Frequency in Relational Mechanics

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ABSTRACT. We analyze the dependence of the frequency of oscillation of several motions as regards the density of distant gravitational matter according to relational mechanics. We conclude that in most situations the frequency is inversely proportional to the square root of this density.

Key words: relational mechanics, gravitational mass.

PACS: 04.50.+h (Unified field theories and other theories of gravitation), 12.25.+e (Models for gravitational interactions).

1 Introduction:

In the last few years there has been a renewed interest in Weber's law as applied to electromagnetism and gravitation, [Sokolskii and Sadovnikov, 1987], [Wesley, 1990], [Phipps, 1992], [Assis, 1994], [Kinzer and Fukai, 1996], [Guala-Valverde, 1998], [Assis, 1999], [Bueno and Assis, 2001]. Weber's force exerted by particle j located at $\vec{r}_j$ on particle i located at $\vec{r}_i$ relative to the origin O of a frame of reference S is given by

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$$\vec{F}_{ji} = D_{ij}\frac{\hat{r}_{ij}}{r_{ij}^2}\left[1+\frac{\xi}{c^2}\left(r_{ij}\ddot{r}_{ij}-\frac{\dot{r}_{ij}^2}{2}\right)\right], \quad (1)$$

Here $r_{ij} = |\vec{r}_i - \vec{r}_j|$ is the distance between the particles, $\hat{r}_{ij} = (\vec{r}_i - \vec{r}_j)/r_{ij}$ is the unit vector pointing from particle j to particle i, $\dot{r}_{ij} = dr_{ij}/dt$ is the radial relative velocity between the particles, $\ddot{r}_{ij} = d^2 r_{ij}/dt^2$ is the radial relative acceleration between the particles and $c = 3\times 10^8 m/s$. In the case of electromagnetism the particles are charges q_i and q_j, $D_{ij} = H_e q_i q_j$ and $\xi = 1$, with H_e being a constant. In the case of gravitation we have gravitational masses m_{gi} and m_{gj}, $D_{ij} = -H_g m_{gi} m_{gj}$ and $\xi = 6$, with H_g being a constant.

Relational mechanics is a mathematical implementation of Mach's principle utilizing Weber's law for gravitation and the principle of dynamical equilibrium according to which the sum of all forces acting on any body is always zero in all frames of reference, [Assis, 1989a] and [Assis, 1999]. Consider a test body of gravitational mass m_{g1} interacting with local bodies and with the distant universe. The force exerted by the local bodies and by anisotropic distribution of mass around it will be represented by $\sum_{j=1}^{N}\vec{F}_{j1}$ The force due to isotropic distribution of masses around body 1 would go to zero according to Newton's inverse square law of gravitation. But according to Weber's law this force is not zero anymore if there is a relative acceleration between the test body and the distant masses, due to the acceleration component of Weber's force (1) which falls only as 1/r. Integrating this gravitational force acting on m_{g1} due to the isotropic distribution of gravitational mass of the distant universe in the universal frame of reference (frame in which the set of distant galaxies are seen without rotation and without linear acceleration) yields, [Assis, 1989a] and [Assis, 1999]: $-\Phi m_{g1}\vec{a}_{1U}$, where

$\vec{a}_{1U}$ is the acceleration of body 1 relative to the universal frame of reference U and $\Phi = 2^n \pi \xi H_g \rho_{go} R_o^2 / 3c^2$ Here ρ_{go} is the average gravitational mass density of the distant universe, R_o is the radius of the known universe related with Hubble's constant H_o by $R_o = c / H_o$. The value of n depends on the cosmological model adopted. Utilizing a finite universe and integrating up to R_o yields $n = 1$. For an infinite and boundless universe we can integrate to infinity utilizing an exponential decay in gravitation, e^{-r/R_o}, yielding $n = 2$.

We will analyze here oscillatory macroscopic motions due to gravitational, electric and elastic interactions. As regards the local forces we will consider only slow motions in which $\dot{r}_{ij}^2 << c^2$ and $r_{ij}\ddot{r}_{ij} << c^2$. The elastic force will be represented by $-K(\vec{\ell}_1 - \vec{\ell}_o)$, where $\vec{\ell}_1$ is the position of body 1 when the spring is compressed or stretched, and $\vec{\ell}_o$ the position of body 1 when the spring is relaxed, with K being the elastic constant in relational mechanics. The general equation of motion of relational mechanics for body 1 of gravitational mass m_{g1} and electrical charge q_1 interacting with gravitational mass m_{g2}, with electrical charge q_2, with a spring of elastic constant K and with the distant universe can then be written as (with the approximation above of slow motions):

$$-H_g \frac{m_{g1} m_{g2}}{r_{12}^2} \hat{r}_{12} + H_e \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} - K\left(\vec{\ell}_1 - \vec{\ell}_o\right) - \Phi m_{g1} \vec{a}_{1U} = 0 \ . \quad (2)$$

Passing the last term to the right hand side and dividing by Φ yields an analogous to Newton's second law of motion, namely:

$$-G \frac{m_{g1} m_{g2}}{r_{12}^2} \hat{r}_{12} + \frac{1}{4\pi \varepsilon_o} \frac{q_1 q_2}{r_{12}^2} \hat{r}_{12} - k\left(\vec{\ell}_1 - \vec{\ell}_o\right) = m_{g1} \vec{a}_{1U} \ , \quad (3)$$

where $G = H_g / \Phi = 3H_o^2 / 2^n \pi \xi \rho_{go}$, $1/4\pi \varepsilon_o = H_e / \Phi$ and $k = K / \Phi$ are the usual constants which appear in classical mechanics.

From Eq. (2) we see that the influence of the distant universe is embodied in the constant Φ. The local gravitational masses m_{g1} and m_{g2} and the local charges q_1 and q_2 are supposed to be independent of the distant universe, that is, we assume that they do not depend on H_o nor on ρ_{go}.

2 Static Situations:

We begin analyzing static situations, that is, situations in which the test body is not accelerated relative to the distant universe, $\vec{a}_{1U} = 0$. Our first goal is to understand the dependence of the constants G, ε_o and k on Hubble's constant and on ρ_{go}.

Comparing Eqs. (2) and (3) we observe that the universal constant of classical physics is proportional to the square of Hubble's constant and inversely proportional to the average gravitational density of distant matter, namely:

$$G = \frac{3}{2^n \pi \xi} \frac{H_o^2}{\rho_{go}} \text{ or } G \; \alpha \; \frac{H_o^2}{\rho_{go}} . \tag{4}$$

That is, G in relational mechanics is proportional to H_o^2 / ρ_{go}.

Consider now two large bodies of gravitational masses m_{gA} and m_{gB} at rest in the universal frame of reference. Their gravitational attraction is balanced by a mechanical force, for instance, by a rigid mechanical bar supporting them. A free test body of gravitational mass m_{g1} may be in static equilibrium between them due only to the gravitational attractions of m_{gA} and m_{gB} provided $G m_{g1} m_{gA} / r_{1B}^2 = G m_{g1} m_{gB} / r_{1B}^2$, that is:

$$\frac{m_{gA}}{m_{gB}} = \left(\frac{r_{1B}}{r_{1A}} \right)^2 , \tag{5}$$

where r_{1A} and r_{1B} are the distances of body 1 to bodies A and B, respectively. Changing the gravitational density of the distant universe will not change the ratio m_{gA} / m_{gB}. From Eq. (5) this means that body 1 will remain in equilibrium and the ratio r_{1B} / r_{1A} will remain the same, no matter the value of ρ_{go}.

Consider now the free test body 1 being held in static equilibrium between the gravitational mass m_{gA} and a charge Q_B. Once more we suppose the bodies A and B at rest in the universal frame of reference, with the electric and gravitational forces between them being balanced by a rigid mechanical bar. The free test body 1 can be in static equilibrium between them (at a distance r_{1A} to the center of body A and r_{1B} to the charge B) provided $Gm_{g1}m_{gA} / r_{1A}^2 = q_1 Q_B / 4\pi \varepsilon_o r_{1B}^2$. This can also be written as $\varepsilon_o G = (1/4\pi)(q_1 Q_B / m_{g1} m_{gA})(r_{1A} / r_{1B})^2$. As the charges, the gravitational masses and the ratio of distances do not depend on the distant universe, the same must be concluded of the left hand side of this equation. From Eq. (4) we then conclude that

$$\varepsilon_o \ \alpha \ \frac{\rho_{go}}{H_o^2} \ . \qquad (6)$$

Consider now the test body being held in static equilibrium due to a gravitational and an elastic force. We can think of body 1 being attracted gravitationally by the gravitational mass m_{gA} at a distance r_{1A} from its center, with a spring compressed and at rest between the surface of body A and body 1. If the relaxed length of the spring is ℓ_o and its length when compressed is ℓ the condition of equilibrium is $Gm_{g1}m_{gA} / r_{1A}^2 = k(\ell_o - \ell)$. Supposing that the distances do not depend on the distant universe yields G/k as constant whatever the value of ρ_{go}. From Eq. (4) we then conclude that

$$k \ \alpha \ \frac{H_o^2}{\rho_{go}} \ . \qquad (7)$$

We are now ready to analyze the influence of the gravitational density of distant matter on the frequency of oscillation of macroscopic bodies according to relational mechanics.

3 Gravitational Interactions:

We first consider two equal bodies of gravitational masses m_g separated by a distance *2R* performing a circular orbit around one another relative to the universal frame of reference. Their acceleration is then the centripetal acceleration given by $a_{1U} = \omega_{1U}^2 R$, where ω_{1U} is the angular rotation of one of the bodies relative to the frame of distant galaxies (each body is at a distance R from the center of the circular orbit). The equation of motion in this case is given by $Gm_g^2 / (2R)^2 = m_g \omega_{1U}^2 R$. The frequency of rotation is then given by $\omega_{1U} = \sqrt{Gm_g / 4R^3}$. With (4) and assuming that m_g and that R do not depend on the distant universe yields

$$\omega_{1U} \; \alpha \; \frac{H_o}{\sqrt{\rho_{go}}} \; . \tag{8}$$

We can also say that if we multiply the gravitational density of the distant universe by 4 then the frequency of rotation will by divided by 2 provided bodies of the same gravitational mass are rotating around one another relative to the distant universe separated by the same distance 2R. That is, if ω_o is the frequency of oscillation when the distant universe has a gravitational mass density ρ_{go} and ω is the frequency of oscillation for the same motion when the universe has a gravitational mass density ρ_g, Eq. (8) can also be written as: $\omega / \omega_o = \sqrt{\rho_{go} / \rho_g}$.

Consider now a spherical body of gravitational mass M_g and radius R with an uniform density $\rho_g = M_g / (4\pi \, R^3 / 3)$. If we have a radial tunnel passing through its center and a test body of gravitational mass m_g is released at a distance r_o $(r_o < R)$ from the center of the sphere the equation of motion will be: $GM_g m_g r / R^3 = m_g \ddot{r}$. The solution of this equation is $r(t) = r_o \cos \omega t$, where $\omega = \sqrt{GM_g / R^3}$. From Eq. (4) and supposing that M_g and R do not depend on the distant universe we obtain once more Eq. (8).

Another situation involving gravitation and rotation is related with the figure of the earth. Due to its diurnal rotation relative to the distant universe with a period of one day the equatorial radius of the earth $R_>$ is bigger than the polar radius $R_<$. The fractional change (its oblateness) according to relational mechanics is given by [Assis, 1999, Section 9.5.1]:

$$\frac{R_> - R_<}{R_<} \approx \frac{5\xi}{8} \frac{\rho_{go}}{\rho_{ge}} \left(\frac{\omega_{eU}}{H_o} \right)^2 . \tag{9}$$

In this expression ρ_{ge} is the gravitational mass density of the earth and ω_{eU} is its angular rotation relative to the distant universe. If the gravitational density of the distant universe goes to zero, the same will happen with the fractional change, that is, the earth will become spherical. This is reasonable because in this case there will be only the earth in the universe, which means that we cannot speak of its rotation. Supposing now a constant gravitational mass density for the earth and a fixed fractional change or oblateness, we obtain from Eq. (9) the same result as Eq. (8).

4 Electromagnetic Interactions:

We now consider two opposite charges $+q$ and $-q$ separated by a distance 2R orbiting around one another relative to the distant universe with angular frequencies ω_{1U}. We suppose they have equal gravitational masses m_g and that the electrical force between them is much larger than the gravitational force between them. The equation of motion is then given by $q^2 / 4\pi \varepsilon_o (2R)^2 = m_g \omega_{1U}^2 R$. Supposing that q, m_g and R do not depend on the distant universe and utilizing Eq. (6) yields once more Eq. (8).

We now consider a test charge q of gravitational mass m_g describing a circular orbit in an uniform magnetic field B. As the source of this field we consider a spherical shell of radius R uniformly charged with a total charge Q and spinning relative to the universal frame of reference U with a constant angular velocity $\vec{\Omega}_{QU} = \Omega_{QU}\hat{z}$. According to classical electromagnetism this system generates an uniform magnetic field inside the shell given by $\vec{B} = \mu_o Q\, \Omega_{QU}\hat{z} / 6\pi R$. We consider the test charge moving in a circular orbit in the xy plane inside the shell, describing a circle of radius $r < R$ centered on the center of the shell due to a magnetic force $q\vec{v} \times \vec{B}$, [Assis, 1989b] and [Assis, 1992]. We are then led to $qvB = m_g a_{1U}$ or $q\omega_{1U} rB = m_g \omega_{1U}^2 r$. This yields $\omega_{1U} = qB / m_g$. With the previous value of the magnetic field we are led to

$$\frac{\omega_{1U}}{\Omega_{QU}} = \frac{\mu_o qQ}{6\pi R m_g} \tag{10}$$

As the left hand side of this equation is a ratio of two frequencies, it cannot depend on H_o^2 / ρ_{go}. The same must then be true of the right hand side. As q, Q and m_g are local quantities and we are supposing that the radius R of the shell does not depend on the distant universe, we conclude that μ_o must be a constant whatever the value of H_o^2 / ρ_{go}:

$$\mu_o = \text{constant whatever the value of } \frac{H_o^2}{\rho_{go}}. \tag{11}$$

We are now able to analyze LC circuits. The frequency of oscillation of these circuits is given by $\omega = 1 / \sqrt{LC}$, where L and C are the self-inductance and capacitance of the circuit, respectively. We consider only a representative case as all the others will behave similarly: a capacitor composed of two large plane areas A separated by a small distance d, such that $C = \varepsilon_o A / d$, in series with a solenoid of length ℓ and radius a composed of N turns of a wire, such that $L = \mu_o (N / \ell)^2 \pi a^2 \ell$. Utilizing (6), (11), $\omega = 1 / \sqrt{LC}$ and supposing once more that all distances and sizes do not depend on the distant universe yields once more Eq. (8).

5 Elastic Interactions:

We now consider a body of gravitational mass m_g describing a circular orbit relative to the universal frame of reference around a central point due to a stretched spring. When relaxed the length of the spring is ℓ_o, while stretched it is given by ℓ. The equation of motion is then given by $k(\ell - \ell_o) = m_g \omega_{1U}^2 \ell$. With Eq. (7) we obtain

$$\frac{\ell - \ell_o}{\ell} = \frac{\Phi}{K} m_g \omega_{1U}^2 = \frac{2^n \pi \xi}{3} \frac{H_g}{K} \frac{\rho_{go}}{H_o^2} m_g \omega_{1U}^2 \ . \tag{12}$$

If ρ_{go} goes to zero, the spring returns to its relaxed length. On the other hand, supposing ℓ, ℓ_o, H_g, K, m_g fixed no matter the value of ρ_{go} and H_o, we then the same result as Eq. (8).

Instead of rotation we now consider a spring fixed at one extremity and with a body of gravitational mass m_g oscillating around the equilibrium position (one dimensional rectilinear motion), relative to the universal frame of reference. The equation of motion is then given by $k(\ell - \ell_o) = m_g \ddot{\ell}$. The solution of this equation gives the position of the test body as a function of time, namely: $\ell(t) = A\cos(\omega_{1U} t + B)$, where $\omega_{1U} = \sqrt{k / m_g}$ is its frequency of oscillation relative to the universal frame of reference. Applying Eq. (7) we obtain once more Eq. (8).

Consider now a simple pendulum oscillating in a vertical plane due to the gravitational force of the earth and to the tension in the string. To clarify the analysis we replace the string by a spring of stretched length ℓ and relaxed length ℓ_o, in such a way that the tension T in the string can be written as

$T = k(\ell - \ell_o)$, *k* being the elastic constant of the spring. Suppose the pendulum is dislocated at an angle θ from the vertical direction and released from rest. We obtain the equations of motion in the radial and tangential directions as, respectively: $m_g g \cos\theta = k(\ell - \ell_o)$ and $m_g g sin\theta = m_g \ell \ddot{\theta}$. Supposing small displacements such that $sin\theta \approx \theta$ this last equation yields the solution $\theta(t) = A\cos(\omega_{1U} t + B)$, where $\omega_{1U} = \sqrt{\ell / g} = \sqrt{\ell R_e^2 / GM_{ge}}$ is the angular frequency of the pendulum relative to the universal frame of reference. Here M_{ge} is the gravitational mass of the earth and R_e its radius. Applying Eq. (4) and supposing once more that M_{ge}, R_e and ℓ do not depend on H_o^2 / ρ_{go} yields once more Eq. (8).

The same conclusion can be obtained for a conical pendulum. In this case we have a test body of gravitational mass m_g moving in a circular orbit in a plane orthogonal to the gravitational field of the earth connected to a string. Once more we replace the string by the spring above. The angle of the spring with the vertical direction is given by a constant ϕ when the body moves in a circular orbit. Calling by *r* the radius of the circle and by *h* the distance of its center to the point of support of the spring we have $\tan\phi = r / h$. The angular rotation of the test body relative to the universal frame of reference is represented by ω_{1U}. The horizontal and vertical components of the equation of motion are given by, respectively: $k(\ell - \ell_o) sin\phi = m_g \omega_{1U}^2 R$ and $k(\ell - \ell_o)\cos\phi = Gm_g M_{ge} / R_e^2$. From these equations and from Eq. (4) we obtain

$$\tan\phi = \frac{2^n \pi \xi}{3} \frac{\rho_{go}}{H_o^2} \frac{\omega_{1U}^2 R_e^2 r}{M_{ge}} . \qquad (13)$$

Supposing as above that the distances $(R_e,\ \ell,\ \ell_o,\ r,\ h)$ do not depend on H_o^2 / ρ_{go}, the same will happen with $\tan\phi$. As M_{ge} is also independent of H_o^2 / ρ_{go}, we derive once more Eq. (8).

6 Alternative Formulation:

There is an alternative formulation of relational mechanics which leads essentially to the same results, [Guala-Valverde, 1999a and 1999b]. Instead of Eq. (2) we obtain

$$-\frac{m_{g1}m_{g2}}{r_{12}^2}\hat{r}_{12}+\frac{1}{4\pi\varepsilon_o}\frac{q_1q_2}{r_{12}^2}\hat{r}_{12}-k\left(\vec{\ell}_1-\vec{\ell}_o\right)-\Phi_G m_{g1}\vec{a}_{1U}=0 \tag{14}$$

where $\Phi_G = 2^n \pi \xi \rho_{go} / 3H_o^2$. Following the same procedures as above yields $G \, \alpha \, H_o^4 / \rho_{go}^2$, ε_o constant whatever the value of H_o^2 / ρ_{go}, $\mu_o \, \alpha \, \rho_{go} / H_o^2$ and the elastic coefficient *k* constant whatever the value of H_o^2 / ρ_{go}. Although these results are different from Eqs. (4), (6) and (7), this is not important. After all we do not measure directly G, ε_o nor *k*. What we measure are distances and periods (or frequencies). And this formulation also leads to Eq. (8) in all cases, supposing as above that the masses, charges and local distances are independent of H_o^2 / ρ_{go}.

7 Conclusion and Experimental Tests:

Supposing that the charges and gravitational masses of local bodies do not depend on H_o^2 / ρ_{go}, and supposing that the same happens with all local distances, we concluded that the frequencies of rotation or of vibration are inversely proportional to the square root of the mean density of gravitational mass of the distant universe, Eq. (8). That is, increasing the density of gravitational mass of distant galaxies will slow down the frequencies of the local motions analyzed here. This seems to be an universal property valid for all oscillatory or vibratory motions. If the frequency of a circular or vibratory motion is given by ω_o in our universe with an average gravitational mass density ρ_{go} and we change this density to ρ_g, the new frequency will be given by: $\omega / \omega_o = \sqrt{\rho_{go} / \rho_g}$.

As the radii of curvature of circular motions are supposed independent of the distant universe and the centripetal acceleration is given by $a_{1U} = (v_{1U}^2 / \mathrm{r})\ \alpha\ (H_o^2 / \rho_{go})$, we conclude that all velocities scale as: $v_{1U}\ \alpha\ H_o / \sqrt{\rho_{go}}$. Even light velocity will scale like this utilizing Eqs. (6) and (11). If the velocity of a particle in our universe with ρ_{go} is v_o, then the new velocity in an universe with ρ_g is given by: $v / v_o = \sqrt{\rho_{go} / \rho_g}$. On the other hand all the accelerations scale as: $a / a_o = \rho_{go} / \rho_g$.

As these effects are universal, it might be thought that it would be impossible to detect it. That is, as all frequencies (and velocities) scale as $1 / \sqrt{\rho_{go}}$, the ratio of any two frequencies (or of any two velocities) is independent of the distant universe. As we only measure frequencies (or velocities) by comparing it with other frequencies (or velocities), the effect will not be detected by changing the density of gravitational mass of the distant universe (for instance, the ratio of the orbital period of the earth of one year divided by its diurnal period of one day, $T_o / T_d = 365.4$, will not change by modifying the gravitational density of the distant universe). But the effect can be detected in principle in the laboratory by modifying the surroundings of some systems but not of other systems. Suppose we have two equal systems with equal oscillatory motions of frequency ω_o. We now surround one of the systems with a spherical shell of gravitational mass M_g and radius *R* at rest relative to the universal frame of reference. According to relational mechanics this shell will exert a gravitational force on any internal test particle which is undergoing an oscillatory motion given by ([Assis, 1999]): $-H_g \xi\ m_g M_g \vec{a}_{1U} / 3c^2 R$. Combining this with Eq. (2) shows that we obtain Eq. (3) with the mass m_g in the right hand side replaced by

$m_g\left(1+\frac{\xi\ GM_g}{3c^2 R}\right)$. That is, as if the inertia of the test body had increased. As the frequency of oscillation is inversely proportional to the square root of the inertia of the test body, the new frequency of oscillation inside the shell will be given by (calling by $\Phi = GM_g / R$ the gravitational potential inside the shell and considering $\Phi << c^2$): $\omega_{new} = \omega_o(1-\xi\ \Phi / 6c^2)$. That is,

$$\frac{\omega_{new}-\omega_o}{\omega_o} = -\frac{\xi}{6}\frac{\Phi}{c^2}. \tag{15}$$

The same will be valid for all velocities, namely:

$$\frac{v_{new}-v_o}{v_o} = -\frac{\xi}{6}\frac{\Phi}{c^2} \tag{16}$$

This allows an experimental verification of the effect. That is, by comparing the frequencies or velocities of the system inside the shell with the other equal system which is outside and far away from the shell (so that we can neglect its influence). This might be detected in principle, although the effect is usually very small. For instance, if $M_g = 1000\ kg$ and $R = 1\ m$ we obtain a fractional change of (with $\xi = 6$ for gravitation): 7×10^{-25}.

Instead of placing a neutral shell around the system it might be put an uniformly charged spherical shell of radius *R* and total charge *Q* at rest relative to the universal frame of reference. In this case Weber's electrodynamics predicts a force on an internal accelerated test charge given by ([Assis, 1992], [Assis, 1993] and [Assis, 1994, Chap. 7]): $\mu_o qQ\vec{a} / 12\pi\ R$, where $\vec{a}$ is the acceleration of the test charge relative to the shell. Combining this with Eq. (3) shows that the test charge should behave as having an effective inertial mass given by $m - q\Phi / 3c^2$, where m is the usual mass of the test charge and $\Phi = Q / 4\pi\varepsilon_o R$ is the electrostatic potential of the shell (choosing zero potential at infinity). If $qQ > 0$ ($qQ < 0$) there is a decrease (increase) in the effective inertial mass of the test charge. This should affect the motion of charged particles, in particular electrons which are very light. The fractional change of mass for an electron due to a potential of 3kV is given by 2×10^{-3}. And this can be detected in the laboratory. The first experiments of this kind known to us are due to Mikhailov, who detected an effect of this order of magnitude and coinciding in sign with the prediction of Weber's electrodynamics, [Mikhailov, 1999] and [Mikhailov, 2001]. In this case the effect is not universal anymore, as it affects only the motion of charged bodies. For instance, it will not affect the frequency of oscillation of a neutral pendulum inside the charged shell, nor the frequency of oscillation of a neutral body connected to a spring. In any event this effect is predicted only by Weber's electrodynamics, not by Lorentz's force. For this reason Mikhailov's experiments are so important. They should be repeated and performed with greater accuracy by other laboratories around the world.

Acknowledgments: One of the authors (AKTA) wishes to thank the Alexander von Humboldt Foundation of Germany for a research fellowship during which this work was completed.

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Dernière mise à jour : lundi 18 décembre 2002.