

On the foundations of electromagnetism

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ABSTRACT. The laws of classical and quantal electromagnetisms are presented here as based on principles whose data are only the isotropic propagation of the electromagnetic action and the Coulomb law. The Maxwell laws are then obtained -via the deduction from these principles of Lorentz's integral formula of the retarded potentials- simply as a consequence.

RÉSUMÉ. Les lois de l'électromagnétisme classique et quantique sont présentées ici comme fondées sur des principes qui reposent sur les seules données que sont la propagation isotrope de l'action électromagnétique et la loi de Coulomb. Les lois de Maxwell s'en déduisent -via l'établissement à partir de ces principes de la formule intégrale des potentiels retardés de Lorentz- comme une simple conséquence.

1 Introduction

Nothing in the present survey, except perhaps the remark made above Eq. (14) in Sect. 3, may be considered as original. Some readers (we hope as many as possible) will consider all that follows, in particular the first part of the conclusion, as evident. Nevertheless we have thought that some points commonly accepted require to be discussed into detail.

At least, as it is said in the Abstract, what is presented everywhere as the foundation of electromagnetism is relegated here to the place, almost accessory, of the consequence of much more simpler principles.

The laws of electromagnetism are presently considered as based on the Maxwell laws in the form of the equation

$$\partial^\nu \partial_\nu A^\mu = 4\pi j^\mu \quad (1)$$

established with the help of the relations

$$\partial_\mu A^\mu = 0, \quad \partial_\mu j^\mu = 0 \quad (2)$$

i.e. the Lorentz condition on the potential A^μ and the conservation of the charge current j^μ .

The current j^μ , source of the potential A^μ , may be considered as constructed by considering a population made (a) in classical theory, of distinct charges or (b) in quantum theory, of the eventualities of presence of an unic charge. Furthermore, the quantization of the field, when one thinks that it is usefull to use it, implies a new object, the Planck constant \hbar , just when \hbar intervenes in the source. But in no case this theory may be used in contradiction with the Maxwell laws.

These laws were established during the 19th century, actually, only by means of considerations on the fields created by free electrons moving inside moving wires. It seems a bit miraculous that these laws, whose origin is macroscopic and classical, can be applied exactly to the microscopic theory of the electrons bound in atoms, with a probability concept about the source opposite to the deterministic one of the classical theory. That leads to think that the substratum of the Maxwell laws is made of some entity which may be considered as well from a quantal as from a classical point of view.

The importance of Maxwell-Lorentz's laws is here absolutely not contested. These laws have been at the origin of the invention of the Relativity. They lead to concepts (Maxwell tensor S , Poynting vector, ...) and properties (as the relation implying S and the Lorentz force density) which are essential for the study of electromagnetism. (A smart presentation of these concepts and properties is made in [1], Part II). Furthermore an explanation of the "useless part" $\epsilon^{\eta\lambda\mu\nu}\partial_\lambda F_{\mu\nu} = 0$ of the Maxwell laws may perhaps lead to the invention of new physical objects. At last, because they are based on the notion of current, and that the notion of probability current is a necessity in quantum mechanics, these laws are indispensable to the application of electromagnetism to this part of physics.

2 The Lorentz integral formula of the retarded potentials

Especially in quantum mechanics (in explanation of the light emitted by atoms, Lamb shift calculation ..) one often uses, instead of the Maxwell

laws, the Lorentz formula of the retarded potentials

$$A^\mu(x^0, x^k) = \int \int \int_V \frac{j^\mu(x^0 - R, a^k)}{R} d\tau, \quad R = [\sum_k (x^k - a^k)^2]^{1/2} \quad (3)$$

In this formula, x^μ and a^μ are the coordinates, in a orthonormal galilean frame $\{e_\mu\}$ of the Minkowski space $M = R(1, 3)$, of the points X and P of M where A^μ and j^μ are considered. V is the volume in the space $E(e_0)$ orthogonal to e_0 which is the orthogonal projection upon $E(e_0)$ of the domain Ω of M containing all the charges which are the source of the potential A^μ .

What is surprising in this formula is the fact that the right hand side of the formula is to be considered as relativistically invariant *whatever the frame $\{e_\mu\}$ may be*.

This formula is presented as deduced from the Maxwell equation (1), but the proofs are not convincing. For example, in [2], Sect.I-1, it is simply said that Eq. (3) may be *easily* deduced from Eq. (1) (I presume that the use of the word "easily" is a joke). In [3], Sect. 6.6, one uses the relativistic Green function but it is applied to the components j^μ of the current, *taken separately* and that cannot prove the invariance (see Eq. (5)). In [4], Sect. XXXII-III, the proof is based on the integration of a partial derivative equation. But it implies an hypothesis of a symmetry on the distribution of the charges, around the point P , which is pointed out with honesty by the author as not very compatible (see also Sect. 3) with the equation (2) of the current conservation. Furthermore as in [3] the proof is relative to the components j^μ taken separately.

3 Invariance of the formula of the retarded potentials

The fact that Eq. (3) might be correctly deduced from Eq. (1) do imply that the right hand side of (3) is invariant as its left hand side, and also as the right hand side of (1). We are going to achieve a direct proof of this invariance but in taking into account this indisputable principle:

All electromagnetic action is propagated at the light velocity.

We apologize for recalling this truism, but it seems that it has not been taken entirely into account (see the question above Eq. (14)) in the presentation of electromagnetism. It means that *all the charge matter contributing to the potential at the point X is situated, in the past of X , inside the isotropic hypercone $H(X)$ whose top is X* . But we will see

that this invariance implies a strong constraint on the definition of the charge current j^μ .

Notations. For simplicity we will write $a.b = a^\mu b_\mu$ (and $a^2 = a.a$) for the scalar product of two vectors $a, b \in M$, $a \wedge b$ for their Grassmann product (whose components are $a^\mu b^\nu - a^\nu b^\mu$) or (simple) bivector. The interior product $F.c$ on the right of a bivector F (or antisymmetric tensor of rank two) by a vector $c \in M$ will be defined by the relation $(a \wedge b).c = (c.b)a - (c.a)b$. We will write $\partial = e^\mu \partial_\mu$ for the gradient operator of M and $E(v)$ for the three dimensional space orthogonal to a timelike vector $v \in M$ (such that $v^2 > 0$).

The isotropic vector \overrightarrow{PX} is written in the frame $\{e_\mu\}$

$$\overrightarrow{PX} = R(e_0 + N), \quad N \in M, \quad N.e_0 = 0, \quad N^2 = -e_0^2 = -1, \quad R = x^0 - a^0 \quad (4)$$

Let us write Eq. (3) in the schematic form

$$A(X) = \sum_V j(P) \frac{d\tau}{R} \quad (5)$$

in which the integral appears as a sum of objects eachone necessarily invariant. We have replaced the symbol $\int \int \int$ by \sum because the right hand side of Eq. (5) will be replaced by a sum of invariant objects situated in the domain Ω of M which contains the charges contributing to the value of $A(X)$ and because this last sum will be able to be considered also as a discrete sum instead of an integral.

A striking point of our proof will be the fact that, by application of the above fundamental principle, Ω must be situated inside the hypercone $H(X)$.

Because $j(P)$ is invariant, *the ratio $d\tau/R$ must be invariant.* On one side $d\tau$ is the measure of a part of the volume V which is the orthogonal projection upon $E(e_0)$ of a small neighbourhood ϖ in M of P . On the other, R may be related to an invariant length in the following way.

The charge current j may be written

$$j = \rho v, \quad v \in M, \quad v^2 = 1, \quad \rho > 0 \quad (6)$$

where the charge density ρ and the vector v are invariant. One can write

$$\overrightarrow{PX} = r(v + n), \quad n \in M, \quad n.v = 0, \quad n^2 = -v^2 = -1 \quad (7)$$

which shows that the length $r = \overrightarrow{P'X}.v$ is invariant as a scalar product of two invariant vectors of M . The comparison between Eqs. (4) and (7) leads to the relations

$$v = \alpha e_0 + \beta N, \quad n = \beta e_0 + \alpha N, \quad \alpha^2 - \beta^2 = 1 \quad (8)$$

$$\overrightarrow{P'X} = r(v+n) = r(\alpha+\beta)(e_0+N) = R(e_0+N), \quad \Rightarrow \quad R = r(\alpha+\beta) \quad (9)$$

The number $\alpha + \beta$ is appeared in the passage from the *arbitrary* frame $\{e_\mu\}$ to the *privileged* frame $L(P)$, associated with the current at the point P , whose vector time is v . This number *must be eliminated, because the frame $\{e_\mu\}$ is arbitrary*. So *it does appear in the expression of the volume $d\tau$* .

The use of the frame $L(P)$ requires a more precise definition of the density ρ . This number is the ratio

$$\rho = \frac{dq}{d\tau_0} \quad (10)$$

of the sum dq of the charges contained in the neighbourhood $\varpi \subset \Omega$ of P and the measure $d\tau_0$ of the orthogonal projection of ϖ upon the space $E(v)$. Eq. (5) becomes

$$A(X) = \sum_V dq \left[\frac{d\tau}{d\tau_0} \right] \frac{v}{R} \quad (11)$$

We have now to take into account the inclusion of ϖ into the hypercone $H(X)$. That may be done by considering ϖ as generated by an isotropic vector $\xi = d\ell(v+n)$ whose origin describes a portion of plane η orthogonal to v and n . If the size of η is small with respect to the length r , then the vector $\overrightarrow{P'X}$ may be considered as isotropic for any $P' \in \varpi$ and one can consider that $\varpi \subset H(X)$. η is situated inside $E(v)$, and since $-\xi.n = d\ell$, the measure of the projection of ϖ upon $E(v)$ is $d\tau_0 = d\sigma d\ell$, where $d\sigma$ is the area of η .

The passage from the frame $L(P)$ to $\{e_\mu\}$ keeps η unchanged. On the other side one can write, in the same way that in (9)

$$d\ell(v+n) = d\ell(\alpha+\beta)(e_0+N) = d\ell'(e_0+N), \quad d\ell' = d\ell(\alpha+\beta)$$

and

$$d\tau = d\sigma d\ell' = d\sigma d\ell(\alpha+\beta) = d\tau_0(\alpha+\beta) \quad \Rightarrow \quad \frac{d\tau}{d\tau_0} = \alpha+\beta \quad (12)$$

After elimination of $\alpha + \beta$, Eq. (11) becomes the relation

$$A(X) = \sum_{\Omega} dq \frac{v}{r} = \sum_{\Omega} dq \frac{v}{\overrightarrow{PX}.v} \quad (13)$$

where all the terms of the right hand side are relativistically invariant.

The summation may be considered as made by means of a partition of $\Omega \subset H(X)$ into elementary domains ϖ_i , eachone centered on a point P_i . Each ϖ_i contains a charge dq_i , and is associated with an unit timelike vector v_i .

But the current $j(P)$ is not defined in the Maxwell laws as depending on the point X . So it is independent, in these laws, of the vector n . The compatibility of these laws with an invariance of the retarded potentials formula, *coherent with the fundamental principle of the isotropic propagation of the electromagnetic action*, requires to suppose that the charge dq contained in the neighbourhood ϖ of P , defined above, is the same whatever the direction of the vector n may be. Is this property of local symetry of the repartition of the charges (distinct from the one used in [4], Sect. XXXII-III, but of the same nature) compatible with Eq. (2) of the conservation of the current? It is not sure. However this last equation is fundamental not only in the Maxwell laws but in the simple considerations on the behaviour of the source in spacetime, implying the fact that all accumulating somewhere of the charges is forbidden. It is a basic datum in quantum mechanics, in particular in the electron theory. A way to elude the difficulty would be to say that dq is *about* the same whatever the direction of the vector n may be.

On the other side, Eq. (13) is a *static* spacetime equation, in which the conservation of the current is necessarily absent. It expresses that $A(X)$ collectes the actions of all the charges placed inside the hypercone $H(X)$ in the domain Ω , by means of the number $1/r$, where the invariant lenght r is defined by the isotropic vector \overrightarrow{PX} , with $P \in \Omega$.

A *kinematical* evolution of the study of electromagnetism, related to the fact that the derivatives of the potential $A(X)$ are to be considered, call in the following question. *When one passes from X to $X + dX$, what happens to $A(X + dX)$, knowing that the new domain Ω' to consider is situated inside the hypercone $H(X + dX)$, and that $P + dP \in \Omega'$ must be in agreement with the relations*

$$\overrightarrow{PX}^2 = 0 \Rightarrow 2\overrightarrow{PX}.d(\overrightarrow{PX}) = 0 \Rightarrow \overrightarrow{PX}.dX = \overrightarrow{PX}.dP ? \quad (14)$$

We think that the fundamental principles of electromagnetism must precede this question, and lie in Eq. (13), whereas the Maxwell laws, which contain the double derivatives of A , must follow.

4 On the principles of electromagnetism

We propose a presentation of the principles of electromagnetism which is a simple prolongation to the geometry of spacetime of the Coulomb law.

Then these principles *will be based only on the two experimental data which are the propagation of the electromagnetic action at the light velocity, and the Coulomb law.* They will lead directly to the expression (13), then (11), then (3) of the potentials, then (see Sect. 5) to the Maxwell equation (1).

As a consequence *all the laws of electromagnetism -except the ones related to the gauge theories of the particles theory (see Sect. 6)- will be able to be considered as based upon these two experimental data.*

Before recalling the Coulomb law, we only mention the relation which is the link between electromagnetism and electrodynamics (see Sect. 6). The force (in its spacetime meaning) $f \in M$ which acts on a charge q' , whose spacetime velocity is w , subjected to an electromagnetic field F , is the interior product $f = q'F.w$, or Lorentz force.

The Coulomb law. If two charges q, q' , situated at points P and X of the spacetime M , are at rest in a galilean frame whose vector time is $u \in M$, the force f acting on the charge q' is defined par the bivector $q'F$ (F is the Coulomb field)

$$q'F = q'q \frac{n \wedge u}{r^2}, \quad \overrightarrow{PX} = r(u + n), \quad u^2 = 1 = -n^2, \quad u.n = 0 \quad (15)$$

where the vector $n \in M$ corresponds to the direction of the oriented straight line which joins the projections of P and X on the space $E(u)$. Because the spacetime velocity of the charge q' is here u one has

$$f = q'F.u = q'q \frac{n}{r^2} \quad (16)$$

We propose three fundamental principles.

P_1 . Let a punctual charge q , situated at a point P of the Minkowski spacetime M , be. One associates with q an unitary timelike vector u

(such that $u^2 = 1$). For all point $X \in M$, in the future of P and such that the vector \overrightarrow{PX} is isotropic, one defines the spacetime vector

$$A(X) = q \frac{u}{\overrightarrow{PX}.u} \quad (17)$$

called retarded potential created at X by the charge q situated at P .

P_2 . Le potentiel A created at a same point X by different charges q_i is the vector sum of their potentials A_i considered separately:

$$A(X) = \sum_i q_i \frac{u_i}{\overrightarrow{P_i X}.u_i} \quad (18)$$

P_3 . The product by the charge q' of a punctual charge situated at X by the bivector F

$$F = \partial \wedge A \quad (19)$$

which is the spacetime curl of A (considered by taking the derivatives of A with respect to the point X), or electromagnetic field at X derived from the potential A , determines the force f acting on the charge q' .

Let us verify that P_1 is in agreement with the Coulomb law. Suppose an unic charge q at P , at rest in a galilean frame whose spacetime vector is u (P describes then a straight line of M) and let $\{e_\mu\}$ be the orthonormal frame such that $e_0 = u$. In a point X such that

$$\overrightarrow{PX} = r(u + n), \quad n.u = 0, \quad n^2 = -u^2 = -1, \quad (u + n).u = 1$$

the potential is, because $\overrightarrow{PX}.u = r$,

$$A = q \frac{u}{r} \quad (20)$$

If X is at rest in the frame, the time coordinate x^0 of X does not intervene and, since $e^k = -e_k$, ($k = 1, 2, 3$), the operator ∂ expressed in spherical coordinates is reduced to

$$\partial = -n \frac{d}{dr}, \quad \text{and} \quad F = -qn \wedge u \frac{d}{dr} \left(\frac{1}{r} \right) = q \frac{n \wedge u}{r^2}$$

The expression (17) is analog to the Lienard and Wiechert potential which has been deduced from the integral formula of the retarded potentials, in the case where the charge is a small sphere whose center

describes a straight line. But here *we only associate to the charge a scalar q , a point P in spacetime, and an unit timelike vector u* . That may be applied to the case where, in classical mechanics, the charge describes a trajectory whose tangent vector at P is u , *as the one, in quantum mechanics, where P is the eventuality of presence of the charge q* .

In the case of a population composed of few charges, one deduces immediatly the expression (13) of the potential $A(X)$, where the symbol \sum corresponds to a discrete summation.

Let us consider the case of a numerous population of punctual charges such that, in a small neighbourhood ϖ of a point P , the vector u of each charge is about the same as an unic vector v . One can then associate to P the total charge dq included in ϖ and so obtain the expression (13) of $A(X)$, then the equivalent expression (11) associated with the choice of an arbitrary galilean frame. This construction is applicable as well, in classical electromagnetism, to a population of distinct charges, as, in quantal electromagnetism, to the population of eventualities of presence of an unic charge. dq corresponds then to the product of a constant charge (e for the electron) by a local presence probability.

Defining a charge density $\rho = dq/d\tau_0$, where dq is the charge contained in a small neighbourhood ϖ of P , one can introduce the notion of charge current $j = \rho v$ and obtain the formula (3) of the retarded potentials, but with the restrictive condition that, for a given measure $d\tau_0$ of the orthogonal projection of ϖ on $E(v)$, the shape of ϖ has no incidence (or a weak incidence) on the value of dq , and so that j is independent of the choice of the point X where the potential A is considered .

5 The Maxwell laws as consequence of the formula of the retarded potentials

If a charge q describes a curve (C) whose vector tangent at P is u , the calculation of $F = \partial \wedge A$ may be made (see [5], 1967-68 or 1967) by means of the relation of equiprojectivity $\overrightarrow{PX}.dX = \overrightarrow{PX}.dP$ (Eq. 14) which allows one to clearly define the derivatives upon the potential $A(X)$ with respect to the situation of the source. (Another calculation, using the Dirac distribution, can be made). Furthermore one can show that for any point $X \neq P$ the equations $\partial.A = 0$ and $\partial^2 A = 0$ (where $\partial^2 = \partial^\mu \partial_\mu$) are verified. As the conservation of the charge is, like an evidence, ensured, one can say that the Maxwell-Lorentz laws are in this case satisfied (except along the curve (C) where they have no sense). One

must emphasize that not only is the use of the relation of equiprojectivity a way to facilitate the calculation, but also it is the answer to the question asked before Eq. (14) in Sect. 3.

In the general case, one can deduce the Maxwell equation (1) from the formula (3) of the retarded potentials, by adding the current conservation equation $\partial \cdot j = 0$, but the following proof does not bring an answer to the above question.

If X is outside Ω the right hand side of Eq. (3) is a proper-Riemann integral and one can derivate under the sign \int . A simple calculation shows that the relation $\partial^2 A = 0$ is verified.

In the case where X belongs to Ω the integral becomes improper. One can associate with any point P of Ω a small neighbourhood ω of P , entirely situated inside Ω , and one can separate Ω in two parts $\Omega - \omega$ and ω , whose projections upon $E(e_0)$ are two volumes V_1 and V_2 . Let A_1, A_2 the potentials, created by the charges situated inside these two parts respectively, at a point X of ω , be.

(a) The integral (3) is proper-Riemann in V_1 and one can write

$$\partial^2 A_1 = 0$$

(b) the integral (3) becomes improper in V_2 .

Suppose ω sufficiently small in such a way that the advance of X with respect to P can be neglected. One can consider the potential A_2 as being the same as in electrostatic. If one uses at the point P the frame whose vector time is v , ∂^2 is replaced by the operator $-\Delta$, because the derivatives with respect to the time are not to be taken into account. One can apply the Poisson formula $-\Delta A_2^0 = 4\pi\rho$ of the newtonian potential ([6], p. 273-276), which allows one to replace an integral on a volume by a relation verified at each point. That gives in an *arbitrary* frame, for this neighbourhood ω of P , $\partial^2 A_2^\mu = 4\pi j^\mu$.

From $A = A_1 + A_2$ upon ω , one deduces that at each point X of ω one has $\partial^2 A = 4\pi j$. Repeating the reasoning for each point P of Ω , one deduces that for all point X inside Ω the relation

$$\partial^2 A = 4\pi j \tag{21}$$

i.e. the Maxwell equation, is verified.

Furthermore one can deduce (see [4], Sect. XXXII-III) from Eq. (3) that the Lorentz relation $\partial \cdot A = 0$ is verified if $\partial \cdot j = 0$ and if the charges remain confined to Ω .

However the hypothesis adopted to achieve this calculation (as those, habitually used to deduce Eq. (3) from Eq. (1)), are not entirely satisfactory. In particular the hypothesis of the existency of a domain ω around each point P , sufficiently small to allow one the use of the electrostatic properties, may be contested.

Note that the factor 4π which appears as associated with the current j in the Maxwell equation (1), has no reason to be used when the source is composed of distinct punctual charges not joined together in a current.

6 The Maxwell-Lorentz electromagnetism and the gauge potentials of the theory of particles

Note that, at no time, we have mentioned the photon. This object (although the word photon is often used about relations implying only the pure Maxwell-Lorentz theory, in particular the retarded potentials formula) obeys laws (as in the Compton effect) which belong to electrodynamics, and so are outside our subject.

However, we cannot end this survey without mentioning the gauge potentials, although these entities belong to electrodynamics and not to electromagnetism, because they are closely related to the Maxwell-Lorentz potentials.

Since the check, in the beginning of the 20th century, of the attempt of interpreting the mass m of the electron as being of an electromagnetic nature (see [2], Sect. I-4), electrodynamics and electromagnetism are two disciplines, often mixed in the treatises, but which are to be carefully distinguished.

The first one studies the behaviour of charged particles, endowed with a mass and eventually a spin, in a given electromagnetic field.

The second one is the study of the properties of the fields created by charged corpuscles (or, in quantum mechanics, eventualities of presence of one corpuscule) eventually joined in a current, without the direct intervention of their mass and spin.

The link between the two disciplines lies in the Lorentz force $f = q'F.w$ mentioned in Sect. 4. And the two equations

$$m\Gamma = eF.w, \quad \partial_\mu S^\mu = -F.j \tag{22}$$

where w and $\Gamma = dw/d\sigma$ are the spacetime velocity and acceleration of a classical electron, m its mass, where $S^\mu = S(e^\mu)$, and S is the

Maxwell tensor associated with the field F defined via the Maxwell laws by a charge current j , present similar forms. But the first depends on electrodynamics, and the second is a pure property of electromagnetism. The equation

$$\partial_\mu T^\mu = F.j \quad (23)$$

deduced from the Dirac equation of the electron, where T is the Tetrad momentum-energy tensor, F the exterior electromagnetic field, $j = e\rho_0 v$ the charge Dirac current (ρ_0 is the presence probability density) is an electrodynamic property of the Dirac theory.

What is called a gauge potential is an object which may be considered in addition with a Maxwell-Lorentz potential (in which intervene expressions of the form q/r). These two kinds of potentials appear as quite different and the first belong to electrodynamics.

Without looking for an illustration in the electroweak theory ([5], 1997), we give a simple example of the difference between the geometrical natures of these two objects in the case of the $U(1)$ gauge of the Dirac electron theory. We recall, as we have made it many times, that the momentum-energy vector $p = T(v)/\rho_0$, may be written ([5], 1971) at the point $x \in M$

$$p = \frac{\hbar c}{2}\omega - eA, \quad \text{with } \omega_\mu = (\partial_\mu n_1).n_2 = -(\partial_\mu n_2).n_1 \quad (24)$$

Here ω is a spacetime vector which represents the infinitesimal rotation upon itself of a plane $\pi(x)$ orthogonal to v , (n_1, n_2) an orthonormal frame of this plane whose direction is nothing else but that of the bivector spin $(\hbar c/2)n_1 \wedge n_2$. A is the exterior potential. A rotation of the axes in the plane $\pi(x)$ through an angle $\chi(x)$ changes ω in $\omega - \partial\chi$ and the gauge invariance is obtained by the addition to A of the spacetime vector $-(\hbar c/2e)\partial\chi$ (see [7], [8], [5], 1988). This vector may be called a gauge potential. However it is placed in an electrodynamics part of the Dirac theory, it has not been generated by a charged current, and its geometrical interpretation is quite different of that of the exterior potential A , in the case where A is, for example, the central potential $A = -(Ze/r)e_0$ created by the nucleus of an atom. But it may be called a potential because it is in addition to an electromagnetic potential, although, because it is a gradient, it disappears in the electromagnetic field F . Certainly it is at the center of the secret of the exchange between the exterior potential A and the energy of the electron.

We recall also that the term *gauge potential* has been given in [9] independently of all reference to the Dirac electron theory (and quite independently to the geometrical interpretation we had given to p some years before in [5], 1971), as the product of $\hbar c/2e$ by the infinitesimal rotation ω upon itself of a plan $\pi(x)$, orthogonal to a timelike vector v . It was associated in [9] with the theory of the strings.

Presently the gauge theories appear as one of the most fundamental aspect of the particles theory. However the importance in quantum mechanics of the Maxwell-Lorentz potentials appears everywhere, in particular in the form of the retarded potentials formula, especially in the explanation of the linear or circular polarizations of the light emitted by atoms ([5], 1993) and in the Lamb shift calculation ([1], Sect. VI-34), and the two aspects of the notion of potential are narrowly bound.

7 Conclusion

Taking into account *incontestable features* contained in the *uncontested* relativistic invariance of Lorentz's retarded potentials formula, we have shown that only two experimental data are enough for the edification of the Maxwell-Lorentz laws of electromagnetism: the isotropic propagation of the electromagnetic action and the Coulomb law. They lead to simple principles which precede these laws and may be applied as well to quantal as to classical electromagnetism. These principles explain the "divine surprise" by which the Maxwell laws, despite their classical, deterministic, macroscopic origin, have been revealed as applicable to the undeterminist, microscopic, quantal electromagnetism.

Due to the simple geometrical aspect of the Coulomb law, and the simple geometrical interpretation we have given elsewhere ([5], 1997) to the cousins of the Maxwell-Lorentz potentials, the gauge potentials, it seems that the key of the laws of electromagnetism lies entirely in elementary properties of the geometry of spacetime.

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