

Radiation reaction 4-force: orthogonal or parallel to the 4-velocity?

CĂLIN GALERIU

Worcester Polytechnic Institute
100 Institute Rd., Worcester, MA 01609, USA
E-mail: cgalериu@wpi.edu

ABSTRACT. In this note we point to some problems related to the classical derivation of the radiation reaction 4-force, and, using Dirac's relativistic energy-momentum balance equation, we derive a new expression for this 4-force, parallel to the 4-velocity.

RÉSUMÉ. Dans cette note nous exposons quelques problèmes liées à la dérivation classique de la 4-force du freinage de rayonnement, et, avec l'aide de l'équation de Dirac pour la balance relativiste de l'énergie et de l'impulsion, nous dérivons une nouvelle expression pour cette 4-force, parallèle à la 4-vélocité.

1 Introduction

In recent years we have seen the emergence of theories challenging the assumption that a particle's rest mass is constant. Assis, working with Weber's electrodynamics, has shown that the inertial mass of a charged particle is affected by the surrounding distribution of electric charge [1, 2]. Costa de Beauregard, using a covariant stationary action, has shown that the inertial mass of a charged particle depends on the 4-potential [3]. Experimental proof that the inertial mass of the electron indeed changes, in good agreement with the theory, has been brought by Mikhailov [4, 5]. Oron and Horwitz, working with the covariant mechanics of Stueckelberg, have derived an equation for the variation of the renormalized (off-shell) mass [6]. The time-symmetric action-at-a-distance theory developed by the author [7] prescribes a variation of the rest mass. We therefore believe it is necessary to reevaluate the implications of the assumption that a particle's rest mass is constant. This

assumption is most apparent in the derivation of the radiation reaction 4-force.

The problem of whether a 4-force F_μ could be not orthogonal to the 4-velocity v_μ has appeared long ago, when the ponderomotive 4-force, in a system which dissipates energy by Joule heating, was considered [8]. Since an inertial mass must be ascribed to every kind of energy, the rest mass m_o of the system has to decrease, corresponding to the energy dissipated. The ponderomotive 4-force must thus have a component parallel to the 4-velocity, and the equation of motion is modified accordingly [9]:

$$F_\mu = \frac{d}{d\tau}(m_o v_\mu) = m_o \frac{dv_\mu}{d\tau} + v_\mu \frac{dm_o}{d\tau}. \quad (1)$$

The rate of energy dissipation, reflected in the variation of the rest mass m_o , is given by

$$F_\mu v^\mu = -c^2 \frac{dm_o}{d\tau} = -\gamma(v) \frac{dE}{dt}. \quad (2)$$

While it is accepted that the electromagnetic radiation can transport rest mass from one system to another [9], this rest mass has been associated with an internal energy of the system, and not with the rest mass of the individual particles. However, Brillouin [10], de Broglie [11], Lucas [12], and Costa de Beauregard [13] have concluded that the rest mass of the interaction energy has to be localized on the particles.

2 Radiation Reaction - Standard Derivation

The radiative reaction force is introduced in order to satisfy an energy balance equation [14]. The work done by the radiative reaction force has to equal the energy dissipated through electromagnetic radiation:

$$\int_{t_1}^{t_2} \mathbf{F} \cdot \mathbf{v} dt = -\frac{2}{3} \frac{q^2}{c^3} \int_{t_1}^{t_2} \dot{\mathbf{v}} \cdot \dot{\mathbf{v}} dt \Rightarrow \frac{2}{3} \frac{q^2}{c^3} \int_{t_1}^{t_2} \ddot{\mathbf{v}} \cdot \mathbf{v} dt. \quad (3)$$

The last part of (3) results from integration by parts, if we assume that the motion is either periodic, or $\dot{\mathbf{v}} \cdot \mathbf{v} = 0$ at the moments t_1 and t_2 . The radiation reaction force extracted this way is thus somehow averaged, and does not reflect the instantaneous damping force.

From (3) the radiation reaction force is extracted as

$$\mathbf{F} = \frac{2}{3} \frac{q^2}{c^3} \ddot{\mathbf{v}}. \quad (4)$$

We have to warn that, since in (3) the force \mathbf{F} is in scalar product with the velocity \mathbf{v} , the only meaningful information that can be extracted is about the component of the force which is parallel to the velocity, $(\mathbf{F} \cdot \mathbf{v})\mathbf{v}/v^2$! Another problem related to expression (4) is that it is not clear whether this force is indeed a *damping* force, pointing in the opposite direction than the velocity. This problem is evident if we consider the 'runaway' solution, in which the velocity, the acceleration and the acceleration's derivative are all parallel, pointing in the same direction, and increasing exponentially. This solution can be eliminated, but with the price of introducing acausal effects [14, 15].

The force from (4) is generalized to the relativistic case by introducing the derivative with respect to the proper time τ .

$$F_\mu = \frac{2}{3} \frac{q^2}{c^3} \frac{d^2 v_\mu}{d\tau^2}. \quad (5)$$

An extra term, specifically needed to ensure the orthogonality between the 4-force and the 4-velocity, is then added. The relativistic 4-force becomes:

$$F_\mu = \frac{2}{3} \frac{q^2}{c^3} \left(\frac{d^2 v_\mu}{d\tau^2} - \frac{1}{c^2} \frac{dv_\nu}{d\tau} \frac{dv^\nu}{d\tau} v_\mu \right). \quad (6)$$

As Dirac [15] pointed out, by analyzing the temporal part of (6), only the last term corresponds to a dissipation of energy, and gives the effect of radiation damping. The first term gives a perfect differential, and is associated with an intrinsic energy of the electron, the acceleration energy. "Changes in the acceleration energy correspond to a reversible form of emission or absorption of field energy, which never gets very far from the electron"[15]. Since the first term cannot be associated to a damping force, its presence in (6) is unjustified.

3 Radiation Reaction - Alternative Derivation

Since the only reason for being of the radiation reaction force is to account for the dissipation of energy, and this dissipation is correlated to a decrease in the rest mass of the system, and furthermore only the

component of the force parallel to the velocity enters the energy balance equation, we can consider the radiation reaction 4-force as being parallel to the 4-velocity. In other words, we make the intuitive assumption that a force parallel to the velocity will generalize to a 4-force parallel to the 4-velocity. We extract from (5) the component parallel to the 4-velocity, but pointing into the opposite direction, sought to describe the radiation reaction 4-force:

$$F_\mu = \frac{2}{3} \frac{q^2}{c^3} \frac{d^2 v_\nu}{d\tau^2} v^\nu \frac{v_\mu}{-c^2} = -\frac{2}{3} \frac{q^2}{c^5} \frac{dv_\nu}{d\tau} \frac{dv^\nu}{d\tau} v_\mu. \quad (7)$$

This is exactly the last term of (6), responsible for the effect of radiation damping. From (2) and (7) we can calculate the rate of energy dissipation:

$$\frac{dE}{dt} = \frac{-1}{\gamma(v)} F_\mu v^\mu = \frac{-1}{\gamma(v)} \frac{2}{3} \frac{q^2}{c^3} \frac{dv_\nu}{d\tau} \frac{dv^\nu}{d\tau}. \quad (8)$$

In the nonrelativistic limit we recover the exact (not averaged!) Larmor power formula:

$$\frac{dE}{dt} = -\frac{2}{3} \frac{q^2}{c^3} \dot{\mathbf{v}} \cdot \dot{\mathbf{v}}. \quad (9)$$

The force (7) also satisfies Dirac's [15] relativistic energy-momentum balance equation:

$$\frac{q^2}{2\epsilon} \frac{dv_\mu}{d\tau} - qv_\nu F_\mu{}^\nu{}_{\text{in}} - \frac{2}{3} \frac{q^2}{c^3} \left(\frac{d^2 v_\mu}{d\tau^2} - \frac{1}{c^2} \frac{dv_\nu}{d\tau} \frac{dv^\nu}{d\tau} v_\mu \right) = \frac{dB_\mu}{d\tau}. \quad (10)$$

As Dirac pointed out, from this equation the radiation reaction 4-force is not uniquely derived, but is determined up to a perfect differential B_μ , subject only to the condition

$$\frac{dB_\mu}{d\tau} v^\mu = 0. \quad (11)$$

The solution (6) is obtained using

$$B_\mu = \left(\frac{q^2}{2\epsilon} - m_o \right) v_\mu, \quad (12)$$

with a constant rest mass m_o . The solution (7) is obtained using

$$B_\mu = \left(\frac{q^2}{2\epsilon} - m_o \right) v_\mu - \frac{2q^2}{3c^3} \frac{dv_\mu}{d\tau}, \quad (13)$$

with a variable rest mass m_o (but still with a constant charge q). In the later case the condition (11) reduces to equation (8).

At this stage one should recall the experimental fact that, at least in an averaged way, the rest mass of the electron is constant. Therefore a 4-force (5) might be added to the radiation reaction 4-force (7) in order to ensure that the total 4-force is orthogonal to the 4-velocity. The physical origin of this 4-force (5), which gives the acceleration energy, is not clear, and the mechanism by which a charged particle acquires rest mass from the field needs more investigation.

4 Conclusions

Without using the assumption that a particle's rest mass is constant, but using Dirac's [15] relativistic energy-momentum balance equation, we have derived the dissipative component of the radiation reaction 4-force. Our derivation avoids some problems of the classical derivation, and stresses the stand-alone nature of this dissipative component, parallel to the 4-velocity.

References

- [1] A. K. T. Assis, "Changing the Inertial Mass of a Charged Particle", *J. Phys. Soc. Japan* **62**, 1418 (1993).
- [2] A. K. T. Assis, *Weber's Electrodynamics* (Kluwer Academic, Dordrecht, 1994).
- [3] O. Costa de Beauregard, "Effet Mikhailov généralisé: inertie induite par un 4-potentiel irrotationnel", *Ann. Fond. Louis de Broglie* **25**, 129 (2000).
- [4] V. F. Mikhailov, "The Action of an Electrostatic Potential on the Electron Mass", *Ann. Fond. Louis de Broglie* **24**, 161 (1999).
- [5] V. F. Mikhailov, "Influence of an electrostatic potential on the inertial electron mass", *Ann. Fond. Louis de Broglie* **26**, 633 (2001).
- [6] O. Oron and L. P. Horwitz, "Radiation Reaction of the Classical Off-Shell Relativistic Charged Particle", *Found. Phys.* **31**, 951 (2001).
- [7] C. Galeriu, "Time-Symmetric Action-at-a-Distance Electrodynamics and the Structure of Space-Time", *Phys. Essays* **13**, 597 (2000).
- [8] M. Abraham, "Zur Elektrodynamik bewegter Körper", *R. C. Circ. Mat. Palermo* **28**, 1 (1909).

- [9] W. Pauli, *Theory of relativity* (Dover, New York, 1981), pp. 99-108.
- [10] L. Brillouin, "L'énergie potentielle et sa masse", *C. R. Acad. Sc. Paris* **259**, 2361 (1964).
- [11] L. de Broglie, "Sur la répartition des potentiels d'interaction entre les particules d'un système", *C. R. Acad. Sc. Paris* **275 B**, 899 (1972).
- [12] R. Lucas, "Sur la répartition de la masse équivalente à l'énergie potentielle et ses conséquences", *C. R. Acad. Sc. Paris* **282 B**, 43 (1976).
- [13] O. Costa de Beauregard, "Distribution de l'énergie potentielle selon Louis de Broglie; Effet Mikhailov, prédiction d'un effet angulaire analogue", *Ann. Fond. Louis de Broglie* **26**, 165 (2001).
- [14] J. D. Jackson, *Classical Electrodynamics* (Wiley, New York, 1975), pp. 783-798.
- [15] P. A. M. Dirac, "Classical theory of radiating electrons", *Proc. Roy. Soc. London* **A167**, 148 (1938). Dirac uses the metric (+1, -1, -1, -1), while we use (-1, +1, +1, +1).

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