On electric charge

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ABSTRACT. A recent extended-particle interpretation or relativistic quantum mechanics is applied to problems of both classical and quantum physics as size and mass of elementary electric charges, possible changes in these concepts in the presence of external electromagnetic fields, answering certain questions of principle, and critique of earlier arguments.

1. Introduction

Ever since the days of Lorentz and Poincaré the problems of the structure and origin of the mass of elementary electric charges, the role of electromagnetic (EM) 4-potentials $A^\mu = (\varphi, \vec{A})(\mu = 0, 1, 2, 3)$ in classical and quantum physics, etc, have been of basic interest for a numbrer of writers. Recently, novel theoretical ideas on the possible local gauge-noninvariant role of field-free potentials A^μ (i.e. potentials $A^\mu \neq 0$ which do not generate EM fields in certain regions of interest) were advanced [1-4] and a striking prediction that constant potentials A^0 can change the inertial mass of charged particles [1], [3] appears to have found experimental verification [5]. More precise experiments are necessary, however, in order to be definite on the matter.

In this note we shall consider certain properties of elementary electric charges and their masses that follow from a recent extended-particle interpretation (EPI) of ours of the gauge-invariant Dirac and Klein-Gordon equations. In combination with the EPI, the quantum character of the approach admits the discussion of possible effects and properties that are non-obvious in both classical electrodynamics (CED) and the conventional interpretation of relativistic quantum mechanics.

For the sake of concreteness we shall concentrate on the Dirac equation (DE) but shall remember that there exists a noteworthy parallelism with the spin 0 Klein-Gordon theory [6].

2. Outline of the EPI

In Gaussian units and the Dirac-Pauli representation the DE is written

$$i\hbar\frac{\partial\psi}{\partial t} = \{c\vec{\alpha}[\hat{\vec{p}} - \frac{e}{c}\vec{A}(t,\vec{x})] + e\varphi + mc^2\beta\}\psi \equiv H\psi, \tag{1}$$

where $\hat{\vec{p}} = -i\hbar\nabla$ and the magnitude H in the braces is treated as the actual Hamiltonian of the elctrically charged spin $\frac{1}{2}$ object (say electron). The existence of both positive and negative eigenvalues of the field-free "Hamiltonian" H_0 (the field-free magitudes being indexed hereafter with zero) induces the thought that H_0 (and H) are just pseudo-Hamiltonians and their change of sign under charge conjugation confirms this [6] : H can be represented as

$$H = \tilde{H}\Lambda \quad ; \quad \Lambda = H/(+H^2)^{\frac{1}{2}}), \tag{2}$$

where $\tilde{H} = H\Lambda$ is the actual positive Hamiltonian and Λ represents the sign operator of electric charge ; $\Lambda = \Lambda^\dagger = \Lambda^{-1}, \Lambda^2 = 1, \Lambda H = H\Lambda$. (The origin of energy is taken, in the middle of the gap between positive and negative pseudo-energy states, so we have an acceptable definition of Λ only as long as such a clear-cut gap exists). All one-particle operators must commute with Λ [6] and one can define in a straightforword fashion one-particle operators as that of electric charge $\tilde{e}$, etc :

$$\tilde{e} = e\Lambda. \tag{3}$$

Defining the actual states of motion of the reinterpreted theory as $\tilde{\psi} = \Lambda\psi$, we can recast the DE in the form

$$i\hbar\frac{\partial}{\partial t}(\Lambda\tilde{\psi}) = \tilde{H}\tilde{\psi}, \tag{4}$$

so the reinterpreted equation of motion has no longer a purely Hamiltonian form. Λ_0, and hence $\tilde{H}_0, \tilde{H}, \Lambda, \tilde{e}$, etc, turn out to be integral operators of (unsharp) range of action $\sim \hbar/mc = \lambda_C$ (the Compton wavelength), which magnitude is interpreted as the characteristic size of the particle-like kernel of the spin $\frac{1}{2}$ object. Since both Λ and $\tilde{H}$ are motion-determining magnitudes (see eq. (4)), the eigenvalue $\pm e$ of $\tilde{e} = e\Lambda$ and mc^2 of $\tilde{H}_0$ (at zero momentum) can be regarded as being the result of matter motions within the kernel, without necessarily being related to the existence of any density functions whose integration could generate them, as they are not obtained this way.

For electrically charged elementary spin $\frac{1}{2}$ particles there also exists in the rest frame their long range electric field but this fact is disregarded by our integral operators, so renormalization (at that with a correct finite order of magnitude) is inevitable [6].

3. Conceptions of charge and mass

The well known standard classical relativistic equation of motion of a point charge e having mass m in a givent classical (external) EM field A^μ is obtained by varying the charge's trajectory at a fixed A^μ [7] (Sec. 17) and contains just the Lorentz force :

$$d\vec{p}/dt = e\vec{E} + (e/c)\vec{v} \times \vec{H}. \qquad (5)$$

The conserved total energy of the particle in static EM fields is $E_{tot} = mc^2(1-v^2/c^2)^{\frac{1}{2}} + e\varphi$. It is then clear that the charge e and mass m of our point particle are not influenced at all by the magnitude of $\vec{E}$ and $\vec{H}$, the charge e being stripped of its own field. Indeed, the said field is nowhere present in the equation ; besides, the said conservation of E_{tot} is just what is to be expected of a charge which possesses no field to be emitted, so e in Eq. (5) simply represents a coupling constant between the EM field and the bare charge. The explanation that Eq. (5) is applicable when the charge is small and its influence on the external field is weak [7] (p. 68-9) is unconvincing since the EM field of a point charge, when considered explicitly, is infinitely large at the point's location. (Emission of EM waves is treated by employing the Maxwell equations but their logic is quite different : they are obtained by varying A^μ at given 4-current densities ([7], Sec. 30) that do generate now EM fields of their own, so it is not really surprising that distinct logics entail distinct physics).

The standard theory based on the DE (1) has features similar to those of Eq. (5) : we have a fieldless (bare) point charge of constant magnitude e and mass m. Indeed, that theory deals with only the external A^{μ} and $\psi^{\dagger}\psi$ is treated as point-position probability density distribution, the mass m being fixed by the A^{μ} -independent term $mc^2\beta$. (This term does not represent any physical operator, as it is non-commutative with Λ.) Besides, the stationary states of the charge are infinitely stable, so energy is conserved and EM emissions - that actually exist - are disregarded.

The common features of Eq. (4) of the EPI with Eqs. (1) and (5) are that the charge generates no EM fields outside the kernel and the energy levels (all of which are now positive) are infinitely stable. Everything else changes since the charge-carrying kernel is extended in the EPI and physical processes in it are to be identified.

As the physical magnitudes of quantum theory are represented by operators, we shall consider operator properties. For a non-zero EM field the operator Λ is represented as $\Lambda = \Lambda_0 + \Lambda_f$, where Λ_f is the part of Λ due to the field. The charge operator $\tilde{e} = e\Lambda$ has the same eigenvalues as in the field-free case but the additional term $e\Lambda_f$ in $\tilde{e}$ shows that there are nevertheless certain structural changes of the charge inside the kernel under the influence of the EM field. Besides, there exists in the EPI no mass-fixing term of the kind of $mc^2\beta$ in Eq. (1) owing to the presence of the additional term $mc^2\beta\Lambda_f$ in $\tilde{H}$. Therefore, we have no longer a sharp conception of paticle rest mass in non-zero external fields. (The term $mc^2\beta\Lambda_0$ in $mc^2\beta\Lambda$ fixes the rest mass only in the field-free case.)

It appears that there is just one reasonable explanation of the latter effect. Namely, the fact that the extended object formed by the interacting classical field and charged kernel is indefinite in some respects means that it is engendered by the merging of field and particle matter and represents a non-autonomous entity (i.e. one that can have no independent free existence). It is thus neither a pure field formation nor a pure particle one that can exist autonomously, so it will lack certain properties of any one of these, e.g. a definite rest mass that characterises free (autonomous) particle motion. Correspondingly, the parameter m in Eq. (4) is to be interpreted as the particle's rest mass in the absence of any EM fields and A^{μ} as the 4-potential we would have in the absence of the particle. A definite rest mass is to be anticipated only of the autonomous overall system formed by the union of the field and particle structures.

The above consideration evinces two distinct types of model interactions : (i) direct interaction without any interpenetration of structures, and (ii) interaction via merging of structures. The EPI of Eq. (4) is an example of a type-(ii) theory, whereas point-charge CED and standard DE theory are type-(i) theories. Indeed, a fieldless point charge is a structureless entity which cannot influence the structure of finite classical EM fields but can be influenced by them. This influence, however, is just external (say imparting acceleration), as such fields cannot introduce energy in a volume $V = 0$. There are thus non reasons for rest-mass variation of a particle of volume $V = 0$ in these models, so it is a requirement of the particular model logic (to which these type-(i) theories conform) that the mass of point charges be constant indeed in arbitrary external EM fields.

We pass now to the discussion of specific items.

4. Brillouin's argument on interaction mass

Brillouin finds it strange that in special relativity one assigns a definite mass to the interaction energy within a system of particles, whereas when the system is placed in a given classical field one assigns no mass to the external potential energy ([8], Sec. 2.1). He examines then a system of two uniformly charged spheres S' and S'' (my notations) at rest of equal radii ($= a$) that are very small in comparison with the distance r between the spheres ([8], Sec. 2.4.) and computes within CED the interaction energy of the respective charges Q' and Q'' by integrating $\vec{E}'\vec{E}''/4\pi$ throughout space, the result certainly being $E_{int} = Q'Q''/r$. In the particular geometry, the density of E_{int} turns out to be considerable just quite close to the surface of the spheres, for which reason Brillouin asserts that the rest mass of each free charged sphere must now be increased by $m_{int} = \frac{1}{2}E_{int}/c^2$ ([8], Sec. 2.5.).

With respect to the point-charge model given by Eq. (5) his first remark is answered by the argument at the end Sec. 3 justifying the standard outlook. We may add that the external potential energy cannot be interpreted as any interaction energy like $E_{int} \sim \int \vec{E}'\vec{E}''d^3x \neq 0$ since $E_{point} = 0$ in Eq. (5). If the charged-point and/or external-field concepts are irrelevant in a givent problem of CED, then one faces a substantially different situation and has no employ, at least inexplicitly, the Maxwell equations (generating concepts as E_{int}, ect). Brillouin's second remark (about M_{int}) applies to a situation of this kind (S' and S'' are explicitly regarded as interacting field sources) but is subject to critique too. Indeed, the fact that in the geometry considered E_{int} is practically equipartitioned among the close vicinities of S' and S''

does not mean that these portions are attached to (carried by) only the respective individual spheres (i.e. that, say, S' and $E'_{int} \approx E_{int}/2$ form a unique body). Really, E'_{int} would just vanish if S'' would recede to infinity, which means that E'_{int} is carried by both S' and S'', the same applying to E''_{int}. CED is a linear theory in which the EM fields obey the principle of linear superposition and what matters in it are just overall vector sums of fields created by (and belonging to) all field sources.

The discussion in Sec. 3 offers a counterargument to the one of Brillouin. Namely, the above case represents one more example of type(ii) interaction : we have here interpenetration of the electrostatic field of S' and S''. Therefore, E_{int} is again a measure of the indefiniteness of the individual-mass conception for systems whose masses M' and M'' were sharply defined at infinite separation. The said indefiniteness be noticeable in principle in pertinent experiments.

5. The size-of-charge parameters

The EPI order-of-magnitude estimate $D \sim \hbar/mc$ of the rest-frame size of elementary charges coincides with the one known from hadron experiments but appears to be strikingly large for electrons : $D_e \sim 3.86.10^{-11}$ cm. Nonetheless, apart from conceptual advantages of such a D_e [6], it can be assessed in experiment too ; Thomson scattering of low energy photons (that could little affect the kernel's size) on electrons is the pertinent method. Indeed, the cross section $6_{Th} = (8\pi/3)(e^2/m_e c^2)^2 = (2/3)\alpha^2 4\pi(\hbar/m_e c)^2, \alpha = e^2/\hbar c \approx 1/137$, and a standard interpretation of α^2 as the probability for absorption and emission of a photon by the charge [9] yields a "radius" $\sim \hbar/m_2 c$ of the electron.

A large D_e as this would not make the quantum electron insensitive to the structure e.g. of the proton (p) whose D_p is by three orders of magnitude smaller than D_e. Really, sensitivity to details is determined by the de Broglie wavelength which tends to zero as $|\vec{v}| \to c$ (and is then of the same order of magnitude as the Lorentz-contracted size of the kernel [6] !), thus rendering the elctron sensitive to arbitrarily small details.

An illustration of the role of the charge-size parameter D for answering questions of principle may also be found in the unique decay mode $n \to p+e+\bar{\nu}_e$ of the neutron : it could be asked whether n consists of p and e, bound by some non-electromagnetic force prior to decay. One standard answer would be : no, since this would lead to non-conservation of lepton number in the decay. The latter requirement, however, is an empiric rule known from experiment. The EPI offers a first-principle answer : no, because D_n is smaller than D_e by three orders of magnitude, whereas any recognizable constituent of well defined mass m of the neutron should have a size $D \sim \hbar/mc \leq D_n = \hbar/m_n c \ll \hbar/m_e c$; for mass we have correspondingly, $m \gg m_e$.

We hope that the above considerations illustrate the useful character of the EPI for discussing problems in various branches of physics, CED in that number.

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