Symmetry Properties of Photon Eigenstates of Generalized de Broglie-Bargmann-Wigner Equations

H. Stumpf

Institute of Theoretical Physics, University Tübingen, Germany

ABSTRACT. Generalized de Broglie-Bargmann-Wigner equations are relativistically invariant quantum mechanical many body equations with nontrivial interaction, selfregularization and probability interpretation. In accordance with de Broglie's fusion theory the photon is assumed to be the fusion product of two fermionic constituents leading to a partonic substructure of the photon. For this case exact photon eigenstates of the generalized fusion equations are studied. In particular the relativistic transformation properties of such states are discussed and it is demonstrated that the internal part of the photon wave function is invariant under these transformations. In addition it is proved that the photon wave function is antisymmetric under permutations of space-time coordinates and algebraic indices which is a necessary property for selfconsistency of the formalism.

1 Introduction

In numerous high energy experiments a partonic substructure of photons has been detected, [1]. Usually this behavior of the photon is qualitatively explained by fluctuations of the photon in other states, [2]. But as was discussed in [3] such an interpretation leads to various difficulties. Thus another hypothesis is likely, namely that the photon possesses a permanent partonic substruture which is revealed in these experiments. This means that theoretically the photon has to be considered as a relativistic bound state.

The idea of the photon as a relativistic bound state has a long history. Seventy years ago de Broglie published his first papers on the theory of fusion based on two Dirac equations for a second order spin tensor, [4]. His aim was to obtain the photon as a fusion product of two neutrinos, i.e., to describe the photon as a relativistic bound state of neutrinos. Later on de Broglie, [5] and Bargmann and Wigner, [6] extended this formalism to the treatment of n Dirac equations for n-th order spin tensors and these fusion equations were evaluated by many authors afterwards, cf., [7], [8]. The principal success of this approach was the derivation of Maxwell's equations by de Broglie as an effective theory for the composite photon, but its drawback was the absence of interactions between the constituents in favor of a purely kinematical treatment of the fusion process. On the other hand with the further development of quantum field theory the de Broglie-Bargmann-Wigner formalism was superseded by genuine quantum field theoretic methods in order to obtain appropriate descriptions of relativistic bound states. The most preferred version of such a quantum field theoretic formalism was the theory of Bethe-Salpeter equations which was started fifty years ago, [9] and was elaborated by numerous authors, [10],[11],[12]. But also this approach leads to various difficulties, [10],[13]. So the construction of relativistic two-body (and many body) equations and the comparison between different methods are active areas of current research, [14].

In particular a quantum field theoretic formalism for relativistic bound states can be based on the idea that de Broglie's spin fusion should be caused by direct interactions of fermions without the assistance of bosons, because in this (in de Broglie's) picture the latter are fusioned objects and not elementary entities.

The corresponding theory which exclusively deals with spinorial interactions is based on a nonperturbatively regularized nonlinear spinor field with canonical quantization and probability interpretation. It can be considered as the quantum field theoretic generalization of de Broglie's fusion theory and as a mathematical realization and physical modification of Heisenberg's approach, [15], and is expounded in [16],[17]. Owing to this generalization the basic ingredients of this theory are assumed to be unobservable subfermions and not neutrinos, and only after having derived the conventional gauge theories as effective theories it is possible to introduce the corresponding (composite) gauge bosons into the dynamical interplay of matter.

In the following sections the results of these calculations and investigations are used, as far as they are of importance for our discussion which is concerned with the derivation of the symmetry properties of single photon eigenstates of generalzed de Broglie-Bargmann-Wigner equations. The information about these symmetry properties is crucial in two different respects: first it has to be demonstrated that these eigenstates are actual relativistic bound states and second a precise knowledge of these properties is needed if by means of these states quantum electrodynamics is to be derived as an effective theory of photons with partonic substructure. In spite of the fact that in this theoretical approach the photon states represent the most simple case of relativistic bound states, the investigation of their symmetry properties is rather extensive. Hence the whole subject cannot be treated in one paper. Complementary additional information to the treatment given here can be found in [18].

2 Partonic photon wave functions

By means of the field theoretic formalism wave equations for single composite particles with partonic substructure can be derived. The general theory is discussed in [16],[17] and details concerning photon states are given in [3],[8],[18]. Referring to these references we start with the hard core equations for composite photon states. We use the following notation in order to obtain clearly organized expressions:

$\mathbf{r} \in R^3$, $x \in M^4$, and $Z = (i, \kappa, \alpha)$ where $\kappa :=$ superspin-isospin index, $\alpha :=$ Dirac spinor index, $i :=$ auxiliary field index. The latter index characterizes the subfermion fields which are needed for regularization.

Let $\varphi_{Z_1 Z_2}(x_1, x_2)$ be the covariant, antisymmetric state amplitude of the composite particle or quantum , respectively. Then within the general formalism for this amplitude the following set of covariant photon equations can be derived:

$$\begin{aligned} &[D^{\mu}_{Z_1 X_1} \partial_\mu(x_1) - m_{Z_1 X_1}] \varphi_{X_1 Z_2}(x_1, x_2) = \\ &\qquad 3U_{Z_1 X_2 X_3 X_4} F_{X_4 Z_2}(x_1 - x_2) \varphi_{X_2 X_3}(x_1, x_1) \end{aligned} \tag{1}$$

and

$$\begin{aligned} &[D^{\mu}_{Z_2 X_2} \partial_\mu(x_2) - m_{Z_2 X_2}] \varphi_{Z_1 X_2}(x_1, x_2) = \\ &\qquad - 3U_{Z_2 X_2 X_3 X_4} F_{X_4 Z_1}(x_2 - x_1) \varphi_{X_2 X_3}(x_2, x_2) \end{aligned} \tag{2}$$

with the following definitions

$$D^{\mu}_{Z_1 Z_2} := i\gamma^{\mu}_{\alpha_1 \alpha_2} \delta_{\kappa_1 \kappa_2} \delta_{i_1 i_2} \tag{3}$$

and

$$m_{Z_1 Z_2} := m_{i_1} \delta_{\alpha_1 \alpha_2} \delta_{\kappa_1 \kappa_2} \delta_{i_1 i_2} \tag{4}$$

and

$$F_{Z_1Z_2}(x_1-x_2) := -i\lambda_{i_1}\delta_{i_1i_2}\gamma^5_{\kappa_1\kappa_2}[(i\gamma^\mu\partial_\mu(x_1)+m_{i_1})C]_{\alpha_1\alpha_2}\Delta(x_1-x_2,m_{i_1}) \tag{5}$$

where $\Delta(x_1-x_2,m_{i_1})$ is the scalar Feynman propagator. The meaning of the index κ can be explained by decomposing it into two parts $\kappa := (\Lambda, A)$ with $\Lambda = 1,2$ superspin index of spinors and charge conjugated spinors and A = 1,2 isospin index, which can be equivalently expressed by κ = 1,2,3,4 .

The vertex terms in equtions (1) and (2) are fixed by the following definitions:

$$U_{Z_1Z_2Z_3Z_4} := \lambda_{i_1}B_{i_2i_3i_4}V^{\kappa_1\kappa_2\kappa_3\kappa_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} \tag{6}$$

where $B_{i_2i_3i_4}$ indicates the summation over the auxiliary field indices and where the vertex V is given by a scalar and a pseudoscalar coupling of the subfermion fields:

$$V^{\kappa_1\kappa_2\kappa_3\kappa_4}_{\alpha_1\alpha_2\alpha_3\alpha_4} := \frac{g}{2}\{[\delta_{\alpha_1\alpha_2}C_{\alpha_3\alpha_4} - \gamma^5_{\alpha_1\alpha_2}(\gamma^5C)_{\alpha_3\alpha_4}]\delta_{\kappa_1\kappa_2}[\gamma^5(1-\gamma^0)]_{\kappa_3\kappa_4}\}_{as[2,3,4]} \tag{7}$$

The parameters λ_i originate from the regularization procedure and fulfill the conditions $\sum_i \lambda_i = 0$ and $\sum_i \lambda_i m_i = 0$ which guarantee the finiteness of the regularized expressions.

Similar equations but without auxiliary fields were treated by the Heisenberg group, [15]. These equations suffer from singularities and negative norm states, difficulties which are avoided by the present formalism. For vanishing coupling constant $g = 0$ de Broglie's original fusion equations for local photons are obtained, and for a solution of the set (1), (2) only equation (1) has to be used if the wave functions are antisymmetric.

Equation (1) admits exact solutions. We only give the result of such calculations and refer for details to [8],[16],[17],[18].

Let φ be a solution of equation (1). Then φ describes a vector boson with definite momentum k , if it is given by the following expression:

$$\varphi_{Z_1Z_2}(x_1,x_2) = T^a_{\kappa_1\kappa_2}exp[-i\frac{k}{2}(x_1+x_2)]A^\mu\chi^\mu_{\substack{i_1i_2\\ \alpha_1\alpha_2}}(x_1-x_2|k) \tag{8}$$

with the antisymmetric tensor $T^a_{\kappa_1\kappa_2}$ and the internal wave function

$$\chi^{\mu}_{\substack{i_1 i_2\\ \alpha_1\alpha_2}}(x) := \frac{2ig}{(2\pi)^4}\lambda_{i_1}\lambda_{i_2}\int d^4p e^{-ipx}[S_F(p+\frac{k}{2}, m_{i_1})\gamma^\mu S_F(p-\frac{k}{2}, m_{i_2})C]_{\alpha_1\alpha_2} \tag{9}$$

(no summation over i_1, i_2) and $S_F(p,m) := (i\gamma^\mu p_\mu - m)^{-1}$.

As equations (1) and (2) are homogenous equations their solutions (8) lead to secular equations for the eigenvalues which in this case are given by k^2 as a function of the value of the coupling constant g. And owing to the special structure of equations (1) and (2) the secular equations are finite and need no further regularization. As a consequence these equations allow a physical interpretation, see [18]. On the other hand the integral in (9) can be evaluated by standard methods and leads to a singular behavior on the light cone. Therefore for a physical interpretation a regularization is needed which has to be consistent with the corresponding secular equations. And concerning the physical interpretation of wave functions this clearly means that these functions must admit a probability interpretation.

For the general theory this problem was treated and solved in [19]. In accordance with [19] we apply this regularization to the special case of photon states under consideration. We define the physical, i.e. regularized state amplitudes by summation over the auxiliary field indices

$$\hat{\varphi}_{\substack{\alpha_1\alpha_2\\ \kappa_1\kappa_2}}(x_1, x_2) := \sum_{i_1 i_2} \varphi_{Z_1 Z_2}(x_1, x_2) \tag{10}$$

which is consistent with the secular equations. If these regularized states are to be physical states they must obey probability conservation. This proof was given in [19] and we refer for details to this paper, see also [3],[18].

In order to derive a probability interpretation for the boson wave functions their single time formulation has to be used. We decompose the index $Z := (\alpha, \kappa, i)$ into $Z = (z, i)$ and sum over i_1, i_2. Afterwards the limit to equal times is performed by means of a special symmetrical limit procedure. For the resulting amplitudes it is possible to derive exact constraints using equations (1) and (2). From these constraints one can derive a current conservation law for the density $\hat{\varphi}^\dagger\hat{\varphi}$, provided that for $m_i = m + \delta m_i$ the limit $\delta m_i = 0$ is considered.

This means: in this limit the physical state amplitudes $\hat{\varphi}$ describe stable bound states and are elements of a corresponding Hilbert space with the norm expression

$$\langle\hat{\varphi}|\hat{\varphi}\rangle = \sum_{z_1 z_2} \int d^3r_1 d^3r_2 \hat{\varphi}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2, t)^* \hat{\varphi}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2, t) \qquad (11)$$

Hence a quantum mechanical interpretation of the photon states is possible. But as the photon is a relativistic particle it has to be shown that the transformation properties of such states are consistent with the requirements of relativity.

3 Transformation properties of states

For $g = 0$ one obtains from (1) and (2) the original fusion equations of de Broglie (apart from the bilocal wave functions and the auxiliary fields). So the question arises whether it is imperative to take over de Broglie's neutrino interpretation to the interpretation of the generalized de Broglie-Bargmann-Wigner equations or whether there exist alternative interpretations and we will show that indeed such an alternative interpretation is possible (and necessary).

Without becoming too much engrossed into the quantum field theoretic background of the generalized de Broglie-Bargmann-Wigner equations, we only point out that the basic nonlinear spinor field is primarily formulated in terms of the field operators $\psi_{\alpha A}(x,i)$ and $\bar{\psi}_{\alpha A}(x,i)$ in order to secure the relativistic invariance of the corresponding Lagrangian. But in order to provide a unique group theoretical structure of the spinor field quantum theory it is necessary to use instead of the adjoint spinor field $\bar{\psi}_{\alpha A}(x,i)$ the charge conjugated spinor field $\psi^c_{\alpha A}(x,i)$. And although on this elementary level no charges can be defined at all and the basic field equations in both representations are completely equivalent, the group structure of the corresponding quantum theories differs. While in the $\psi, \bar{\psi}$ representation no meaningful permutation symmetry can be established, such a symmetry can be defined in the ψ, ψ^c representation.

To verify this conjecture we consider the transformation properties of spinor fields under infinitesimal Lorentz transformations. These transformations are defined by

$$\psi'(x') = S(a)\psi(a^{-1}x'); \qquad \bar{\psi}'(x') = \bar{\psi}(a^{-1}x')S^{-1}(a) \qquad (12)$$

with

$$S(a) = 1 - \frac{i}{4}\sigma_{\mu\nu}\omega^{\mu\nu}; \qquad S^{-1}(a) = 1 - \frac{i}{4}\sigma_{\mu\nu}\omega^{\mu\nu} \tag{13}$$

and $a_{\mu\nu} = \eta_{\mu\nu} + \omega_{\mu\nu}$, see [13]. With the definition

$$\psi^c(x) = C\bar{\psi}(x)^T \tag{14}$$

and the relations $\sigma_{\mu\nu}C = -C\sigma^T_{\mu\nu}, \quad \sigma^T_{\mu\nu} = -\sigma_{\nu\mu}$ one obtains

$$\psi^{c\prime}(x') = S(a)\psi^c(a^{-1}x') \tag{15}$$

Hence a superspinor $\psi_{\alpha A\Lambda}(x,i)$ can be introduced by the definition

$$\psi_{\alpha A1}(x,i) := \psi_{\alpha A}(x,i), \qquad \psi_{\alpha A2}(x,i) := \psi^c_{\alpha A}(x,i) \tag{16}$$

for $\Lambda = 1,2$ with the transformation property

$$\psi'_{\alpha' A\Lambda}(x',i) = S_{\alpha'\alpha}(a)\psi_{\alpha A\Lambda}(a^{-1}x',i) \tag{17}$$

and the set (A,Λ) can be replaced by κ defined in the preceding section. In this representation the basic spinor field equation reads [16],[17]

$$(D^\mu_{Z_1Z_2}\partial_\mu - m_{Z_1Z_2})\psi_{Z_2}(x) = U_{Z_1Z_2Z_3Z_4}\psi_{Z_2}(x)\psi_{Z_3}(x)\psi_{Z_4} \tag{18}$$

with $\psi_Z(x) := \psi_{\alpha\kappa}(x,i)$ and the definitions of section 2 .

The generalized de Broglie-Bargmann-Wigner equations (1) and (2) are derived from this basic spinor field equation and thus must be interpreted in the same way, i.e., de Broglie's neutrino interpretation is replaced by an interpretation in terms of spinor fields and charge conjugated spinor fields. Owing to the complete homogeneity of the transformation law and the dynamical law for superspinors, the latter quantities can be considered as indistinguishable and thus are objects to which the permutation group can be applied in quantum theory. This will be discussed in section 5.

In the covariant quantum version of the spinor field the basic quantities of the theory are defined by time ordered matrix elements $\langle 0|T\psi_{Z_1}(x_1)...\psi_{Z_n}(x_n)|k\rangle$ where $\langle 0|$ is the groundstate of the quantized spinor field and $|k\rangle$ any excited state. With respect to our problem we consider the transformation property of $\varphi_{Z_1Z_2}(x_1,x_2|k) =$

$\langle 0|T\psi_{Z_1}(x_1)\psi_{Z_2}(x_2)|k\rangle$ for orthochronous Lorentz transformations which preserve time ordering. Then for $t_1 > t_2$ one obtains

$$\varphi'_{Z'_1 Z'_2}(x'_1, x'_2) = S_{\alpha'_1\alpha_1}(a) S_{\alpha'_2\alpha_2}(a) \varphi_{\alpha_1\kappa_1 i_1, \alpha_2\kappa_2 i_2}(a^{-1}x'_1, a^{-1}x'_2) \quad (19)$$

and although the eigenstates of equations (1) and (2) are no eigenstates of the full theory we assume that they obey the same transformation law for consistency.

Owing to this tensor transformation law (19) no discrimination between spinors and charge conjugated spinors can be made by means of space-time transformations. However the spinor field equation is invariant under $U(1) \otimes SU(2)$ transformations [16],[17] and one easily realizes that if the spinor field operator has the fermion number *f*, then the conjugated spinor field has the fermion number -*f* . So by means of this transformation one can classify the content of both kinds of spinor fields in the matrix element under consideration. In combination with the isospin quantum numbers this information is exclusively contained in the tensor $T_{\kappa_1\kappa_2}$ which was discussed in detail in [3],[16],[17].

4 Relativistic invariance of photon states

In the preceding section it was explained that owing to the introduction of charge conjugated spinor fields the superspin-isospin transformations can be decoupled from space-time transformations. For photon states this fact is expressed by equation (8) which we equivalently write in the form

$$\varphi_{Z_1 Z_2}(x_1, x_2) = T^a_{\kappa_1\kappa_2} \frac{2ig}{(2\pi)^4} \varphi_{\substack{i_1 i_2 \\ \alpha_1\alpha_2}}(x_1, x_2) \quad (20)$$

with

$$\begin{aligned}\varphi_{\substack{i_1 i_2 \\ \alpha_1\alpha_2}}(x_1, x_2) = {} & e^{-i\frac{k}{2}(x_1+x_2)} \lambda_{i_1}\lambda_{i_2} \int d^4p e^{-ip(x_1-x_2)} f_{i_1}(p + \frac{1}{2}k) f_{i_2}(p - \frac{1}{2}k) \times \\ & \{[(p_\rho + \frac{1}{2}k_\rho) A_\mu (p_\lambda - \frac{1}{2}k_\lambda)\gamma^\rho\gamma^\mu\gamma^\lambda + (p_\rho + \frac{1}{2}k_\rho) A_\mu m_{i_2}\gamma^\rho\gamma^\mu \\ & A_\mu(p_\lambda - \frac{1}{2}k_\lambda) m_{i_1}\gamma^\mu\gamma^\lambda + A_\mu m_{i_1} m_{i_2}\gamma^\mu] C\}_{\alpha_1\alpha_2} \quad (21)\end{aligned}$$

where the f_i are the Lorentz invariant denominators $f_i(p) := (p^2 - m_i^2)^{-1}$. If one applies a Lorentz transformation $x'_\mu = a^\nu_\mu x_\nu$ to (21) one obtains according to (19)

$$\varphi'_{\substack{i_1 i_2 \\ \alpha'_1 \alpha'_2}}(x'_1, x'_2) = S_{\alpha'_1\alpha_1}(a) S_{\alpha'_2\alpha_2}(a) \varphi_{\substack{i_1 i_2 \\ \alpha_1 \alpha_2}}(a^{-1}x'_1, a^{-1}x'_2) \tag{22}$$

To evaluate this expression, i.e., to show its relativistic invariance we derive the following relation

$$S_{\alpha\beta}(a) S_{\delta\gamma}(a)(\gamma^{\mu_1}...\gamma^{\mu_n}C)_{\beta\gamma} = (a^{-1})^{\mu_1}_{\mu'_1}...(a^1)^{\mu_n}_{\mu'_n}(\gamma^{\mu'_1}...\gamma^{\mu'_n}C)_{\alpha\delta} \tag{23}$$

by means of the transformation law for γ^μ and by use of infinitesimal transformations.

Transformation of (21) according to (22) and substitution of (23) into this expression leads with

$$p' = p(a^{-1}), \qquad k' = k(a^{-1}), \qquad A' = A(a^{-1}) \tag{24}$$

and $f_i(p + \frac{1}{2}k) = f_i(p' + \frac{1}{2}k')$ to the transformed photon state

$$\begin{aligned} \varphi' = & exp[-i\frac{1}{2}k'(x'_1 + x'_2)]\lambda_{i_1}\lambda_{i_2} \int d^4p' exp[-ip'(x'_1 - x'_2)] f_{i_1} f_{i_2} \times \\ & \{(p'_\rho + \frac{1}{2}k'_\rho)A'_\mu(p'_\lambda - \frac{1}{2}k'_\lambda)(\gamma^\rho\gamma^\mu\gamma^\lambda C) + (p'_\rho + \frac{1}{2}k'_\rho)A'_\mu m_{i_2}(\gamma^\rho\gamma^\mu C) \\ & m_{i_1} A'_\mu (p'_\lambda - \frac{1}{2}k'_\lambda)(\gamma^\mu\gamma^\lambda C) + m_{i_1} m_{i_2} A'_\mu(\gamma^\mu C)\}_{\alpha'_1\alpha'_2} \end{aligned} \tag{25}$$

where the primes on the spinor indices in the internal summations have been omitted. Comparison with (21) shows that (21) is an invariant expression under homogeneous Lorentz transformations.

Finally we consider inhomogenous Lorentz transformations. Let us define such an inhomogeneous transformation by $\mathcal{L}(h, a)$ where a denotes the homogenous part, while h denotes the translation vector. Then the following decomposion holds, [20]:

$$\mathcal{L}(h, a) = \mathcal{L}(h, 1)\mathcal{L}(0, a) \tag{26}$$

If such a transformation is applied to (21) then according to the decomposition (25) the transformation $\mathcal{L}(0, a)$ leads to the result (24), while the translation part $\mathcal{L}$ acts exclusively on the plane wave part $exp[-i\frac{1}{2}k(x_1+x_2)]$ in the usual manner. Hence the relativistic covariance of (21) under inhomogeneous Lorentz transformations is demonstrated.

5 Permutation symmetry

Although not explicitly recorded the generalized de Broglie-Bargmann-Wigner equations (1) and (2) are referred to antisymmetric wave functions. This property is expressed by antisymmetrizers,[18], which for brevity have been omitted in (1) and (2). Hence for reasons of consistency the solutions (8) must be antisymmetric because otherwise they are worthless.

Owing to the decomposition into superspin-isospin part and spin part, one can both parts discuss separately. According to [8],[3],[18] or [16],[17], respectively, the superspin-isospin tensors $T^a_{\kappa_1\kappa_2}$ are antisymmetric for photon states. Hence the spin part (21) must be symmetric under permutations of the remaining general coordinates $(i_1, \alpha_1, x_1), (i_2, \alpha_2, x_2)$ in order to secure the antisymmetry of the whole wave function.

For convenience we rewrite (21) in the following form:

$$
\begin{aligned}
(21) = e^{-ik'(x_1+x_2)} \int d^4p e^{-ip(x_1-x_2)} \{ & [R_\nu(p+k', i_1) R_\tau(p-k', i_2) \gamma^\nu \gamma^\mu \gamma^\tau \\
& S(p+k', i_1) R_\tau(p-k', i_2) \gamma^\mu \gamma^\tau + R_\nu(p+k', i_1) S(p-k', i_2) \gamma^\nu \gamma^\mu \\
& S(p+k', i_1) S(p-k', i_2) \gamma^\mu] C \}_{\alpha_1\alpha_2} A_\mu
\end{aligned} \tag{27}
$$

with $k' = 1/2k$ and

$$
R_\nu(p, i) := p_\nu \lambda_i (p^2 - m_i^2 + i\epsilon)^{-1}; \qquad S(p, i) := m_i \lambda_i (p^2 - m_i^2 + i\epsilon)^{-1} \tag{28}
$$

Then it has to be verified that for (21) the permutation symmetry

$$
\varphi_{\substack{i_1 i_2 \\ \alpha_1 \alpha_2}}(x_1, x_2) = \varphi_{\substack{i_2 i_1 \\ \alpha_2 \alpha_1}}(x_2, x_1) \tag{29}
$$

holds.

Now consider the first term in the bracket in (27). After permutation (29) this term reads

$$
R_\nu(p+k', i_2) R_\tau(p-k', i_1) (\gamma^\nu \gamma^\mu \gamma^\tau C)_{\alpha_2\alpha_1} e^{-ip(x_2-x_1)} \tag{30}
$$

We replace in (30) p by $-p'$ and obtain the equalities

$$
R_\nu(-p'+k', i_2) = -R_\nu(p'-k', i_2) \tag{31}
$$

and

$$R_\tau(-p'+k',i_1) = -R_\tau(p'+k',i_1) \tag{32}$$

Thus it follows with $\tau \to \nu$ and $\nu \to \tau$

$$(30) = R_\nu(p'+k',i_1)R_\tau(p'-k',i_2)(\gamma^\tau\gamma^\mu\gamma^\nu C)_{\alpha_2\alpha_1}e^{-ip'(x_1-x_2)} \tag{33}$$

Furthermore it can be verified that

$$(\gamma^\tau\gamma^\mu\gamma^\nu C)_{\alpha_2\alpha_1} = (\gamma^\nu\gamma^\mu\gamma^\tau C)_{\alpha_1\alpha_2} \tag{34}$$

holds, and with $d^4p = d^4p'$ one eventually obtains

$$\begin{aligned}&\int d^4pR_\nu(p+k',i_2)R_\tau(p-k',i_1)(\gamma^\nu\gamma^\mu\gamma^\tau C)_{\alpha_2\alpha_1}e^{-ip(x_2-x_1)} \\ =&\int d^4p'R_\nu(p'+k',i_1)R_\tau(p'-k',i_2)(\gamma^\nu\gamma^\mu\gamma^\tau C)_{\alpha_1\alpha_2}e^{-ip'(x_1-x_2)}\end{aligned} \tag{35}$$

i.e., the first term of (27) is symmetric under permutation of the general coordinates $(i_1\alpha_1x_1, i_2\alpha_2x_2)$.

To verify the symmetry of the other terms in (27) one has to proceed in an analogous manner.

Consider the second term and the third term of (27). Permutation leads to

$$\begin{aligned}&[S(p+k',i_2)R_\tau(p-k',i_1)(\gamma^\mu\gamma^\tau C)_{\alpha_2\alpha_1} \\ &\qquad + R_\nu(p+k',i_2)S(p-k',i_1)(\gamma^\nu\gamma^\mu C)_{\alpha_2\alpha_1}]e^{-ip(x_2-x_1)}\end{aligned} \tag{36}$$

If now p is replaced by $-p'$, and the definitions (28) and (31),(32) are applied, then after relabeling $\tau \to \nu, \nu \to \tau$ one obtains from (36)

$$\begin{aligned}&-[S(p'+k',i_1)R_\tau(p'-k',i_2)(\gamma^\tau\gamma^\mu C)_{\alpha_2\alpha_1} \\ &\qquad + R_\nu(p'+k',i_1)S(p'-k',i_2)(\gamma^\mu\gamma^\nu C)_{\alpha_2\alpha_1}]e^{-ip'(x_1-x_2)}\end{aligned} \tag{37}$$

Owing to the relation

$$(\gamma^\nu\gamma^\mu C)_{\alpha_2\alpha_1} = -(\gamma^\mu\gamma^\nu C)_{\alpha_1\alpha_2} \tag{38}$$

which holds for $\mu = \nu$ too and the invariance of the volum element $d^4p = d^4p'$ also these terms in (27) are symmetric. In the same manner the symmetry of the last term in (27) is shown.

Hence the photon wave functions possess the symmetry properties which are required for the self consistency of the solution procedure.

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(Manuscrit reçu le 24 juin 2002)