

**On the essence of slightly generalized  
Maxwell classical electrodynamics,  
reply on the  
“Comment on the paper  
Slightly generalized Maxwell  
classical electrodynamics  
can be applied to inneratomic phenomena”**

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One of our results from the paper [1] has been commented in [2] immediately after appearance of [1]. In our opinion, the comment under consideration twists the essence of our paper [1], contains non-correct assertions, which may lead the reader of our paper into the fallacy. Therefore, we have to revise the comment [2], trying to give minimal explanations in the essence of the subject of [1].

To understand the main point of our propositions one has to forget for a moment about the Dirac equation. A priori we start in [1] from the *slightly generalized Maxwell equations* (in the improved Gauss system of units)

$$\begin{aligned} \frac{\partial \vec{E}}{\partial ct} &= \text{curl} \vec{H} - \text{grad} E^0, & \frac{\partial \vec{H}}{\partial ct} &= -\text{curl} \vec{E} - \text{grad} H^0, \\ \text{div} \vec{E} &= -\frac{\partial E^0}{\partial ct}, & \text{div} \vec{H} &= -\frac{\partial H^0}{\partial ct}. \end{aligned} \quad (1)$$

Equations (1) are the *classical Maxwell equations* in the vacuum, i. e. with electric and magnetic permeabilities  $\varepsilon = \mu = 1$ . Nevertheless, the Eqs. (1) are not free ones. They contain the densities of both electric and magnetic charges and currents

$$\rho_e = -\partial_0 E^0, \quad \vec{j}_e = \text{grad} E^0, \quad \rho_{mag} = -\partial_0 H^0, \quad \vec{j}_{mag} = \text{grad} H^0. \quad (2)$$

Namely due to the presence of the magnetic charge and magnetic current densities ( $\rho_{mag}, \vec{j}_{mag}$ ) we have used the notion “*slightly generalized Maxwell equations*” (SGME). Further, in the Eqs. (1) there are no sources generated by “*material (electric or magnetic) charged particles*” (such as electrons, protons, monopoles, etc.). As it is seen from (2) the scalar field ( $E^0, H^0$ ) generates the correspondent charge and current densities. The last circumstances just mean that the electromagnetic ( $\vec{E}, \vec{H}$ ) and scalar ( $E^0, H^0$ ) fields obeying Eqs. (1) are coupled together in the 8-component real field  $(\vec{E}, \vec{H}, E^0, H^0) = (E^\mu, H^\mu)$ .

The Eqs. (1) are the normal system (8 equations for 8 functions ( $E^\mu, H^\mu$ )) of first-order differential equations with constant coefficients. Therefore, we call them conventionally as system of free SGME. The solution of such system has the form (53), (52) in [1] (presented in terms of eigenvectors  $\vec{e}_j(\vec{k})$ , of quantum-mechanical photon helicity  $h \equiv \vec{s}\vec{k}/\omega$ ). From the Eqs. (1) (with charge and current densities (2)) and the explicit form of their solutions one can see without any doubt that we deal just with (generalized) classical Maxwell’s equations and not with Dirac equation.

An interesting and meaningful form of SGME is their form in terms of the complex quantity  $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$  (introduced by E. Majorana) and, of course, in terms of  $\varphi = E^0 - iH^0$ . In terms of the quantities  $\mathcal{E}^\mu = E^\mu - iH^\mu$ , ( $\mathcal{E}^j = E^j - iH^j$ ,  $\mathcal{E}^0 = \varphi$ ) the SGME have the form

$$\partial_0 \vec{\mathcal{E}} = i \text{curl} \vec{\mathcal{E}} - \text{grad} \varphi, \quad \text{div} \vec{\mathcal{E}} = -\partial_0 \varphi, \quad \partial_0 \equiv \frac{\partial}{\partial ct}. \quad (3)$$

The transition from Eqs. (1) to Eqs. (3) is not a formal procedure, it has a group-theoretical grounds. The field  $\vec{\mathcal{E}} = \vec{E} - i\vec{H}$  is Poincaré irreducible one,  $\vec{\mathcal{E}} \in (0, 1)$ , whereas the field  $(\vec{E}, \vec{H})$  is reducible,  $(\vec{E}, \vec{H}) \in (0, 1) \otimes (1, 0)$ . Namely by using the form (3) it was comfortably to prove the important theorem [1], which asserts the following. *The SGME are invariant with respect to the three different local representations of the Poincaré group P(1,3), namely, to the vector  $P^V$ , tensor-scalar  $P^{TS}$  and spinor  $P^S$  representations of the group P(1,3), generated by the irreducible vector  $(\frac{1}{2}, \frac{1}{2})$ , reducible tensor-scalar  $(0, 1) \otimes (0, 0)$  and spinor representation  $(0, \frac{1}{2}) \otimes (\frac{1}{2}, 0)$  of the Lorentz group SL(2,C), respectively.* Namely this theorem is a group-theoretical

ground for the assertion that the states of fermions can be constructed from the states of bosons (as it was explicitly given by the formulae (63), (62) in [1]).

Practically dealing with SGME, one can use either (1) or (3) form of these equations.

Of course, the case  $\varepsilon = \mu = 1$  is a limit of more informative case. Taking instead of  $\varepsilon = \mu = 1$  some functions  $\varepsilon(x)$ ,  $\mu(x)$  one receives not free SGME. For our purposes we have taken just Sallhofer's permeabilities [3]

$$\varepsilon(\vec{x}) = 1 - \frac{\Phi(\vec{x}) + m_0c^2}{\hbar\omega}, \quad \mu(\vec{x}) = 1 - \frac{\Phi(\vec{x}) - m_0c^2}{\hbar\omega}, \quad \Phi(\vec{x}) \equiv -\frac{Ze^2}{r}, \quad (4)$$

i. e. we have considered the non-free SGME

$$\begin{aligned} \text{curl}\vec{H} - \partial_0\varepsilon\vec{E} &= \text{grad}E^0, & \text{curl}\vec{E} + \partial_0\mu\vec{H} &= -\text{grad}H^0, \\ \text{div}\vec{E} &= -\partial_0\mu E^0, & \text{div}\vec{H} &= -\partial_0\varepsilon H^0. \end{aligned} \quad (5)$$

In this case the densities of both electric and magnetic charges and currents have the form

$$\begin{aligned} \rho_e &= -\varepsilon\mu\partial_0E^0 + \vec{E}\text{grad}\varepsilon, & \vec{j}_e &= \text{grad}E^0, \\ \rho_{mag} &= -\varepsilon\mu\partial_0H^0 + \vec{H}\text{grad}\mu, & \vec{j}_{mag} &= -\text{grad}H^0. \end{aligned} \quad (6)$$

It is clear from (6) that the sources in Eqs. (5) are generated not only by the field  $(E^0, H^0)$  but also by the permeabilities (4).

In the Sallhofer's permeabilities (4) including into Eqs. (5) one sees Planck's constant  $\hbar$  among other parameters ( $m_0$ ,  $e$ ,  $c$ ) with definite dimensions. Of course, Planck's constant is typical for equations, describing some quantum effects. However, let us emphasize that due to the presence of constant  $\hbar$  the Eqs. (5) have not changed their nature as the *classical slightly generalized Maxwell equations*. This assertion is true for the Eqs. (5) with coefficient functions  $(\varepsilon, \mu)$  containing arbitrary parameters (and, in particular, Planck's constant  $\hbar$ ).

In this connection the important circumstances are following. The Eqs. (5) (the classical Maxwell equations in specific medium (4))

turns to be applicable to the description of such microscopical systems like hydrogen atom. Indeed, the stationary solutions  $\mathcal{E}^{stat}(\vec{x}) = (\vec{E}, \vec{H}, E^0, H^0)^{stat}$ , defined by the formulae (5), (6) of [1] in the domain

$$0 < \omega \leq \frac{m_0 c^2}{\hbar} \quad (7)$$

of the parameter  $\omega$ , exist only for the values of this parameter given by the Sommerfeld - Dirac formula (19) in [1], which coincides with the relativistic hydrogen atom spectrum. And the Dirac's stationary states  $\Psi_{nlm}(\vec{x})$  of the relativistic hydrogen atom are expressed uniquely via the stationary states  $\mathcal{E}^{stat}(\vec{x})$  of the coupled system of electromagnetic and scalar fields according to the formulae (32), (33) in [1]. Further, all other quantum observables of the electron in the external field  $\Phi = -Ze^2/|\vec{x}|$  are expressed in terms of the same stationary states  $E^{stat}(\vec{x})$  of the system of classical electromagnetic  $(\vec{E}, \vec{H})$  and scalar  $(E^0, H^0)$  fields.

With taking into account the results presented above one can assert that here one deals with specific (bosonic) realization of the old idea (Thomson, Abraham, etc) about the electromagnetic nature of the material reality. The specification of this realization consists in attraction not only pure electromagnetic but also the scalar field. Can one reject this assertion?

Unfortunately, the comment of Gersten [2] ignored the above-considered essence of our results [1]. Therefore, we were forced to exhibit them here. Not going into details of comment [2] let us mark that the main wrong conclusions of Gersten are caused by his taking into account only a part of our mutually connected results [1]. Indeed, the author of [2] considered only the one-to-one correspondence and unitary relation between the solutions of non-free SGME and Dirac equation in stationary case, which were illustrated in Sec.3 of our paper [1] only as special additional arguments for our generalization of the Maxwell equations. As it is evident, the appropriate interpretation can not be derived from the single fragment (e. g., Sec. 3. of [1]) of the model under consideration. Thus, the attempt to reinterpret our model in [2] deals with only one puzzle and is not complete.

Further, we were not the first in using in considerations like [1] the system of units  $\hbar = c = 1$ . Such convenient system of units is used widely in modern handbooks and monographs on quantum field theory,

atomic and particle physics. This using assumes that everybody on every step may easy restore the natural atomic system of units. Therefore, the comments in [2] like “The authors use units, which hide the quantum effects,” are non-correct. Firstly, the equations (1), (2) in [1] contain the Planck’s constant after the transition to the appropriate system of units. Nevertheless, the presence of such constants as ( $\hbar$ ,  $c$ ) in the equations for the classical field (1) in [1], and in the different consequences of the Eqs. (1) of [1], does not mean any transition to the quantum theory (we hope that we explained it briefly in this text above). Such transition is performed, as it is well known, by replacing physical values by the operators and by another special procedures, which are not even touched in the paper [1].

Finally, in paper [1] we emphasize many times that different interpretations of our results may be developed. Therefore, the comment [2] about our work [1] does not contain any new information.

Nevertheless, we are much grateful to the author of [2] for the attention to our work.

## References

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