Single-particle Bell-type Inequality

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ABSTRACT. It is generally believed that Bell's inequality holds for the case of entangled states, including two correlated particles or special states of a single particle. Here, we derive a single-particle Bell's inequality for two correlated spin states at two successive times, appealing to the statistical independence condition in an ideal experiment, for a locally causal hidden variables theory. We show that regardless of the locality assumption, the inequality can be violated by some quantum predictions.

1 Introduction

After Bell derived his well- known inequality for a Bohmian version [1] of EPR [2] (hereafter called EPRB) thought experiment and showed its inconsistency with quantum mechanics [3], most authors considered local realism to be untenable, and attributed this inconsistency to the non-locality present in nature. The entangled states, in these experiments, are assumed to play a crucial role in the derivation of Bell's inequality.

In recent years, certain generalizations of Bell's inequality has been proposed in which locality is supposed to be violated [4]. Some people, e.g. Elitzur and Vaidman, have tried to prove non-locality without any appeal to any inequality [5], and Hardy has extended this idea to the case of single particles [6].

Although most of the works done on the single-particle case have been in the direction of denying locality, there has been some attempts in the opposite direction too. Works of Leggett and Garg [7], as well as Home and Sengupta [8] are of this category. The former authors assume locality, but challenge the applicability of quantum mechanics to the macroscopic phenomena. The latter try to show that Bell's inequality is derivable as a general consequence of non-contextual hidden variables theories. To show this, they have considered an entangled wavefunction which is a superposition of two factorized states in the general form of $\Psi = \sum_{i=1}^{2} c_i u_i v_i$, where the u_i and the v_i are eigenstates of the orbital and spin angular momentum, respectively, of a single valence electron. It is claimed that it is possible to drive Bell's inequality for every entangled state and in this sense, there exists a particular way of preparing single particle states [9].

In our proposed experiment, however, we consider a source of microscopic spin 1/2 particles for which the quantum state can be expressed as a sum of two individual spin states and is changed at two successive times. Then, we derive Bell's inequality as a consequence of the statistical independence condition for the ideal joint probability functions of a locally causal hidden variables theory. The meaning of this condition will be made explicit in the following section.

2 Argument

Let us consider a primary source which emits spin 1/2 particles that are polarized along the x-axis, i.e., $\mid \Psi_0\rangle = \frac{1}{\sqrt{2}}[\mid z+\rangle + \mid z-\rangle]$ where $\mid z+\rangle$ and $\mid z-\rangle$ are the two base vectors which correspond to the two eigenvectors of σ_z. There is a relatively large time interval between the emission of successive particles. Thus, we assume that only one particle passes in sequence through two analyzers (Stern-Gerlach apparatuses) $\mathbf{M}_1(\widehat{a})$ along the angle $\widehat{a}$ at t_1 and $\mathbf{M}_2(\widehat{b})$ or $\mathbf{M}_2^{'}(\widehat{b})$ along the angle $\widehat{b}$ at t_2 ($\mathrm{t}_2 > \mathrm{t}_1$), relative to the z-axis. Then, the particle coming out of $\mathbf{M}_2(\widehat{b})$ or $\mathbf{M}_2^{'}(\widehat{b})$, is detected by one of the detectors D_{++}, D_{+-}, D_{-+} or D_{--} (at a time larger than t_2) and we see one of these detectors to be flashing (Fig.1). One can assign a value A and a value B, respectively, to the spin components of the particle along $\widehat{a}$ at t_1 and $\widehat{b}$ at t_2 ($A, B = \pm 1$, in units of $\hbar/2$), when one of the detectors flashes. A flash in D_{++}, e.g., means that $A = +1$ and $B = +1$. No component is filtered or blocked at $\mathrm{t}_1$and t_2, and the result becomes known only at a time after t_2. From the detector that flashes, one can infer the values of A and B which correspond with the spin up (+1) or down (−1) of the particle at t_1 and t_2, respectively. We assume that the spin vector is a constant of motion in all stages of our experiment.

In quantum mechanics, we represent the physical states (spin states) of each particle at t_1 and t_2 by $| \varphi_A^{(t_1)}\rangle$ and $| \varphi_B^{(t_2)}\rangle$, respectively. These individual spin states are defined as

$$| \varphi_+^{(t_i)}\rangle = \cos\frac{\widehat{\theta}_i}{2} \mid z+\rangle + \sin\frac{\widehat{\theta}_i}{2} \mid z-\rangle$$

and

$$| \varphi_-^{(t_i)}\rangle = -\sin\frac{\widehat{\theta}_i}{2} \mid z+\rangle + \cos\frac{\widehat{\theta}_i}{2} \mid z-\rangle$$

where $i = 1, 2$; $\widehat{\theta}_1 = \widehat{a}$ and $\widehat{\theta}_2 = \widehat{b}$.

In an ideal experiment, the probability that we have the value A at t_1 and the value B at t_2, as a result of the joint analysis of the spin components $\sigma_a^{(t_1)} = \overrightarrow{\sigma}^{(t_1)}.\widehat{a}$ and $\sigma_b^{(t_2)} = \overrightarrow{\sigma}^{(t_2)}.\widehat{b}$, respectively, is

$$P^{(t_1,t_2)}(\sigma_a^{(t_1)} = A, \sigma_b^{(t_2)} = B|\widehat{a}, \widehat{b}, \Psi_0) = |\langle\Psi_0|\varphi_A^{(t_1)}\rangle|^2|\langle\varphi_A^{(t_1)}|\varphi_B^{(t_2)}\rangle|^2 \quad (1)$$

This probability depends on the state preparation of the source (denoted by Ψ_0) and the orientation of the Stern-Gerlach (SG) apparatuses at t_1 and t_2. It will be abbreviated as $P^{(t_1,t_2)}(A, B|\widehat{a}, \widehat{b}, \Psi_0)$, and can be derived to be

$$P^{(t_1,t_2)}(A, B|\widehat{a}, \widehat{b}, \Psi_0) = \frac{1}{4}\left(1 + A\sin\widehat{a}\right)\left[1 + AB\cos\left(\widehat{a} - \widehat{b}\right)\right] \quad (2)$$

One can obtain the probabilities for the values at t_1 or t_2 by summing both sides of (2) over appropriate parameters. In this way, the probability of having the value $\sigma_a^{(t_1)} = A$ at t_1, is

$$P^{(t_1)}(A|\widehat{a}, \Psi_0) = \sum_{B=\pm 1} P^{(t_1,t_2)}(A, B|\widehat{a}, \widehat{b}, \Psi_0) = \frac{1}{2}\left(1 + A\sin\widehat{a}\right) \quad (3)$$

Regardless of the result at t_1, the probability of having the value $\sigma_b^{(t_2)} = B$ at t_2, is

$$\begin{aligned} P^{(t_2)}(B|\widehat{a},\widehat{b},\Psi_0) &= \sum_{A=\pm 1} P^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\Psi_0) \\ &= \frac{1}{2}\left[1 + B\sin\widehat{a}\cos\left(\widehat{a}-\widehat{b}\right)\right] \end{aligned} \tag{4}$$

It is also obvious from the relations (2) and (3) that the conditional probability of the value B at t_2 is equal to

$$P^{(t_2)}(B|\widehat{a},\widehat{b},A,\Psi_0) = \frac{1}{2}\left[1 + AB\cos\left(\widehat{a}-\widehat{b}\right)\right] \tag{5}$$

After the particle came out of the first SG apparatus, its spin state changes to a new state $\varphi_A^{(t_1)}$. Since no result is detected at t_1, it is not obvious at this step that the new state is $\varphi_+^{(t_1)}$ or $\varphi_-^{(t_1)}$. However, we can generally interpret the relation (5) in terms of the new preparation as

$$P^{(t_2)}(B|\widehat{a},\widehat{b},A,\Psi_0) = P^{(t_2)}(B|\widehat{b},\varphi_A^{(t_1)}) \tag{6}$$

The probability distributions (4) and (5) which are defined at t_2, depend on the orientation of the SG apparatus at t_1. This can be explained by the correlation which exists between the statistical values of the spin components of the particle at two successive times t_1 and t_2 [10]. This correlation is due to the non-factorized form of the joint probability (1) and it originates from a new state preparation of the particle at t_1, which is denoted by $\varphi_A^{(t_1)}$ in the relation (6). There is no room for non-locality in this experiment, because the events at t_1 and t_2 are time-like separated, and when the particle is coming out from $\mathbf{M}_1(\widehat{a})$ at t_1, there is no particle at t_2, and the communication of information from t_1 to t_2 is done by the particle itself. Thus, the problem of non-locality does not arise.

In an actual experiment and for a massive spin 1/2 particle (like an electron), if we assume that the SG apparatuses are sufficiently efficient, the probability of detecting a result by one of the detectors D_{AB} ($A = \pm 1$ and $B = \pm 1$) can be given by the following relation

$$P_{\exp}(D_{AB}) = \eta_D F P^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\Psi_0) \tag{7}$$

where, η_D is the efficiency of the detector D_{AB} (the efficiencies of all the detectors are assumed to be the same), and F is the overall probability that a single particle emitted by the source will enter one of the detectors D_{AB}.

This probability is equal to

$$F = f_1 f_{21} f_{D2} \tag{8}$$

where, f_1 is the probability that a particle will enter $\mathbf{M}_1(\widehat{a})$ at t_1; f_{21} is the conditional probability that the particle will enter $\mathbf{M}_2(\widehat{b})$ or $\mathbf{M}_2^{'}(\widehat{b})$ at t_2, after it has passed through $\mathbf{M}_1(\widehat{a})$ at t_1; and f_{D2} is the conditional probability that the particle will reach a detector, when it has already passed through $\mathbf{M}_2(\widehat{b})$ or $\mathbf{M}_2^{'}(\widehat{b})$ at t_2. These functions are, in fact, the collimator efficiencies and are proportional to the collimator acceptance solid angles. A detection is present, when all the functions f_1, f_{21}, and f_{D2} are different from zero.

Here, we have not introduced a correlation factor within the probability of detection $P_{\text{exp}}(D_{AB})$, because unlike the case of two particles involved in the regular Bell-type experiments, the initial state is not an entangled one. There can be a complete correlation, however, between the values of the spin components in the same directions at t_1 and t_2, because the spin of the particle is assumed to be conserved.

Now, we consider a locally causal hidden variables theory, as used by Bell and others [11]. In this context, we assume that the spin state of particle is described by a function of a collection of hidden variabless called λ, which belongs to a space Λ. The parameter λ contains all the information which is necessary to specify the spin state of the system. Using the spin state of the particle, it would be possible to define the probability measures and the corresponding mean values on Λ. In this way, we can define the mean value of the product of the values of the spin components for the particle at times t_1 and t_2 along $\widehat{a}$ and $\widehat{b}$, respectively, as

$$E^{(t_1,t_2)}(\widehat{a},\widehat{b},\lambda) = \sum_{A,B=\pm 1} AB \; \wp^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\lambda) \tag{9}$$

where, $\wp^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\lambda)$ is the joint probability of the values A and B, corresponding to the spin components of the particle along $\widehat{a}$ at t_1

and $\widehat{b}$ at t_2, respectively. As a consequence of the principles of the probability theory, the joint probability $\wp^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\lambda)$ is equivalent to the following product form

$$\wp^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\lambda) = \wp^{(t_1)}(A|\widehat{a},\lambda)\wp^{(t_2)}(B|\widehat{a},\widehat{b},A,\lambda) \tag{10}$$

Now, we define the statistical independence condition, for an ideal case at the hidden variables level, as the conjunction of the following two assumptions:

> $\underline{\mathbf{C}_1}$. For definite settings of the two SG apparatuses at t_1 and t_2, the probability of having a value B at t_2 is independent of the value A at t_1.
>
> $\underline{\mathbf{C}_2}$. The probability of having a value B at t_2 is independent of the setting of the SG apparatus at t_1.

The assumptions $\mathbf{C}_1$ and $\mathbf{C}_2$ will be structurally the same as the outcome independence and parameter independence, respectively, in Shimony's terminology for a two-particle Bell state [12], if in their definitions the times t_1 and t_2 are replaced by two spatially separated locations L_1 and L_2 where the values A and B are respectively assigned to the spin components of particle 1 along $\widehat{a}$ and particle 2 along $\widehat{b}$. As is the case for a two-particle entangled state, the conjunction of these two assumptions leads to the factorization of the corresponding joint probability. Consequently, the joint probability (10) takes following form

$$\wp^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\lambda) = \wp^{(t_1)}(A|\widehat{a},\lambda)\wp^{(t_2)}(B|\widehat{b},\lambda) \tag{11}$$

Both the assumptions $\mathbf{C}_1$ and $\mathbf{C}_2$ are violated by quantum mechanics, as is obvious from the relations (4) and (5). The negation of $\mathbf{C}_1$ and $\mathbf{C}_2$ at the quantum level is caused by the statistical dependence of the probability functions at t_2 on the condition(s) generated as a result of the preparation made for the spin state of particle at t_1.

We assume, however, that the hidden probabilities have a classical character [13]. That is, for a definite system, there exist hidden statistical distributions for the values of the spin components of a particle along definite directions which depend *only* on the initial state of the system (represented by some hidden variables) and do not depend on any preparation procedure before the measurement.

As an example, one can suppose that the spin state of a particle depends on its path and has certain projections, e.g., along $\widehat{a}$ at t_1 and $\widehat{b}$ at t_2 which we call, respectively, $s(\overrightarrow{x}(t_1), \widehat{a})$ and $s(\overrightarrow{x}(t_2), \widehat{b})$. Here, the hidden variables (denoted by λ) are the initial position coordinates $\overrightarrow{x}(0)$ by which $\overrightarrow{x}(t)$ can be determined at any arbitrary time. Now, one can build a statistics for the values of the spin components $s(\overrightarrow{x}(t_1), \widehat{a})$ at t_1 and $s(\overrightarrow{x}(t_2), \widehat{b})$ at t_2, based on the different possible initial positions $\overrightarrow{x}(0)$. This representative example shows what we really mean by introducing the hidden probabilities $\wp^{(t_1)}(A|\widehat{a}, \lambda)$ and $\wp^{(t_2)}(B|\widehat{b}, \lambda)$.

Accordingly, the validity of the assumptions $\mathbf{C}_1$ and $\mathbf{C}_2$ in a locally causal hidden variables theory is a consequence of the fact that any information about the values of the spin components of particle at t_1 and t_2 originates from λ and that the first apparatus $\mathbf{M}_1(\widehat{a})$ does not affect the *spin state* of the particle. This means that the past history of the particle is based on λ alone, and the spin values as well as the setting of the SG apparatus at t_1, have no role in specifying the spin state of particle at t_2. By averaging over λ, however, a new description may be needed in which the state preparation of the system at any time plays an important role in the description of the state of the system. Our argument shows that it cannot be expected *a priori* that the same situation holds at a sub-quantum level, too.

Inserting (11) into (9), one gets

$$E^{(t_1,t_2)}(\widehat{a}, \widehat{b}, \lambda) = E^{(t_1)}(\widehat{a}, \lambda)\ E^{(t_2)}(\widehat{b}, \lambda) \tag{12}$$

where

$$E^{(t_1)}(\widehat{a}, \lambda) = \sum_{A=\pm 1} A\wp^{(t_1)}(A|\widehat{a}, \lambda)$$

and

$$E^{(t_2)}(\widehat{b}, \lambda) = \sum_{B=\pm 1} B\wp^{(t_2)}(B|\widehat{b}, \lambda)$$

Here, $E^{(t_1)}(\widehat{a},\lambda)$ and $E^{(t_2)}(\widehat{b},\lambda)$ are, respectively, the mean values of the spin components of particle along $\widehat{a}$ at t_1, and $\widehat{b}$ at t_2.

In an actual experiment, one can define the following correspondence relation

$$P_{\exp}(D_{AB}) = \int_{\Lambda} \wp_{\exp}(D_{AB},\lambda)\rho(\lambda)d\lambda \tag{13}$$

where the probability density $\rho(\lambda)$ is defined over the space Λ ($\int_{\Lambda}\rho(\lambda)d\lambda = 1$) and $\wp_{\exp}(D_{AB},\lambda)$ is defined by

$$\wp_{\exp}(D_{AB},\lambda) = \eta_D F \wp^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\lambda) \tag{14}$$

This relation shows that regardless of what is assumed in the context of the hidden variables theory, the events in the future may depend on what was occurred in the past. This situation happens, when at least one of the conditional probabilities f_{21} or f_{D2} is not equal to one; rather it is less than one. Thus, in an actual experiment, the statistical independence condition cannot be satisfied, in general, even if the assumptions $\mathbf{C}_1$ and $\mathbf{C}_2$ hold at the hidden variabless level. Since F is independent of the content of the theory defining λ and is only determined experimentally, this possibility remains open to use the relation (11) for our next purposes.

In an ideal experiment, if we consider the statistical independence condition at the hidden variables level (the relation (11)), it is generally possible to reproduce the quantum mechanical predictions (see appendix). Then, for the definite settings of the SG apparatuses along $\widehat{a}$ or $\widehat{a'}$ at t_1 and $\widehat{b}$ or $\widehat{b'}$ at t_2, one can obtain Bell's inequality- in Shimony's way of deriving [12]- in the following form

$$\mid E^{(t_1,t_2)}(\widehat{a},\widehat{b},\lambda) + E^{(t_1,t_2)}(\widehat{a},\widehat{b'},\lambda) + E^{(t_1,t_2)}(\widehat{a'},\widehat{b'},\lambda) - E^{(t_1,t_2)}(\widehat{a'},\widehat{b},\lambda) \mid \leq 2 \tag{15}$$

Multiplying (15) through the probability density $\rho(\lambda)$ and integrating over Λ, we get the following inequality at the quantum level

$$\mid \langle\sigma_a^{(t_1)}\sigma_b^{(t_2)}\rangle + \langle\sigma_a^{(t_1)}\sigma_{b'}^{(t_2)}\rangle + \langle\sigma_{a'}^{(t_1)}\sigma_{b'}^{(t_2)}\rangle - \langle\sigma_{a'}^{(t_1)}\sigma_b^{(t_2)}\rangle \mid \leq 2 \tag{16}$$

where, e.g., we have set the quantum expectation values as

$$\langle \sigma_a^{(t_1)} \sigma_b^{(t_2)} \rangle = \int_\Lambda E^{(t_1,t_2)}(\widehat{a}, \widehat{b}, \lambda)\ \rho(\lambda)\ d\lambda \tag{17}$$

Using the definition of $\langle \sigma_a^{(t_1)} \sigma_b^{(t_2)} \rangle_{\text{exp}}$ for an actual experiment, as

$$\langle \sigma_a^{(t_1)} \sigma_b^{(t_2)} \rangle_{\text{exp}} = \sum_{A,B=\pm 1} AB\ P_{\text{exp}}(D_{AB})$$

and the relations (2) and (7), one gets

$$\langle \sigma_a^{(t_1)} \sigma_b^{(t_2)} \rangle_{\text{exp}} = \eta_D F \cos\left(\widehat{a} - \widehat{b}\right) \tag{18}$$

Other experimental expectation values are similarly obtained. Now, if we choose all angles $\widehat{a}, \widehat{a'}, \widehat{b}$ and $\widehat{b'}$ in the xz-plane and let $|\widehat{a} - \widehat{b}| = |\widehat{a} - \widehat{b'}| = |\widehat{a'} - \widehat{b'}| = \alpha$, and $|\widehat{a'} - \widehat{b}| = 3\alpha$, then, for an actual experiment, (16) reduces to

$$\eta_D F \mid 3\cos\alpha - \cos 3\alpha \mid \leq 2$$

This can be violated, if $\eta_D F > \frac{1}{\sqrt{2}}$. If we assume that $\eta_D \simeq 1$ (which could be achived in actual experiments), this means that the overall probability of detection should be greater than 71%. Thus, under these conditions, the factorizability relation (11) for a locally causal hidden variables theory leads to inconsistency with quantum predictions. This shows that one cannot base the past history of the system on λ alone, and it is possible that the concept of state preparation is an intrinsic property of microscopic states.

3 Conclusion

As was indicated by Shr ö dinger [14], *the quantum entanglement is the characteristic trait of quantum mechanics.* Emphasizing the significance of the quantum entanglement, Shimony argued that outcome independence is violated for a two-particle singlet state [12]. The incompatibility of the quantum mechanical predictions with the local realistic hidden variables theories has been frequently reported for one [15], two [16] and more than two-particle *entangled* states [17, 18].

The entangled states lead to the correlation between different eigenvalues corresponding to the factorized eigenstates, but, it is important to notice that the existence of correlation is not limited to the entangled states. Here, we have shown another possibility. The correlation between the statistical values of the spin components of a single particle at two successive times can be related to the statistical dependence of the probability distributions on the earlier preparation. In this sense, there is a point of similarity between all the experiments concerning Bell's inequality, if one uses the state preparation point of view. The difference appears when we distinguish what kind of state preparation is the source of correlation. For quantum systems which are described by an entangled wavefunction, the correlation of the corresponding components originates from the state preparation of *the primary source.* In our proposed experiment, however, the correlation between the statistical values of the spin components at t_1 and t_2 is a result of *the past history* of the particle which is due to the preparation of a new state at t_1 (denoted by $\varphi_A^{(t_1)}$).

If we regard the violations of the Bell inequality for any quantum system (including the case of two particles or the case of one particle) as a consequence of the dependence of the state of the system on the preparation conditions at a hidden variables level, the interpretation of Bell's theorem on a unique basis would be possible. Our work demonstrates the significance of such an interpretation.

Appendix

According to the statistical independence condition, which leads to the relations (11) and (12) at the hidden variables level, one can see how it is possible to reproduce the quantum predictions.

To begin with, there are some elementary relations which are valid as in the two particle case, and they are given as follows

$$P^{(t_1,t_2)}(A,B|\widehat{a},\widehat{b},\Psi_0) = \int_\Lambda \wp^{(t_1)}(A|\widehat{a},\lambda)\wp^{(t_2)}(B|\widehat{b},\lambda)\ \rho(\lambda)\ d\lambda \qquad \text{(A-1)}$$

$$P^{(t_1)}(A|\widehat{a}, \Psi_0) = \int_\Lambda \wp^{(t_1)}(A|\widehat{a}, \lambda)\ \rho(\lambda)\ d\lambda \tag{A-2}$$

$$P^{(t_2)}(B|\widehat{a}, \widehat{b}, A, \Psi_0) = \frac{1}{P^{(t_1)}(A|\widehat{a}, \Psi_0)} \int_\Lambda \wp^{(t_1)}(A|\widehat{a}, \lambda) \wp^{(t_2)}(B|\widehat{b}, \lambda)\ \rho(\lambda)\ d\lambda \tag{A-3}$$

$$\langle \sigma_a^{(t_1)} \sigma_b^{(t_2)} \rangle = \int_\Lambda E^{(t_1)}(\widehat{a}, \lambda)\ E^{(t_2)}(\widehat{b}, \lambda)\ \rho(\lambda)\ d\lambda \tag{A-4}$$

and

$$\langle \sigma_a^{(t_1)} \rangle = \int_\Lambda E^{(t_1)}(\widehat{a}, \lambda)\ \rho(\lambda)\ d\lambda \tag{A-5}$$

where $\langle \sigma_a^{(t_1)} \rangle$ is the expectation value of the spin component of the particle along $\widehat{a}$ at t_1. The correspondence relations for $P^{(t_2)}(B|\widehat{a}, \widehat{b}, \Psi_0)$ and $\langle \sigma_b^{(t_2)} \rangle_a$ (the expectation value of the spin component along $\widehat{b}$ at t_2, when the particle has been already passed through $\mathbf{M}_1(\widehat{a})$ at t_1), however, can be defined in a different way. To show this, we use the following relations which hold for any dichotomic observables (here, $\sigma_a^{(t_1)}$ and $\sigma_b^{(t_2)}$), taking the values ± 1,

$$P^{(t_2)}(B|\widehat{a}, \widehat{b}, \Psi_0) = \frac{1}{2}(1 + B\ \langle \sigma_b^{(t_2)} \rangle_a) \tag{A-6}$$

and

$$\begin{aligned} P^{(t_2)}(B|\widehat{a}, \widehat{b}, A, \Psi_0) &= \frac{P^{(t_1,t_2)}(A, B|\widehat{a}, \widehat{b}, \Psi_0)}{P^{(t_1)}(A|\widehat{a}, \Psi_0)} \\ &= \frac{1}{2}\left[1 + \frac{B\ \langle \sigma_b^{(t_2)} \rangle_a + AB\ \langle \sigma_a^{(t_1)} \sigma_b^{(t_2)} \rangle}{1 + A\ \langle \sigma_a^{(t_1)} \rangle}\right] \end{aligned} \tag{A-7}$$

According to relation (5), the conditional probabilities at t_2 do not change, if we exchange the values A and B. This means that

$$P^{(t_2)}(B|\widehat{a},\widehat{b},A,\Psi_0) = P^{(t_2)}(A|\widehat{a},\widehat{b},B,\Psi_0) \tag{A-8}$$

If we impose the condition (A-8) on (A-7), we get

$$\frac{B\ \langle\sigma_b^{(t_2)}\rangle_a + AB\ \langle\sigma_a^{(t_1)}\sigma_b^{(t_2)}\rangle}{1 + A\ \langle\sigma_a^{(t_1)}\rangle} = \frac{A\ \langle\sigma_b^{(t_2)}\rangle_a + AB\ \langle\sigma_a^{(t_1)}\sigma_b^{(t_2)}\rangle}{1 + B\ \langle\sigma_a^{(t_1)}\rangle} \tag{A-9}$$

which leads to a new relation

$$\langle\sigma_b^{(t_2)}\rangle_a = \langle\sigma_a^{(t_1)}\rangle\ \langle\sigma_a^{(t_1)}\sigma_b^{(t_2)}\rangle \tag{A-10}$$

Now, it is possible to define the correspondence relations for $P^{(t_2)}(B|\widehat{a},\widehat{b},\Psi_0)$ and $\langle\sigma_b^{(t_2)}\rangle_a$, using the relations (A-6), (A-10), (A-4) and (A-5), as follows

$$\langle\sigma_b^{(t_2)}\rangle_a = \iint_\Lambda E^{(t_1)}(\widehat{a},\lambda)\ E^{(t_1,t_2)}(\widehat{a},\widehat{b},\lambda')\ \rho(\lambda)\ \rho(\lambda')\ d\lambda\ d\lambda' \tag{A-10}$$

$$P^{(t_2)}(B|\widehat{a},\widehat{b},\Psi_0) = \frac{1}{2}\iint_\Lambda \left[1 + BE^{(t_1)}(\widehat{a},\lambda)\ E^{(t_1,t_2)}(\widehat{a},\widehat{b},\lambda')\right] \rho(\lambda)\rho(\lambda')\ d\lambda\ d\lambda' \tag{A-11}$$

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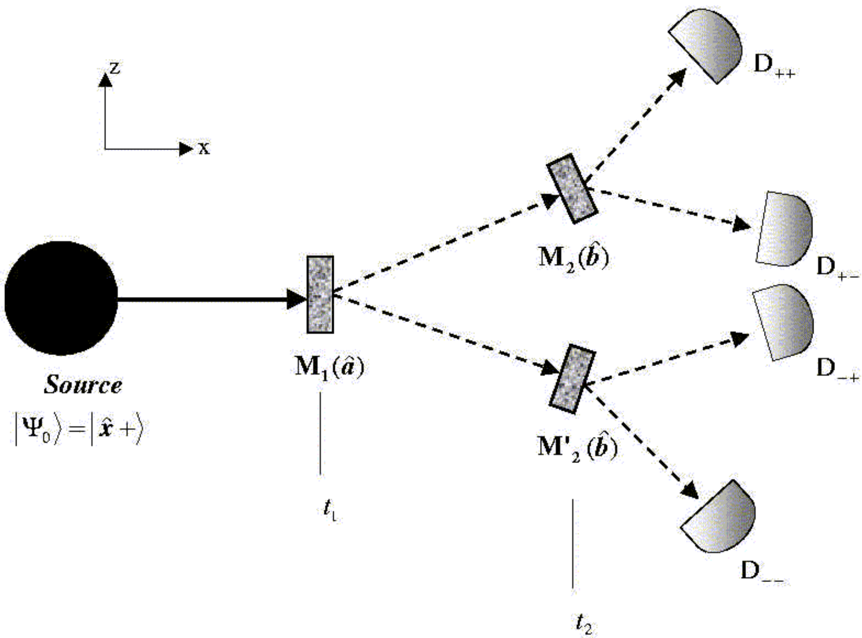

Figure 1. A single-particle passes through two Stern-Gerlach apparatuses $\mathbf{M}_1(\widehat{a})$ at t_1 and $\mathbf{M}_2(\widehat{b})$ or $\mathbf{M}_2'(\widehat{b})$ at t_2 and the result of measuremen is detected by one of the detectors D_{++}, D_{+-}, D_{-+} or D_{--}, at a time after t_2.