Bell's Locality Assumption in Clauser-Horne Inequality

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ABSTRACT. The author previously identified lack of required symmetry and ill-defined nature of Bell's inequality. Violation of quantum mechanical predictions in Bell's type inequalities, here Clauser-Horne's, had been treated as a proof of the inapplicability of given assumptions, such as Bell's locality, or hidden variable theory, the latter also implying EPR's notion of physical reality, to quantum phenomena. This has led to implausible speculation of possible communication between detectors at a speed faster than light. No critical examination has been made on Clauser-Horne inequality which were actually defined in terms of conditional probabilities given polarizer orientations fixed. The corresponding subensembles of linearly polarized photon pairs over which they were respectively defined have not been identified. Unless these subensembles were exactly identical, different conditional probability values could not be intermingled in numerical sense in inequalties. In the special case of interest examined herein, the specified subensembles were found to be all different. The conditional probabilities defined given specific subensembles including mirror image subensembles indicated ill-defined nature of Bell's locality assumption.

RÉSUMÉ. Dans une communication précédente, l'auteur avait identifié une lacune dans la symétrie requise et une nature mal définie de l'inégalité de Bell. La violation des prédictions de mécanique quantique dans les inégalités du type Bell, plus précisément ici du type Clauser-Horne, a été utilisée comme preuve de l'inapplicabilité pour les phénomènes quantiques des hypothèses données, telle que la localisation de Bell, ou la théorie des variables cachées, cette dernière impliquant aussi notion EPR de réalité physique. Cette situation a donné naissance à une spéculation peu plausible de communications possibles entre détecteurs a une vitesse plus grande que la vitesse de la lumière. Aucun examen critique n'a été fait des inégalités de Clauser-Horne, qui étaient en fait définies en termes de probabilités conditionnelles, des orientations fixes de polarisation étant données. Les *sous-ensembles correspondants de paires de photons linéairement polarisés (sur lesquels ces sous-ensembles étaient respectivement définis) n'ont pas été identifiés. A moins que ces sous-ensembles ne soient exactement identiques, des valeurs différentes des probabilités conditionnelles ne pouvaient pas être calculées dans toutes ces inégalités: ces valeurs ne pouvaient pas avoir une signification mathématique. Dans le cas spécial examiné ici, les sous-ensembles spécifiés se sont avérés être tous différents. Les probabilités conditionnelles définies à partir de sous-ensembles spécifiques (comprenant des sous-ensembles qui sont des images inverses l'un de l'autre) exemplifient un autre aspect de la nature mal définie de l'hypothèses de localité de Bell.*

1 Introduction

Bell's type inequality [1-4] was initially considered a tool for demonstrating that predictions in quantum mechanics (QM) were incompatible with local hidden variable theory [5], thus by implication also the notion of physical reality as proposed by Einstein-Podolsky-Rosen (EPR) [6]. When violations of the inequalities were found in idealized cases, some doubted applicability of Bell's locality assumption, but no positive proof existed for the acceptability or rejection of the simple postulate. In this paper the author identifies numerous mathematical problems and deficiencies found in Clauser-Horne inequality and demonstrates that Bell's locality assumption was ill-defined and logically inconsistent with the standard method of specifying a probability model.

In order to refute the applicability of hidden variable theories in general or in particular cases, Bell should have specified the exact identities of the said hidden variable(s) and how such variable(s) could operate only locally with a specific probability distribution in paired particle experiments. Without such required specifications Bell's type inequalities should have been criticized for the vagueness and insupportable mathematical development. For some reasons, perhaps influenced by the standard theory, a strong tilt towards acceptance of Bell's work has existed in academic circles to this date. The author could fondly remember an incidence of receiving a referee's comment stating that Bell's inequality was inviolable, perhaps for his personal reasons, from a prominent journal. Canals-Frau stated that Bell's inequality was based on speculations, and was not an expression of verifiable physical facts [7-9]. Lochak noted that Bell's contention about hidden variables theories placed de Broglie's wave mechanics under suspicion, because an indeterminate singularity point on u wave was supposed to represent an exact position of a particle obviously forming a hidden variable [10-12]. He correctly identified the need to distinguish a subensemble of particle pairs from another, but he focused his investigation on aspects of hidden probability which was assumed to apply to Bell's argument. Because of these experiences he strongly urged readers to investigate and clarify the underlying logical problems. It has been well known that photon pairs maintained the matched polarizations intact until they reached the polarizers/detectors, as experimentally verified in agreement with QM prediction by Aspect [13-14] and subsequently by Gisin, et al, over extended distances [15]. Some QM theorists tried to explain this fact by suggesting a possible communicating between each photon pair or between detectors at a speed faster than that of light at the time of actual measurements. Note that this argument is based on the locality assumption being held between each photon pair. Therefore a questioning Bell's locality assumption becomes suddenly an important logical inquiry by itself to clarify such a myth.

Against these backgrounds the author recently identified lack of required symmetry and ill-defined nature of Bell's 1964 inequality showing that it was mathematically meaningless [16,17]. Instead of Bell's locality assumption he introduced a classical assumption that an undisturbed photon pair should maintain the respective polarizations until they reached polarizers/detectors consistently with known experimental results avoiding the speculative notion of superluminous communication. The analysis of conditional subensembles of spin ½ particles given orientations of detectors fixed could not be used to investigate Clauser-Horne inequality defined with respect to correlated photon pairs. A different model was developed in the following. In this paper the inequality obtained by Clauser, Horne, Shimony, and Holt in testing QM predictions is examined as presented in Shimony [18]. In this simple example, six probability values appear as functions of polarizer orientation angles. Different subensembles of photon pairs must then be identified in order to define conditional probability values given the polarizer settings fixed. This is the place where Bell's locality assumption, i.e. the independence between two detector outcomes, as one detector outcome does not influence the outcome of the other, and vice versa, becomes entangled with the standard practice of specifying a probability model.

2 Preliminary

In case of a correlated photon pair, QM predicts with certainty that, if measurement on the **x**-component α_x (1) of the photon-1 polarization yields a detection result (+), or 1, the measurement on α_x (2) also yields (+), or 1, for the partner photon-2. However, the amplitude of an actual measurement yielding a positive detection result decreases in proportion to the cosine $\alpha(i)\cdot\mathbf{x}$ of the angle between the true polarization orientation unit vector $\alpha(i)$ of the i-th particle, i=1,2, and the polarizer orientation unit vector **x**. This fact is incorporated in QM treatment of photon pairs in the comparison between the predictions using a hidden variable and the QM.

As stated previously the exact role of the hidden variable λ appearing in $p_\lambda(x)$ and in $p_\lambda(x,y)$ in Clasuer-Horne inequalities was not identified by Bell or by others. Shimony described three types of hidden variables, "simple, contextual, and stochastic", loc. cit., but EPR never introduced such a notion. In this paper the author makes a classical assumption that each undisturbed classical photon pair has the polarization orientation values $\alpha(i)$, i=1,2, which remain unchanged until they reach polarizers and detectors. The $\alpha(i)$'s may be thought of representing the unidenfied λ in the author's version of simplification. This simple assumption does not violate EPR's notion of physical reality, but it is not in agreement with and obviously violates Bell's locality assumption.

To identify subensembles of photon pairs in four positive detection pairs, the incorporation of the cosine detection amplitude simply complicates the issue. It is the best to specify the angular range of photon polarization orientation as within $\pm\pi/2$ of the polarizer orientation for yielding positive detections. In this way, the derivation of probability with respect to random polarization orientation of photon pairs for a given polarizer/detector can be eliminated and substituted by an interval statement, which identifies a specific ensemble of polarization pairs with no regards to actual measurement outcomes. When a polarizer is positioned perpendicular to the photon propagation paths spreading in opposite directions, the true polarization orientation $\alpha(1) = \alpha$ measured from a reference axis of the first particle can be assumed uniformly distributed over $[-\pi/2, \pi/2]$, and similarly for the second particle with $\alpha(2) = \alpha+\pi$. (The convention of $\alpha(2) = \alpha + \pi$ is used here to associate with the wrap around in excess of 2π in mirror images as discussed later.) However, the assumption of uniform distribution over α's is not required so long as the assumed density is continuous. Even with any arbitrary density of photon pairs the following argument still holds. If two different polarizer settings are used for detecting polarization orientation of each individual photon of pairs, a joint detection positive on both can occur only in the intersection of the two angular ranges defined above in this analysis scheme. The distinction between two subensembles can be visualized by applying the cosine envelop of the polarizer throughputs over the two different angular ranges.

In the next sections all mathematical or notational problems are listed under the heading of "Note" with a consecutive number assigned for the ease of identification.

3 Clauser-Horne inequality

For the purpose of avoiding misrepresentations, the general outline of the argument presented in Shimony is used in the following [4,18-20]. "Let x,y,x',y' denote four polarizer orientation angles measured from a reference line in the (x,y)-plane perpendicular to the photon paths spreading towards the opposite directions. The detection systems 1 and 2 are spatially separated and each system comprised of a polarizer and detector registering (+) or (-) (1 or 0) measurement outcomes for each incoming photon. A hidden variables theory assumes that every complete set λ of 1+2 assigns a definite probability $p^1_\lambda(X)$ to the (+) outcome of the test on 1 when the parameter has value X, independently of what test is performed on 2 and independently of the outcome of that test; and likewise it assigns the probability $p^2_\lambda(Y)$ to the (+) outcome of the test on 2 when the parameter has value Y independently of the choice of X or the outcome of the experiment on 1. Shimony further presented three comments, two of which are briefly outlined below:

Comment (i): By standard probability theory the probability of a (-) outcome of the experiment on 1 is $1-p^1_\lambda(X)$, and similarly $1-p^2_\lambda(Y)$ on 2. Because of the independence conditions the probability of a joint (+)(+) outcome in an X-test on 1 and Y-test on 2 is $p^1_\lambda(X)p^2_\lambda(Y)$.

Comment (ii): The definition of a local hidden variables theory makes no reference at all to the QM characterization of parts 1 and 2 and the projection operators on these spaces. The definition merely refers to two families of bivalent empirical tests … (More comments on QM predictions appear.)"

Clauser-Horne showed that for any set of four real numbers, r',r,s',s; $0 \leq$ r',r,s',s ≤ 1, the inequality

$$-1 \leq r's' + r's + rs' - rs - r' - s' \leq 0 \qquad (3.1)$$

holds. If x' and x are two values of X, and y' and y are two values of Y in the definition of a local hidden variables theory, then the $p_\lambda^1(x')$, $p_\lambda^1(x)$, $p_\lambda^2(y')$, $p_\lambda^2(y)$ are said to be probabilities lying between 0 and 1, which may be substituted for r', r, s', s of (3.1).

> Note #1: In truth the $p_\lambda^i(u)$'s, i=1,2, u=x',x,y',y, are conditional probabilities given one of the orientation angles x', x, y', or y fixed, and they do not constitute unconditional probabilities free from all the conditioning i's and u's. Such conditional probabilities defined on different i's and u's cannot be multiplied or added as suggested into the form of (3.1) without having the u's exactly identical for i=1,2.

> Note #2: The notation is confusing. Since the standard expression of a conditional probability p of an event A given B is denoted $p(A|B) = p_B(A)$, the above $p_\lambda^i(u)$ should represent a conditional probability of an event u given the λ fixed. Since the u is a prechosen fixed orientation of polarizer setting, it cannot be a random variable, while the said hidden variable λ is assumed to be the only plausible random variable under the model consideration. If so, the required mathematically correct notation should be $p_u^i(\lambda)$ instead of $p_\lambda^i(u)$ as given here.

Under the locality assumption (i) the probability of joint (+)(+) outcome is given as $p_\lambda(X,Y) = p_\lambda^1(X)p_\lambda^2(Y)$. Then (3.1) yields

$$-1 \leq p_\lambda(x',y') + p_\lambda(x',y) + p_\lambda(x,y') - p_\lambda(x,y) - p_\lambda^1(x') - p_\lambda^2(y') \leq 0 \qquad (3.2)$$

It is then said that "in an experimental situation it is impossible to control the choice of the complete state λ of the pair 1+2, but it may be assumed that experimental arrangement determines a probability distribution ρ over the space of complete states. The averages of $p^1_\lambda(X)$, $p^2_\lambda(Y)$, $p_\lambda(X,Y)$ using the distribution ρ are respectively denoted as $p^1(X)$, $p^2(Y)$, and p(X,Y). Averaging over λ's using such a distribution ρ and applying the new notation to (3.2), the final inequality

$$-1 \leq p(x',y') + p(x',y) + p(x,y') - p(x,y) - p^1(x') - p^2(y') \leq 0 \qquad (3.3)$$

is obtained.

Note #3: The fact that x', x, y', y are prechosen fixed values and λ is the only random variable to deal with as stated in Note #2 becomes apparent in this paragraph. Even at this stage the exact identity of the λ is still kept under shroud. No probability distribution ρ is specified either. Without having them completely specified, the said averaging operation is only vacuously defined. Therefore the stated transition from (3.2) to (3.3) must be deemed at most a figment of imagination.

Note #4: Under the Notes #2 and #3, the said independence $p_\lambda(X,Y)=p_\lambda^1(X)p_\lambda^2(Y)$ must be rewritten as $p_{X\wedge Y}(\lambda)=p_X^1(\lambda)p_Y^2(\lambda)$ implying that the conditional probability of given X∧Y fixed is the same as the product of two conditional probabilities of given X and given Y, totally different from the original equation. Therefore (3.2) can never be derived as shown.

Note #5: In a particular situation satisfying x'=x=y'=y as described in Note #1, which makes the manipulation of (3.2) possible, $p_\lambda(X,X) = p^1_\lambda(X)p^2_\lambda(X) = p_\lambda(X)^2$ should hold for some $p_\lambda(X)$. However, since $p^1_\lambda(X)=p^2_\lambda(X)=p_\lambda(X)$ for every photon pair for a given X fixed under the author's classical assumption, the sum of the six terms becomes zero satisfying (3.2) trivially leading to a meaningless result.

In case of an equivalent QM situation the state of each photon pair is represented by the superposition of unit vectors

$$\psi = [u_\alpha(1)\, u_\alpha(2) + u_\beta(1)\, u_\beta(2)]/\sqrt{2} \tag{3.4}$$

where α and β are chosen orthogonal to each other. The QM probabilities for (+) outcomes in an α-test on 1 and a β-test on 2 are then calculated to be

$$p^1_\psi(\alpha) = p^2_\psi(\beta) = ½ \tag{3.5}$$

and the QM probability for joint detection in an X-test on 1 and a Y-test on 2 after passing through two polarizers in arbitrary orientations X and Y is given by

$$p_\psi(X,Y) = (1/2)\cos^2(Y\text{-}X). \tag{3.6}$$

In particular, if the polarizer settings are chosen as x'=π/4, x=0, y'=π/8, and y=3π/8 respectively, then a numerical contradiction of (3.3) in

$$\begin{aligned} &p_\psi(x',y') + p_\psi(x',y) + p_\psi(x,y') \\ &- p_\psi(x,y) - p^1_\psi(x') - p_\psi(y') = 0.207 \end{aligned} \tag{3.7}$$

is obtained using (3.6) showing a violation of the upper bound of (3.3) according Shimony, loc. Cit

4 Subensembles of photon pairs

The same four polarizer settings as specified for (3.7) are used in the following analysis. The angular ranges of photon polarization over which a positive detection can be obtained for the given x', x, y', and y are then:

for $x=0$,	$(x) = [-\pi/2, \pi/2]$	
for $y'= \pi/8$,	$(y') = [-3\pi/8, 5\pi/8]$	(4.1)
for $x'= \pi/4$,	$(x') = [-\pi/4, 3\pi/4]$	
for $y=3\pi/8$,	$(y) = [-\pi/8, 7\pi/8]$	

respectively, and $p^1(x')$, $p^1(x)$, $p^2(y')$, and $p^2(y)$ can tentatively be defined over these specific ranges separately, provided that no joint probabilities, such as p(x',y'), p(x',y), p(x,y') or p(x,y), do not appear simultaneously with them. (The said requirement stems from the way a bivariate probability model is specified as explained below.)

Joint detection outcomes with respect to pairs of polarizer settings, (x',y'), (x',y), (x,y'), or (x,y) must be defined conditionally given the specific combinations of selected polarizer orientations. For instance (+)(+) detections for (x',y') can only occur over the intersection of the two angular intervals of (x') and (y'), while other detections (+)(-), (-)(+), and (-)(-) can occur over the union of these two intervals of (x') and (y'). Therefore it is necessary to examine all the unions and intersections derived from these four intervals. Let $(X\vee Y)$ and $(X\wedge Y)$ denote the union and intersection of the respective angular intervals for the polarizer settings of X and Y, where X=x',x, and Y=y',y, in (4.1). Then

$(x \vee y) = [-\pi/2, 7\pi/8]$	$(x \wedge y) = [-\pi/8, \pi/2]$	
$(x \vee y') = [-\pi/2, 5\pi/8]$	$(x \wedge y') = [-3\pi/8, \pi/2]$	(4.2)
$(x' \vee y) = [-\pi/4, 7\pi/8]$	$(x' \wedge y) = [-\pi/8, 3\pi/4]$	
$(x' \vee y') = [-3\pi/8, 3\pi/4]$	$(x' \wedge y') = [-\pi/4, 5\pi/8]$	

hold, clearly demonstrating that the events of joint detections are defined over different subensembles of photon pairs in terms of the unions and intersections. Note that the probability values, p(X,Y), $p^1(x')$, $p^2(y')$ in (3.2) or (3.3) are now found conditionally defined over the respective subensembles of $(X\vee Y)$, (x'), and (y'). (A potential problem of this tentative statement is further discussed in Section 6.) Accordingly they can never be added or subtracted numerically as given in (3.3) as if they are unconditional probability values free from the conditioning on subensembles of $(X\vee Y)$, (x'), or (y'). This was the most critical problem of (3.3) making it mathematically meaningless as previously cautioned in Note #1.

5 Wrap around

In the above (4.1) and (4.2) the angular intervals were specified in the first, second, and fourth quadrants. By adding π to each bound, the mirror images of the above angular intervals reflected upon the axis perpendicular to x', x, y', or y respectively can be constructed. These mirror image angular intervals define exactly identical subensembles of photon pairs as defined in (4.1) and (4.2) due to the fact that the created photon pairs must have polarization orientation of $(\alpha,\alpha+\pi)$. The mirror image subensembles are identified as:

for $x=\pi$:	$(x) = [\pi/2, 3\pi/2]$
for $y'=9\pi/8$:	$(y') = [5\pi/8, 13\pi/8]$
for $x'=5\pi/4$:	$(x') = [3\pi/4, 7\pi/4]$
for $y=11\pi/8$:	$(y) = [7\pi/8, 15\pi/8]$

and (5.1)

$(x\vee y) = [\pi/2, 15\pi/8]$	$(x\wedge y) = [7\pi/8, 3\pi/2]$
$(x\vee y') = [\pi/2, 13\pi/8]$	$(x\wedge y') = [5\pi/8, 3\pi/2]$
$(x'\vee y) = [3\pi/4, 15\pi/8]$	$(x'\wedge y) = [7\pi/8, 7\pi/4]$
$(x'\vee y') = [5\pi/8, 7\pi/4]$	$(x'\wedge y') = [3\pi/4, 13\pi/8]$

respectively. An examination shows that these intervals of the union, $(u\vee v)$, are actually overlapping with those given in (4.2). The overlapped angular intervals are

$(x\vee y)$'s:	$[\pi/2, 7\pi/8]$	and	$[3\pi/2, 15\pi/8]$	
$(x\vee y')$'s:	$[\pi/2, 5\pi/8]$	and	$[3\pi/2, 13\pi/8]$	(5.2)
$(x'\vee y)$'s:	$[3\pi/4, 7\pi/8]$	and	$[7\pi/4, 15\pi/8]$	
$(x'\vee y')$'s:	$[5\pi/8, 3\pi/4]$	and	$[13\pi/8, 7\pi/4]$	

respectively. In each case these two overlapped angular intervals both have the same size angular range, and the overlap of the $(X\vee Y)$'s span over the ranges of size $3\pi/8$, $\pi/8$, $\pi/8$, and $\pi/8$ respectively. The overlaps cause the following subtle mathematical problems.

6 Inconsistency with standard probability model specification

When a joint distribution of X and Y is defined in a standard probability model, the univariate distribution of X (or Y) must be obtained by integrating out the other (so-called 'nuisance' meaning unneeded) variable Y (or X) in derivation of the marginal distribution of x' (or y'). This implies that $p^1(x')$ or $p^2(y')$ must be derived as the marginal distribution from (X∨Y) given in (4.2) and (5.1), not directly from (x') or (y') in (4.1) as previously indicated in Section 4. However, these (X∨Y)'s have the overlapping angular intervals identified in (5.2). The overlap implies that the photon pairs which have polarization orientations falling into those intervals of (5.2) must be counted twice. Such a practice is not permitted in standard specification of a probability model.

In general a probability distribution must be assigned with respect to a field (a mathematical jargon, not in the sense of a familiar physical field, but it means an exhaustive class of all possible unions and intersections of distinct disjoint subintervals including a null set in this case) generated from a set of subintervals of a given interval. In the present case a class of angular subintervals is generated over the 2π range of [-π,π]and each one of disjoint subintervals must be counted only once. This practice of having each subinterval counted no more than once is critically important to maintain a logical consistency of a probability model. So, the double counting of photon pairs over the overlapping angular intervals of (5.2) in the marginal distribution of $p^1_\lambda(x')$ or $p^2_\lambda(y')$ derived from the subensemble (x'∨y'), (x'∨y), or (x∨y') clearly violates the standard practice of probability model specification.

The trouble started when the local independence assumption of $p_\lambda(X,Y) = p^1_\lambda(X)p^2_\lambda(Y)$ is introduced under (i), while the $p_\lambda(X,Y)$'s are integrated over λ's without specifying what the λ is and what the probability distribution ρ is. Note that no similar argument can be found in the original EPR paper. More significantly both (3.2) and (3.3) have become mathematically meaningless due to lack of attention to the detailed probability specification. The Notes #1-5 further identified deficiencies of the mathematical presentation found in Shimony's text. In particular Note #4 showed that (3.2) can never be derived. The findings of the present paper shows that violations of Bell's type inequalities are not due to the observable physical processes of quantum phenomena analyzed using hidden variables theories as originally intended, but instead due to mathematical mistakes made by those investigators who reported the findings.

7 Summary

A classical assumption of initial conditions for photon polarization pairs $(\alpha,\alpha+\pi)$ was first introduced specifying that undisturbed correlated photon pair should maintain the original polarization orientation values until they reached polarizers/detectors. The derivation of the Clauser-Horne inequality in (3.3) required examination of the subensembles of photon pairs over which the conditional probabilities were defined. A particular case of interest with the polarizer settings at $x'=\pi/4$, $x=0$, $y'=\pi/8$, and $y=3\pi/8$ showed that the probabilities, $p(x',y')$, $p(x',y)\ldots$, $p^1(x')$, $p^2(y')$, appearing in (3.3) were actually conditional probabilities defined over given subensembles, $(x'\vee y')$, $(x'\vee y)$, etc., either as the joint probabilities or as marginal probabilities. Values of conditional probabilities defined over different subensembles could never be added or subtracted as given in (3.3). Consequently the validity of Clasuer-Horne inequality could not be upheld. A further investigation revealed that the angular intervals of unions, $(x'\vee y')$, $(x'\vee y)$, etc., including the mirror image angular intervals, overlapped as shown in (5.2) beyond the 2π range. This implied that $p^1(x')$ and $p^2(y')$ counted those photon pairs falling into those intervals in (5.2) twice in disagreement with the standard specification of a probability model. This was a practice not permitted in probability theory. One concludes that Bell's locality assumption itself has led to this logical inconsistency.

8 Acknowledgment

The author wishes to thank Dr. Canals-Frau for his helpful comments and providing reprints for reference. He also wishes to express his thanks to the late Dr. L. de Broglie and Dr. G. Lochak, contacting him to join activities of the Foundation, and giving him an opportunity to pursue his unorthodox inquiries. Thanks are also due to many other friends, including Dr. Fargue and Dr. Salmon. It is hoped that this paper can meet their expectations and vindicate their past devoted efforts trying to identify deficiencies of Bell's type inequalities.

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(Manuscrit reçu le 12 janvier 2001, révisé en février 2002)