq E_{0s} \\ (E/E_{0s})^2, & E_{0s} < E \end{cases} \;. \tag{9}$$

with

$$E_{0s} = 367.5 \pm 0.4 GeV \;. \tag{10}$$

Let us stress that, in this case, contrarily to the electromagnetic and the weak ones, *a deformation of the time coordinate occurs* (whence the lack of isochrony); moreover, *the three-space is anisotropic* , with two spatial parameters constant (but different in value) and the third one variable with energy in an *"over-Minkowskian"* way[2,6] (namely, it approaches the Minkowskian limit from above ($E_{0s} < E$)).

We want also to stress that a still open problem, from a theoretical point of view, is represented by the description of processes that are ruled by more interactions at the same time. Of course, one has first of all to check explicitly if the energy E_p of the process considered lies in the Minkowskian region for some of the interactions involved (i.e $E_p > E_{0i}$, $i = el, w$, or $E_p < E_{0i}$, $i = s$). There is obviously no difficulty if only one of the interactions involved exhibits a non-Minkowskian behaviour at the process energy. Otherwise, it is expected that a superposition of the metrics describes the total effect of the involved deformation of space-time. We will deal with this topic on a formal basis in our future work.

3 Energy-dependent metric for gravitation

3.1 Analysis of the experimental data on clock rates

Let us derive the form of the gravitational metric by fitting the experimental data on clock rates (a more detailed discussion will be given elsewhere) obtained by Alley[11] comparing (by short pulses of laser light) the rate of cesium beam atomic clocks, raised to a higher gravitational potential by an aircraft, with the rate of similar clocks on the ground. Alley and co-workers verified to about 1.5% the validity of the time-dilation formula derived from the metric (2) for a moving clock, i.e.

$$\Delta\tau = \Delta t\sqrt{1+\frac{2\phi}{c^2}-\frac{v^2}{c^2}}, \tag{11}$$

where v is the clock speed. They, therefore, utilized the data for a *line* measurement. On the contrary, we will use them for a *spectrum* analysis, due to the fact that we are interested in deriving the expression of the time metric parameter b_0 as a function of the gravitational energy E . The experimental data can be found in ref.[17].

The difference in elapsed proper time between the clock on the aircraft and the clock on the ground is

$$\Delta T = \sqrt{\eta_{00}}\Delta\tau - \Delta\tau = \Delta\tau(\sqrt{\eta_{00}} - 1) \tag{12}$$

or, on account of eq.(4):

$$\Delta T = \Delta\tau(b_0 - 1). \tag{13}$$

We take, for the test time parameter, the ansatz

$$b_0(E) = \left[1 + \left(\frac{E}{E_0}\right)^n\right] \tag{14}$$

on the basis of an assumed analogy between the strong and the gravitational metric(6); the factor 1 follows from the requirement $\Delta T = 0$ for $E = 0$. Therefore, eq. (13) becomes

$$\frac{\Delta T}{\Delta \tau} = \left(\frac{E}{E_0}\right)^n \tag{15}$$

Moreover, we assume

$$E \equiv E(q) \tag{16}$$

where q is the aircraft quote over Earth surface in Alley's experiment. Since it must be $E = 0$ for $q = 0$, we can put

$$E(q) \propto q^k. \tag{17}$$

The most natural hypothesis is postulating for the energy the form

$$E = \varepsilon \left(\frac{g}{c^2}\right) q \tag{18}$$

where g is the gravity acceleration (for q in feet and ε in eV, $\frac{g}{c^2} = 3.324 \times 10^{-17} (\text{feet})^{-1}$) and ε is an energy characteristic of the system considered. This agrees with the standard form of the energy of a body of mass m at height q in the Earth gravitational field , $E = mgq$, if we identify ε with the rest energy of the body, $\varepsilon = mc^2$.

The most natural choice for ε (which, in our case, represents a characteristic energy of the time measuring device) is the energy gap $h\nu$ of the Cs^{133} hyperfine transition ($h = 4.136 \times 10^{-15} eV \cdot \sec$; $\nu = 9.193 \times 10^9 \sec^{-1}$), because it provides the time standard (which triggers the time-interval measurements, amplified by the atomic clock device). So, eq.(18) becomes

$$E = \frac{h\nu}{c^2} gq \tag{19}$$

Thus, eq. (15) takes the form

$$\frac{\Delta T_{grav}}{\Delta \tau} = \left[h\nu_0 \left(\frac{g}{c^2}\right) \left(\frac{1}{E_0}\right) q \right]^n \tag{20}$$

where $\Delta\tau = 1$ day$= 8.640 \times 10^{13} ns$, and the suffix "*grav*" means that the time dilation is due *only* to gravitational effects.

Actually, the experimental data on ΔT do contain a term of kinetic origin ΔT_{kin} , which must be subtracted out according to the formula (see fig. 47 of first ref. [11]):

$$\Delta T_{grav} = \Delta T + \Delta T_{kin} \tag{21}$$

with

$$\Delta T_{kin} = \left(\frac{v_0}{c}\right)^2 \frac{\Delta \tau}{2} \tag{22}$$

With this proviso, we can rewrite eq.(24) as

$$\frac{\Delta T_{grav}}{\Delta \tau} = A^n q^n \tag{23}$$

with

$$A = h\nu_0 \left(\frac{g}{c^2}\right) \left(\frac{1}{E_0}\right) . \tag{24}$$

Putting $y = \dfrac{\Delta T_{grav}}{\Delta \tau}$, $x = q$, we get therefore the fit function

$$y = A^n x^n \tag{25}$$

The result of the fit is shown in Fig.1. The values of the parameters n , A obtained by the fit are

$$\begin{gathered} n = 0.9375 \pm 0.0047; \\ A^n = (54.000 \pm 1.149) \times 10^{-4} (feet)^{-n}, \end{gathered} \tag{26}$$

with $R^2 = 0.9519$.

3.2 Discussion of the results

Let us discuss the physical consequences of the fit we carried out.

According to the results of the fit, we can conclude that the energy-dependent time coefficient of the gravitational metric reads (cf. eqs. (14),(15))

$$b_0(E) = 1 + \frac{E}{E_0} \tag{27}$$

where the gravitational energy threshold E_0 is given by (cf. eq. (28))

$$E_0 = E_{0grav} = \frac{1}{A} h\nu_0 \left(\frac{g}{c^2}\right). \tag{28}$$

With the value of A obtained by the fit, we have

$$E_{0grav} = 20.2 \pm 0.1 \mu eV. \tag{29}$$

Intriguingly enough, E_{0grav} is approximately of the same order of magnitude of the thermal energy corresponding to the $2.7^o K$ cosmic background radiation in the Universe.

The value (37) is to be compared with the values of the threshold energies for the electromagnetic, weak and strong interactions, given by eqs. (7),(10). We have

$$E_{0el} < E_{0grav} < E_{0w} < E_{0s} \tag{30}$$

i.e. an increasing arrangement of E_0 from the electromagnetic to the strong interaction. Moreover

$$\frac{E_{0grav}}{E_{oel}} = 4.49 \pm 0.02 \quad ; \quad \frac{E_{0s}}{E_{0w}} = 4.57 \pm 0.01, \tag{31}$$

namely

$$\frac{E_{0grav}}{E_{oel}} \simeq \frac{E_{0s}}{E_{0w}} \tag{32}$$

an intriguing result indeed.

The time-dilation formula corresponding to the time coefficient (35) reads now (cf. eq.(4))

$$d\tau = \left(1 + \frac{E}{E_0}\right) dt \tag{33}$$

at variance with the Einsteinian one (derived from metric (2))

$$d\tau = \left(1 + \frac{E}{E_0}\right)^{1/2} dt \tag{34}$$

As to the explicit form of the *spatial* part of the gravitational metric (on which the experimental data do not provide any information), two possibilities are open:

1) *the spatial, 3-dimensional metric is Euclidean*, i.e. $b_1 = b_2 = b_3 = 1$; we have therefore

$$\eta_{grav} = diag\left[\left(1 + \frac{E}{E_{0grav}}\right)^2, -1, -1, -1\right]. \tag{35}$$

2) *the spatial metric is anisotropic and energy-dependent*, i.e. the 4-dimensional metric has a structure similar to the strong one (8),(9), namely

$$\eta_{grav} = diag\left[\left(1 + \frac{E}{E_{0grav}}\right)^2, -b_1^2, -b_2^2, -\left(1 + \frac{E}{E_{0grav}}\right)^2\right] \tag{36}$$

with, in general, $b_1^2 \neq b_2^2$ [2].

As a final remark, we want to stress that both metrics (43) and (45) can be derived as exact solutions of the five-dimensional Einstein equations in the framework of the Kaluza-Klein-like scheme (with energy as fifth dimension) introduced in ref. [10].

4 Evidence for a voltage induced by a stationary magnetic field: A possible LLI breaking effect

As already stressed in the Introduction, the analysis of some physical processes, carried out by means of the DSR formalism, seems *to provide*

[2] Let us stress that the possible spatial anisotropy of the gravitational metric is a mere conjecture, because, as already emphasized above, the experimental data do not provide any information about the other metric coefficients. At present the only experimental evidence for a space anisotropy concerns the strong metric (see eqs.(8),(9)), and therefore the length scale involved is $\sim 10Fm$.

a first (although preliminary)*, indirect evidence for a breakdown of local Lorentz invariance for all fundamental interactions.*

Quite recently the present authors, together with U. Bartocci[12], proposed a new electromagnetic experiment aimed to testing LLI and able to providing *direct evidence* for its breakdown. The results obtained in a first, preliminary experimental run carried out in June 1998 - essentially aimed to provide new upper limits to the LLI breakdown parameter by an entirely new class of electromagnetic experiments - admit as the most natural interpretation the fact that *local Lorentz invariance is in fact broken*[12,13].

Let us give a brief account of such experiment, which, in our opinion, allows us to establish an intriguing connection between the electromagnetic and the gravitational metrics.

The test is based on the possibility of detecting a non-zero Lorentz force between the magnetic field **B** generated by a stationary current I circulating in a closed loop γ, and a charge q, in the hypothesis that both q and γ are at rest in the same inertial reference frame. Such a force is zero, according to the standard (relativistic) electrodynamics.

The experimental setup was devised in order to put new upper limits on the breakdown of LLI, by means of such an entirely new class of electromagnetic experiments, and also to test possible anisotropic effects in such limits.

The experimental device used is schematically depicted in Fig.2. It consisted of a solenoid γ and a Cu conductor R placed inside it on a plane orthogonal to the γ axis. The conductor R was connected in series to a capacitor C, and a voltmeter was connected in parallel to the capacitor, so to measure the voltage due to a possible gradient of charge across R. The conductor could change its orientation in the plane from 0 to 2π. Moreover, the whole system of the RC circuit and the solenoid could turn so letting its plane coincide with one of the coordinate planes. The coordinate system was chosen as follows: the (x, y) plane tangent to the Earth surface, with the y-axis directed as the (local) Earth magnetic field $\mathbf{B}_T$; the z-axis directed as the outgoing normal to the Earth surface, and the x-axis directed so that the coordinate system is left-handed. A stationary current I circulating in the solenoid generated a stationary magnetic field **B** in which the RC circuit is embedded. The circuit and the solenoid were mutually at rest in the laboratory frame.

Measurements of the voltage V across the capacitor were carried out for the system lying in the different coordinate planes (x, y) , (x, z), (y, z) , and at different values of the orientation angle α of the circuit in the plane considered (spaced by $\pi/4$). The orientation of the coil γ and the verse of the current I were chosen so that, when γ lies on (x, y) , its magnetic field $\mathbf{B}$ is directed as z ; when γ is on (y, z), $\mathbf{B}$ is directed as x; for γ on (x, z), $\mathbf{B}$ is directed as $\mathbf{B}_T$. The last arrangement of the apparatus is shown in Fig. 3 .

The measurement runs were carried out in three different days (each day with a different orientation of the apparatus plane), two times a day. Every run consisted of five measurements of the voltage taken at the same orientation angle α , for eight values of $\alpha \in (0, 2\pi)$. For a fixed angle, the five measurements of V were taken at time intervals of 60 sec from each other.

Measurement of the zero level of the voltmeter fixed such a level to the value $V_0 = (0.015 \pm 0.010) mvolt$ (notice the pessimistic evaluation of the error). The measured values of the voltage V were assumed to represent a physically acceptable, non-zero signal *only if external* to the above interval. Clearly, this permitted to get rid of (at least most of) the fluctuation contributions and other spurious effects connected with the background .

The measurements performed with the system lying on the planes (x, y) and (y, z) gave values of V compatible with the instrument zero. Indeed, in such cases the statistical tests of correlation showed that each of the points[3] outside the zero-voltage band is uncorrelated with the preceding and the subsequent point either, and the whole set of points was shown to be uncorrelated ($R^2 < 30\%$). As to the measurements in the plane (x, z) , it was shown instead that *the four points outside the zero band are statistically correlated* ($R^2 > 80\%$) *, and so they represent a valid candidate for a non-zero signal.*

A polynomial interpolating curve for these points is shown in Fig.4. Such an interpolating procedure was essentially aimed at finding the angle $\alpha_{\max}$ corresponding to the maximum value of V , $V_{\max}^{xz} = (3.6 \pm 1.0) \times 10^{-5} volt$. The value found was $\alpha_{\max} = 3.757 rad$. The knowledge of $\alpha_{\max}$ is needed in order to determine the value of the anisotropic LLI violation parameter in our case[(12)] .

[3]Let us recall that - as streessed before - each point is the average of five measurements, taken at the same angle.

A detailed discussion of possible spurious effects which could simulate the results obtained, and of the precautions taken to avoid false signals, can be found in refs. [12,13].

The experiment was repeated in the summer of 1999 in a different place, with a different apparatus and with a sensitivity improved by two orders of magnitude. The analysis of the data is being carried out, and *it seems to confirm the positive evidence for the effect* (at least as far as the plane xz is concerned).

We want to stress that the estimated amount of breakdown of LLI ensuing from our experiments is in agreement with the existing limits(18). A detailed discussion of this point is given in ref.[13]. Here, we confine ourselves to summarize the main results.

We recall that two different kinds of LLI violation parameters exist: The isotropic (essentially obtained by means of experiments based on the propagation of e.m. waves, e.g. of the Michelson-Morley type), and the anisotropic ones (obtained via experiments of the Hughes-Drever type(18), which test the isotropy of the nuclear levels). The smallest upper limit obtained in the former case is(18) $\delta < 10^{-8}$, whereas the upper limits on the anisotropic parameter range from $\delta < 10^{-18}$ of the HD experiment to $\delta < 10^{-27}$ of the Washington experiment(18). In either case, one has to consider, for the evaluation of δ , a *phenomenological LLI invariance breakdown speed* v (e.g., the speed of a hypothetical preferred frame), such that the new speed of light is $u = c + v$ [4].

In our framework, an effective LLI breakdown speed v can be introduced, by defining it as the relative speed between the coil γ and the conductor R, needed to provide, via the Lorentz force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, the maximum measured voltage across R. A lengthy but straightforward calculation yields (for $V = V^{xz}_{\max} = 3.8\times10^{-5} volt)^{(12,13)}$ $v \simeq 6\times10^{-2} m/\sec$.

Then, it is possible to show that the isotropic LLI parameter corresponding to our effect has the value(13) $\delta \simeq 4 \times 10^{-10}$, which is lower by two orders of magnitude than the upper limit for the isotropic case. In the anisotropic case, the parameter δ is in the range(13) $2\times10^{-29} < \delta < 6\times10^{-20}$, and therefore compatible with the anisotropic upper limits .

[4]Notice that u is nothing but the "*maximal causal speed*" of the electromagnetic interaction, in Deformed Special Relativity(6) , or the "*maximum attainable speed*", by Coleman and Glashow(16).

5 Discussion and conclusions

We want now to show that these two experimental facts we discussed, seemingly completely uncorrelated, have actually intriguing connections, if just viewed from the point of view of the DSR formalism.

The first question to be answered is: Why is the effect observed in the e.m. test of Lorentz invariance anisotropic? Indeed, a valid signal was observed only in the plane *xz.* In our opinion, this is strictly related to the fact *a preferred direction actually does exist in the experiment*, namely that of the gravitational field. Indeed, due to the low values of the energies involved (of the order of $10\mu eV$) gravitational contributions can *a priori* no longer be neglected.

From the standpoint of DSR, this implies that two energy-dependent metrics are involved, the electromagnetic and the gravitational one. This hypothesis allows us to provide an interesting interpretation of the instrumental zero, which we recall is $V_0 \simeq 15\mu volt$ (in the second experiment, we found $V_0 = (16.5 \pm 1.5)\,\mu volt$), and *seems actually to be independent of the instrument.* Namely, we conjecture that *the instrumental zero is related to the energy interval ΔE in which both the electromagnetic and the gravitational metric are fully minkowskian* (and therefore no breakdown of Lorentz invariance occurs). We recall (see ref. [6] and Sect.2) that the energy-dependent electromagnetic metric is sub-minkowskian (i.e. it approaches the Minkowski metric from below), with threshold energy $E_{0el} \simeq 5\mu eV$ (cfr. eq.(7))[5], whereas the gravitational metric is over-minkowskian (i.e. it approaches the Minkowski metric from above), with threshold energy $E_{0grav} \simeq 20\mu eV$ (cfr. eq.(29)). The energy interval of minkowskian behaviour for either metric is therefore

$$\Delta E = E_{0grav} - E_{0el} \simeq 15\mu eV \tag{37}$$

in surprising agreement with the experimental value of the instrumental zero V_0.

This interpretation - which would support the validity of DSR, at least as far as the electromagnetic and the gravitational interactions are concerned - can find confirmation by the just performed repetition of the experimental test of LLI. Indeed, the improved sensitivity of the apparatus might enable us to determine if a valid signal is present in the

[5]Let us recall that the explicit form (5),(6) of the energy-dependent electromagnetic metric, and the corresponding value of E_{0el}, were derived[4,6] from the analysis of the Cologne experimental data on superluminal tunneling in waveguides (see first ref.[9]).

other two planes, too. If so, *we expect a difference in signal between the vertical planes and the horizontal one, xy, of the order of the minimal energy required for gravitational breakdown of LLI in the DSR framework*, namely $E_{0grav} \simeq 20\mu eV$.

A last remark concerns the possible pattern of interactions ensuing from DSR. According to the results of sect.2 and subsect.3.2, we have two pairs of interactions: i) electromagnetic and gravitational; ii) weak and strong , ordered by the increasing arrangement of the threshold energies (see eq.(30)). Moreover, in each pair the former interaction is sub-Minkowskian, and the latter is over-Minkowskian. The first question is: Does this pattern end with the second pair, or not? If a third pair exists, we can assume that the threshold energies of the new pair, E_{05} and E_{06}, are related to the threshold energies of the previous sub-Minkowskian and over-Minkowskian metrics according to

$$\frac{E_{0,n+2}}{E_{o,n}} = \frac{E_{0,n+4}}{E_{0,n+2}}, \qquad n = 1, 2 \tag{38}$$

(with $E_{0el} = E_{01}$; $E_{0grav} = E_{02}$; $E_{0w} = E_{03}$; $E_{0s} = E_{04}$). In such hypothesis, with the values (7), (10), (29) of the threshold energies for the known interactions, we get

$$\begin{aligned} E_{05} &\simeq 1.3 \times 10^{18} GeV; \\ E_{06} &\simeq 6.7 \times 10^{18} GeV. \end{aligned} \tag{39}$$

Such a pattern may repeat itself again, or not, and it's of course a matter of experiment to check the real existence of these new pairs of interactions. What we exclude is that it repeats *ad infinitum*. In this connection, we recall that it was shown that the maximum possible force in Nature is provided by the *Kostro constant* K, given by[(19)]

$$K = \frac{c^4}{G} = 7.556 \times 10^{51} GeV/cm \tag{40}$$

where G is the gravitational constant ($G = 1.072 \times 10^{-10} cm^5 / \left[GeV \sec^4\right]$). The corresponding maximum energy, i.e. the energy of the whole Universe, is therefore

$$E_{\max} = KR_0 \sim 10^{80} GeV \tag{41}$$

where we take $R_0 \sim 10^{11} light - years$ for the observed Universe radius [6].

Either in the case of new interactions (besides the known ones) or not, we deem that the interaction pattern in the DSR scheme is bounded from above at least by the value $E_{\max}$ (41) related to the Kostro limit. This holds, in particular, for the asymptotic behaviour of the over-Minkowskian metrics.

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[6]However, let us notice that, according to the inflationary theory, the real estimated size of the Universe is many orders of magnitude greater than R_0 (which is actually the horizon radius, i.e. the distance travelled by light since the big bang). Therefore, the value of $E_{\max}$ is to be regarded only as a lower limit for the maximum energy.

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Figures

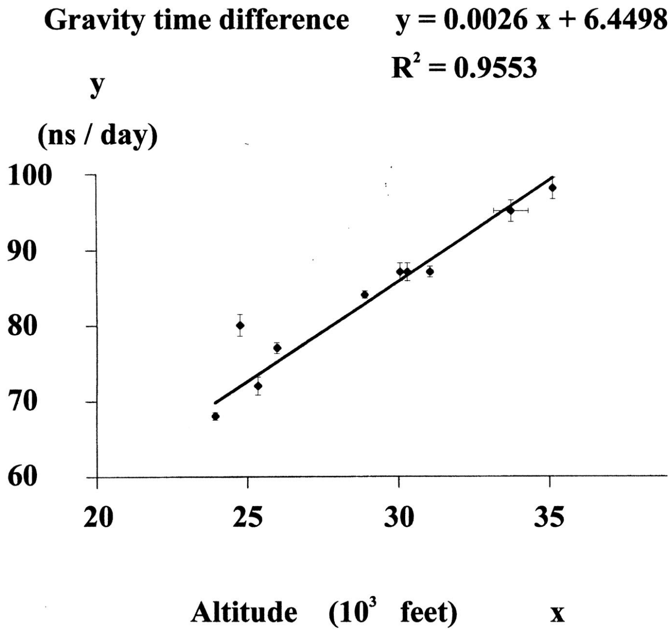

Figure 1. Fit to Alley's experimental data by the fit function $y = A^n x^n$. $R^2 = 0.9519$ is the correlation coefficient. See the text.

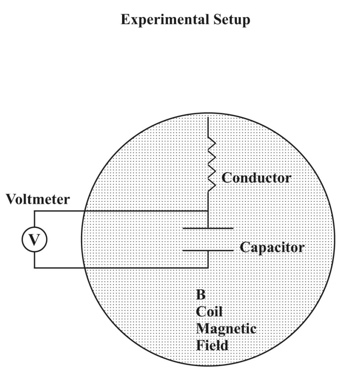

Figure 2. Schematic setup of the Helmholz coil experiment. See the text.

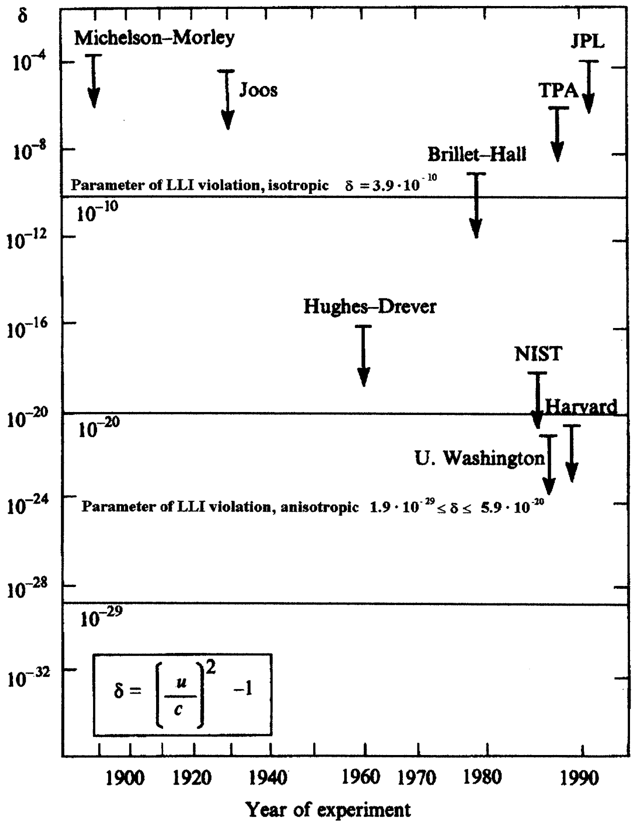

Figure 3. Schematic view of the orientation of the apparatus

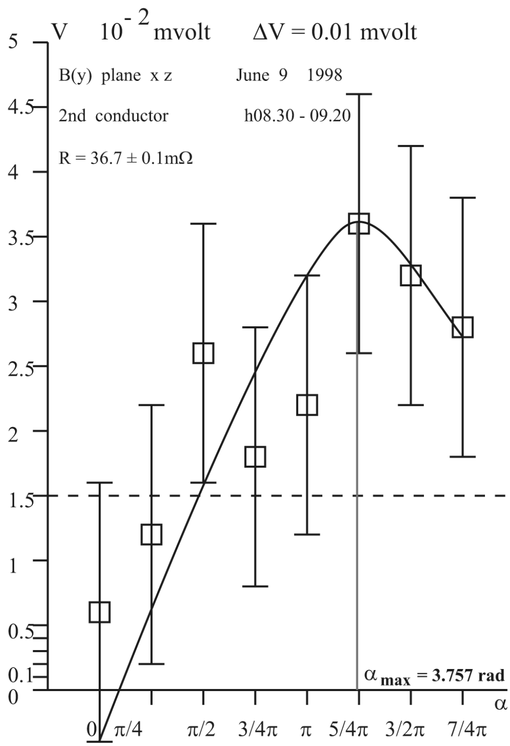

Figure 4. Curve interpolating the data obtained with the apparatus in the $(x,\ z)$ plane, showing the angle of maximum signal $\alpha_{\max} = 3.757 rad$.

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