The Principle of Physical Proportions

A. K. T. Assis

Permanent address: Instituto de Física 'Gleb Wataghin', Universidade Estadual de Campinas - Unicamp, 13083-970 Campinas, São Paulo, Brasil.
Institut für Geschichte der Naturwissenschaften
Universität Hamburg, Bundesstr. 55, D-20146 Hamburg, Germany
E-mail: assis@ifi.unicamp.br, homepage: http://www.ifi.unicamp.br/˜assis

ABSTRACT. We propose the principle of physical proportions, according to which all laws of physics can depend only on the ratio of known quantities of the same type. An alternative formulation is that no dimensional constants should appear in the laws of physics; or that all "constants" of physics (like the universal constant of gravitation, light velocity in vacuum, Planck's constant, Boltzmann's constant etc.) must depend on cosmological or microscopic properties of the universe. With this generalization of Mach's principle we advocate doing away with all absolute quantities in physics. We present examples of laws satisfying this principle and of others which do not. These last examples suggest that the connected theories leading to these laws must be incomplete. We present applications of this principle in some fundamental equations of physics.

Key Words: relative and absolute magnitudes, Mach's principle, Weber's electrodynamics, relational mechanics, principle of physical proportions.

PACS: 01.55.+b (General physics), 01.70.+w (Philosophy of science).

1 Introduction

The idea of dimension had its origins in ancient Greek geometry. It was then considered that lines had one dimension, surfaces had two dimensions and solids had three dimensions, [1, Vol. 1, pp. 158-9, 169-170 and Vol. 3, pp. 262-3], [2] and [3]. These dimensions were related to the rule or principle of homogeneity, according to which only magnitudes of the same kind could be added or equated, and only these had a numeric ratio (we could not divide a volume by a length, for instance), [2]. This principle was also called by Heath the principle of similitude and he also spoke of the theory of proportions, [1, Vol. 1, pp. 137 and 351; Vol. 2, pp. 112-113, 115-129, 187, 280-281 and 292-293]. The geometrical dimensions were also linked to the concept of similar figures, [1, Vol. 2, pp. 187-188].

The geometrical notion of dimension was extended by Fourier to include physical dimensions, [4, §§160-161], words in square brackets added: "It must now be remarked that every undetermined magnitude or constant has one *dimension* proper to itself, and that the terms of one and the same equation could not be compared, if they had not the same *exponent of dimension*. We have introduced this consideration into the theory of heat, in order to make our definitions more exact, and to serve to verify the analysis; it is derived from primary notions on quantities; for which reason, in geometry and mechanics, it is the equivalent of the fundamental lemmas which the Greeks have left us without proof. In the analytical theory of heat, every equation (E) expresses a necessary relation between the existing magnitudes [length] x, [time] t, [temperature] v, [capacity for heat] c, [surface conducibility] h, [specific conducibility] K. This relation depends in no respect on the choice of the unit of length, which from its very nature is contingent, that is to say, if we took a different unit to measure the linear dimensions, the equation (E) would still be the same."

Dimensional analysis grew out of these ideas. It is applied, for instance, to check the correctness of equations (in the sense that all terms should have the same dimensions). Moreover, all equations should be invariant as regards any change in the system of units employed. It is also utilized in the derivation of relations between physical magnitudes applying the principle of homogeneity. For instance, it is possible to derive (except from a dimensionless constant) the dependence of the frequency of oscillation of a pendulum near the earth's surface on the pendulum's length and on the earth's gravitational field by considering the dimensions or units of these physical terms. Apparently the first person to utilize this principle in physics was Foncenex in 1761, before Fourier's works, [2]. Reynolds, Lodge, FitzGerald, Rücker, Jeans and especially Lord Rayleigh made many contributions to dimensional analysis along these lines during the last century, see [5, p. 10] for references and discussion.

In 1914 Tolman presented a "principle of similitude", [6]: "The fundamental entities out of which the physical universe is constructed are of such a nature that from them a miniature universe could be constructed exactly similar in every respect to the present universe." But Buckingham and Bridgman showed that this principle was already included in the "principle of dimensional homogeneity", which had been utilized in geometrical and physical equations for a very long time, [7] and [8]. Buckingham's paper was very important in bringing the attention of the scientific community to the so-called Π-theorem, which had already been enunciated by Vaschy in the 1890's, [5, Chapter VI] and [9, Vol. 2, p. 712]. This theorem had been employed implicitly by many scientists since Fourier's works.

For a critical discussion of the systems of units, of the concepts of physical similarity and of dimensional analysis, with many references, see Chapters XIV (The Symbols of Physics) and XV (Units and 'Dimensions') of O'Rahilly's book, [9].

Here we present a new principle which is not embodied in these previous ones, as will be clarified in the examples below. Just to show the novelty of the principle we can consider the equation describing an ideal gas, presented by Buckingham as: $pv/R\theta = N$. Here p is the pressure, v the specific volume, θ the absolute temperature, R a dimensional constant and N is a dimensionless constant. This is considered a "complete" equation by Buckingham, [7]. It is also invariant as regards any change in the system of units employed. Despite these facts, it does not comply with the principle of physical proportions presented here. For this reason it should be considered correct but incomplete. Below we will see what a complete equation for an ideal gas should look like.

2 The Principle of Physical Proportions

Newton based his dynamics as presented in the Principia (1687) on the concepts of absolute time, absolute space and absolute motion. According to him absolute time flows equably without relation to anything external, absolute space remains always similar and immovable without relation to anything external and absolute motion is the translation of a body from one absolute place into another [10, Definitions]. Leibniz, Berkeley and Mach were against these concepts and proposed that only relative time, relative space and relative motion could be perceived by the senses. For this reason only these relative concepts should appear in physical laws. Mach expressed these ideas clearly in his book The Science of Mechanics, of 1883 [11]. Excellent reviews of Mach's principle and of the distiction between relative and absolute motions at different times in the development of mechanics can be seen in [12], [13]. Relative time is a measure of duration by means of the motion of material bodies (like the angle of rotation of the earth relative to the fixed stars), relative space is a measure of dimension by means material bodies (as the distance between two bodies measured by a material rule; or the relative order of three bodies A, B and C along a straight line: ABC, or ACB, or ...).

We agree with Leibniz, Berkeley and Mach on this topic and propose the principle of physical proportions (PPP) as a generalization of their ideas, [14] and [15]. Mach advocated doing away with all absolute quantities of motion (reducing local, absolute quantities to global, relational ones). Here we advocate doing away with all absolute quantities whatever. As we will see, in classical physics not only space and time are absolute, but also gravitational mass, electrical charge etc. It is our point of view that none of these absolute quantities should appear in the laws of physics. In this work we discuss relational mechanics, [16] and [17], an alternative to standard theory that implements both Mach's principle and the PPP.

We formulate the principle as follows: (1) All laws of physics can depend only on the ratio of known quantities of the same type. This principle can also be understood in four further ways in order to clarify its meaning: (2) In the laws of physics no absolute concepts should appear, only ratios of known magnitudes of the same type should be included; (3) Dimensional constants should not appear in the laws of physics; (4) The universal constants (like G, c, h, k_B, ...) must depend on cosmological or microscopic properties of the universe; (5) All laws of physics and all measurable effects must be invariant under scale transformations of any kind (that is, under scale transformation of length, of time, of mass, of charge, ...).

We consider the PPP as an intuitive principle of nature, which should lead to a better understanding of the physical laws. In particular we believe that equations which do not satisfy the principle should be incomplete. That is, hidden connections of the properties of bodies with the distant universe will be hopefully clarified with the implementation of the principle.

The five statements of the PPP are not totally equivalent to one another. Despite this fact there are connections between them, as we will see below. The first formulation is our preferred one. By quantities of the same type we mean quantities with the same units and embodying the same physical concepts. That is, in the laws of physics there should appear only ratios of lengths, of periods of time, of electrical charges, of frequencies, of gravitational masses etc. The word "known" means that we can identify to which body the property belongs to. In the second statement the expression "absolute concepts" should be understood in the Newtonian terms above, that is, concepts which do not depend on the material external world (like Newton's absolute space, time and motion). As we will see, in classical mechanics there is also an absolute concept of mass, of electrical charge etc. In our point of view all of these concepts should not be included in physical laws. When we have only laws expressed in terms of ratios of masses, of electrical charges, of electrical currents etc. then the goal of eliminating absolute concepts will be reached. In the fourth statement by universal constants we mean constants which do not depend on the properties of bodies, in order to contrast them with the normal characteristics of bodies. For instance, the electrical resistivity varies from metal to metal, while Boltzmann's constant is the same for all gases. For this reason constants like k_B are normally designated as absolute or fundamental ones. But it is our belief that these constants should somehow be related with the properties of the distant bodies in the universe, so that it might be possible to modify or control their values by changing the environment around the measuring devices. By the words "of any kind" in the fifth statement we want to express all types of magnitudes (time, mass, charge, ...) and not only lengths (usually by scale transformation it is understood only a change of distances or linear magnitudes). The meaning of this statement is that no measurable or detectable effect should appear if we, for instance, double the electrical charges of all bodies in the universe. As we will see when analysing the acceleration between two charges, classical physics does not implement this idea.

Some authors in the past have expressed their point of view that no effect should be detected by a length transformation (that is, if all the bodies in the universe, including the atoms, increased in size by the same amount, the same happening with all distances). To our knowledge the first to generalize this idea to include time and motion was Boscovich in 1755, [18]. Some of his typical statements: "A motion which is common to us and the world cannot be recognized by us - not even if the world as a whole were increased or decreased in size by an arbitrary factor." (...) "It even is conceivable that this whole world before our eyes contracted or expanded in a matter of days - with the magnitude of the forces contracting or expanding in unison. Even if this occurred, there would be no change of the impressions in our minds and hence no perception of this kind of change." We agree with these ideas and extend it to all magnitudes (charges, temperatures etc.)

The meaning of the principle is illustrated by the examples below. With these examples we will have a clarification of these five alternative formulations.

3 Laws Satisfying this Principle

The law of the lever is the first example of a relation satisfying this principle. Propositions 6 and 7 of Archimedes's work "On the equilibrium of planes" reads as follows, [19] and [20]: "Two magnitudes, whether commensurable [Prop. 6] or incommensurable [Prop. 7], balance at distances reciprocally proportional to the magnitudes." We can write this as follows: two weights P_1 and P_2 at distances d_1 and d_2 from a fulcrum remain in horizontal static equilibrium (relative to the surface of the earth) when $P_1/P_2 = d_2/d_1$. Only ratios of local weights and local distances are relevant here. No fundamental constants appear in this law. Doubling all lengths or all weights (or gravitational masses) in the universe does not affect the equilibrium of the lever.

The law of the inclined plane also satisfies this principle. Stevin proved this law considering a triangle ABC with its plane perpendicular to the horizon and its base AC parallel to it. By hanging two weights D and E on sides AB and BC, respectively, he showed by the principle of the impossibility of perpetual motion that the two bodies connected by a string would be in equilibrium if $D/E = AB/BC$, [21].

Consider now floating bodies. Archimedes discovered the main principle of hydrostatics and presented it in his work "On floating bodies", [19] and [20]. The fifth proposition of this work reads as follows, our words in square brackets: "Any solid lighter than a fluid [that is, of smaller relative or specific weight] will, if placed in the fluid, be so far immersed that the weight of the solid will be equal to the weight of the fluid displaced." Considering a homogeneous solid, its weight is proportional to its density, ρ_S, multiplied by its total volume, V_T. Analogously, the weight of the fluid displaced is proportional to its density, ρ_F, multiplied by the volume of the solid which is below the surface, V_B. The condition for static equilibrium can then be stated as

$$\frac{V_B}{V_T} = \frac{\rho_S}{\rho_F} \ . \tag{1}$$

This equation, which can be considered as the law describing the static equilibrium of homogeneous solids in fluids of higher relative weight, also satisfies completely the PPP. Only ratios of known volumes and known densities appear here. No fundamental constants are involved in this law. Doubling all densities in the universe will not affect the ratio V_B/V_T.

Another example involves communicating vessels filled with liquids. If the cross-sectional area of vessel 1 (2) is A_1 (A_2) and we apply on their free surfaces the forces F_1 (F_2), respectively, equilibrium (no motion relative to the surface of the earth) will result if $F_1/F_2 = A_1/A_2$. In the case of a hydraulic jack these forces may be two weights P_1 and P_2. Once more this relation satisfies completely the PPP.

There are also dynamical laws which satisfy this principle. One example is Kepler's second law of planetary motion: Areas swept out by the radius vector from the sun to the planet in equal times are equal, [22, p. 135]. In other words, the area is proportional to the time. In algebraic terms if one planet describes an area A_1 in time t_1 and area A_2 in time t_2 then $A_1/A_2 = t_1/t_2$.

Another example is Newton's second law of motion coupled with his third law. Consider two bodies of inertial masses m_{i1} and m_{i2} interacting with one another along a straight line. If they suffer accelerations a_1 and a_2 relative to an inertial system of reference we obtain from Newton's laws (considering constant inertial masses): $m_{i1}/m_{i2} = -a_2/a_1$.

4 Laws Not Satisfying this Principle

The law of elastic force was first presented by Hooke in 1678 in terms of proportions, namely: "the power of any spring is in the same proportion with the tension thereof: that is, if one power stretch or bend it one space, two will bend it two, and three will bend it three, and so forward," [23]. In this form it complies with the principle of physical proportion, as it can be written as $power_1/power_2 = space_1/space_2$. But nowadays it is expressed in terms of an equality involving an elastic dimensional constant, so that it does not satisfy any longer the PPP. Consider a spring of relaxed length ℓ_o and elastic constant k. If it is compressed or stressed by a force F to a length ℓ, the law states that the displacement or lengthening of the spring is proportional to this force. Expressing this by means of an equality, as usually done, yields the condition of static equilibrium as $F = k(\ell - \ell_o)$. This relation does not satisfy the PPP as there is no ratio of forces on the left-hand side and no ratio of lengths on the right-hand side. Moreover, there appears the elastic constant which has no relation with the force that the spring is supporting. This law is correct in the sense that it describes the behavior of springs (it is valid so long as the lengthening of the spring is not so large as to become irreversible). But because it does not satisfy the PPP, it is to be regarded as incomplete.

If the spring is replaced by a string or a bar made of homogeneous material with cross-sectional area A_o this law would be written as (if the strain is not too large):

$$F = YA_o \frac{\ell - \ell_o}{\ell_o} \ , \tag{2}$$

where Y here is called Young's modulus. This expression of the law is better than the previous one because in the right hand side there is a ratio of lenghts. But it still does not satisfy completely the PPP. Although a ratio of lengths appear on the right-hand side, there is no ratio of forces on the left-hand side and no ratio of areas on the right-hand side. Moreover, Young's modulus is not a unitless constant. Its units are those of pressure and its value is characteristic of each material, although it does not depend on the cross-sectional area nor length of the string made of an specific material. For this reason we might say that it is incomplete. As Young's modulus has a different value for each material, it must depend on microscopic properties of the material (like being inversely proportional to the cross-sectional area of the molecules composing the string, or to the square of the average distance between these molecules etc.) We might hope that when a better understanding of the origin of the elastic properties of bodies is found, it will be possible to write Hooke's law as

$$\frac{F}{F_*} = \alpha \frac{A_o}{A_*} \frac{\ell - \ell_o}{\ell_o} \ , \tag{3}$$

Here F_* and A_* are a force and an area of some yet unknown origin and α is a dimensionless constant yet to be determined.

Consider now the acceleration of free fall near the surface of the earth, which is given by

$$a = G\frac{M_e}{R_e^2} \ . \tag{4}$$

Here $G = 6.67 \times 10^{-11}\ Nm^2/kg^2$ is the constant of gravitation, $M_e = 5.98 \times 10^{24}\ kg$ is the earth's mass and $R_e = 6.37 \times 10^6\ m$ is its average radius. This acceleration of free fall depends only on the mass of the earth, and not on the ratio of this mass to other masses in the universe. It also depends on the distance of the test body to the center of the earth and not on the ratio of this distance to other distances in the universe. According to classical mechanics the constant G is not dependent on other bodies in the universe. This means that it is considered a universal constant of nature, which can not be modified nor influenced by external means. If we double all masses in the universe this expression suggests that the free fall acceleration will also double, in such a way that this might be perceived or detected. This shows that not only space and time are absolute in classical mechanics, but also mass. All of these things are against the principle of physical proportions.

We now analyse the flattening of the earth. Due to its diurnal rotation around the North-South direction the earth takes essentially the form of an ellipsoid of revolution. With a period of one day the angular rotation of the earth relative to an inertial frame of reference is given by $\omega = 7.29\times10^{-5}\ rad/s$. Its equatorial radius $R_>$ becomes bigger than the polar radius $R_<$. According to classical mechanics the fractional change f is given by (as essentially first obtained by Newton):

$$f \equiv \frac{R_> - R_<}{R_<} \approx \frac{15\omega^2}{16\pi G\rho_e} \approx 0.004 \ . \tag{5}$$

Here $\rho_e = 5.5 \times 10^3\ kg/m^3$ is the earth's average mass density. There are several aspects of this result which are questionable. In the first place this fractional change depends on the angular rotation of the earth relative to absolute space or to an inertial frame of reference. In principle the distant universe composed of stars and galaxies can disappear without affecting f. This consequence is not intuitive. After all if the earth were alone in the universe it would not make sense to speak of its rotation. Consequently its flattening should disappear when the distant stars and galaxies also disappear according to the Machian perspective. If the earth remained stationary in an inertial frame of reference and the distant universe rotated around its North-South direction in the opposite direction (compared with the previous situation), the earth would not be flattened. This is against Mach's point of view. Moreover, the fractional change depends only on the density of the earth, but not on the density of distant matter. If it were possible to double the average matter density of the distant universe, without affecting the matter density of the earth, the previous result would not be affected. This shows that not only space and time, but also mass or matter density are absolute quantities in classical mechanics. All of these aspects are against the principle of physical proportions.

The great majority of physical laws do not comply with the principle of physical proportions. Whenever there are physical laws expressed in terms of equalities, instead of proportions, and in which there appear some local constants (like the spring constant k, the dielectric constant ε of the material, ...) or some universal constants (like G, ε_o, k_B, h, ...) they must be incomplete, although correct. Some examples: the law of ideal gases, $PV = k_B NT = RnT$ (P being the pressure, V the volume, $k_B = 1.38 \times 10^{-23}$ J/K Boltzmann's constant, N the number of atoms or molecules, T the temperature, $R = 8.3$ $J/Kmol$ the universal gas constant and n the number of moles), the velocity of sound, $v_s = \sqrt{B/\rho}$ (B being the bulk modulus of the fluid with density ρ), Ohm's law, $V = RI$ (where V is the voltage or potential difference between two points A and B of a conductor of resistance R where flows the constant current I), etc.

5 Implementation of the Principle of Physical Proportions

We now discuss how to implement this principle in order to complete the laws. We first consider hydrostatics and Archimedes's principle. Although Eq. (1) satisfies the principle, we will discuss an incomplete form of this law.

It is easy to imagine how people unaware of Archimedes's results might arrive at a correct but incomplete law when experimenting with floating bodies. They might put ice, cork, wood etc. floating only in water and observe that the ratio of the submersed to the total volume was proportional to the density of the material, namely

$$\frac{V_B}{V_T} = A\rho_S \ , \tag{6}$$

where A would be a constant of proportionality with dimensions of the inverse of density. This constant would be the same for all solid bodies specified above. This equation is correct dimensionally and is invariant under unit transformation (the numerical value of A will depend on the system of units employed, for instance $A = 1.0 \times 10^3 kg/m^3$ or $A = 1.6 \times 10^{-5} g/in^3$, but the form of the equation will be the same in all systems of units). It would count as a "complete" equation judged against Buckingham's standards.

Although this law describes correctly the behavior of floating bodies in water, it is incomplete. In order to transform this law into one that is compatible with the PPP it would be necessary to discover if A was of cosmological, local or microscopic origin. Specifically, it would be necessary to discover if $1/A$ was proportional to the mean density of mass in the universe, to the density of the local fluid where the solid was floating, or to the density of the molecules composing the fluid, for instance. By floating the same solids in different fluids like liquid mercury, gasoline and alcohol it would be possible to arrive at $A = 1/\rho_F$. The situation might then be described by Eq. (1) and the law might be considered complete.

Relational mechanics satisfies completely Mach's principle and the more general PPP. A presentation and discussion of this theory can be found in several places: [16], [24], [25], [26], [27], [28], [29], [30, Chapt. 6], [31, Chap. 3], [32], [33], [34] and [17]. It is based on Weber's law for gravitation and electromagnetism: [35]. Weber's force depends only on the relative distance between the interacting charges, on their relative radial velocity and on their relative radial acceleration, so that it is completey relational. For a modern discussion of Weber's electrodynamics see: [36], [37], [38], [39], [40], [30], [41], [42], [43], [44], [45], [46], [47], [17, Sections 11.2 and 11.3], [48], [49] and [50]. Relational mechanics is based also on the principle of dynamical equilibrium, [16] and [17, Section 8.1]: The sum of all forces of any nature (gravitational, electric, magnetic, elastic, nuclear etc.) acting on any body is always zero in all frames of reference. As the sum of all forces is zero, only ratios of forces will be detectable or measurable. The system of units (MKSA, cgs etc.) to be employed is not relevant. Moreover, the unit or dimension of the forces can be whatever we wish.

According to relational mechanics the acceleration of free fall towards the earth, a_{mU}, is given by [17, Sections 8.4, 8.5 and 9.2]:

$$a_{mU} = \frac{3c^2}{2\alpha\pi\rho_{go}R_o^2}\frac{M_{ge}}{r^2} \ , \tag{7}$$

or

$$\frac{a_{mu}}{a_o} = \frac{2}{\alpha}\frac{M_{ge}}{M_{go}}\frac{R_o^2}{r^2} \ . \tag{8}$$

We now discuss this relation in detail, first explaining its terms. The gravitational mass of the earth is represented by M_{ge}. The distance between the test body and the center of the earth is given by r. The gravitational mass of the test body does not appear, only its acceleration relative to the universal frame of reference U, a_{mU}. The universal frame of reference is the frame in which the set of distant galaxies are seen as essentially at rest (apart from random or peculiar velocities) without any rotation nor linear acceleration. The cosmological properties appear in the radius of the known universe given by $R_o \approx 10^{26}\ m$ and in $\rho_{go} \approx 3 \times 10^{-27}\ kg/m^3$, which is the average gravitational mass density of the distant universe. If the universe is infinite, R_o may represent a characteristic length of gravitational interactions, namely, the effective length of gravitational interactions due to an exponential decay. The gravitational mass in the known universe is given by $M_{go} = 4\pi\rho_{go}R_o^3/3 \approx 10^{52}\ kg$. If the universe has an infinite size and an infinite gravitational mass, this M_{go} may represent a characteristic gravitational mass (that is, the gravitational mass in the characteristic volume $4\pi R_o^3/3$) which would exert effects on local bodies. We also related the light velocity $c = 3 \times 10^8\ m/s$ with Hubble's constant $H_o \approx 3 \times 10^{-18}\ s^{-1}$ by $R_o = c/H_o$. A characteristic cosmological acceleration is given by $a_o \equiv R_oH_o^2 \approx 6 \times 10^{-10}/ms^{-2}$. Moreover, α is a dimensionless number with value 6 if we work with a finite universe and integrate Weber's law for gravitation until Hubble's radius R_o. If we work with Weber's law and an exponential decay in gravitation we can integrate up to infinity and in this case $\alpha = 12$.

The important aspect of this result is that only ratios of gravitational masses, of distances and of accelerations are relevant here. Doubling the earth's gravitational mass while keeping the characteristic gravitational mass M_{go} of the distant universe unaltered is equivalent of keeping the earth's gravitational mass unaltered while halving the characteristic gravitational mass of the distant universe. In both cases the acceleration of free fall doubles compared to its present value of 9.8 m/s^2. Halving the distance between the center of the earth and the test body increases four times the acceleration of free fall. According to the expression above, the same will happen by keeping a_o, M_{ge}, M_{go}, r unaltered and doubling R_o. Values as small as a_o happen on a cosmological scale in the centripetal acceleration of rotating galaxies. This may indicate that gravitational effects locally are connected with the rotation of distant galaxies. For instance, doubling the rotation of all galaxies in the universe may double the acceleration of free fall near the surface of the earth. Or maybe a_o may represent the average acceleration of all bodies in the universe relative to the universal frame of reference U. Comparing the equation above with those of classical mechanics we can see that the gravitational constant G can be seen as a function of the cosmological properties of the universe, namely: $G = 3c^2/(2\alpha\pi\rho_{go}R_o^2)$. All of these aspects are in consonance with the PPP.

With this example we can also illustrate the physical content of the PPP. Historically Galileo discovered first that the acceleration of free fall near the surface of the earth is independent of the weight or chemical composition of the falling bodies. Later on Newton showed that it is proportional to the mass of the attracting body. But we could imagine these discoveries being made in the reverse order. That is, in the first place scientist A could discover that the acceleration of free fall is proportional to the mass of the attracting body. According to the PPP we could then write this law as: $a_1/a_o = m_e/m_o$, where a_1 is the acceleration of the test body of mass m_1, m_e represents the gravitational mass of the attacting body, and a_o and m_o would be the acceleration and masses of other bodies yet to be determined. In this form the equation is compatible with the PPP, but it is not yet complete because we still did not identify the bodies to which m_o and a_o refer to (comparing the first statement of the PPP, we still do not "know" these quantities). One reasonable suspect for the mass m_o would be that $m_o = m_1$, that is, to suppose that it refers to the mass of the test body. But following the paths of Galileo or by independent experimental researches it would be shown that the acceleration of free fall does not depend on the mass of the test body, so eliminating this suspect. As the two obvious candidates (the attracting and the test bodies) have already been considered, it would only remain the distant bodies in the universe. That is, somehow m_o must be a representative mass of the distant stars and galaxies. As we have seen, relational mechanics shows that this is indeed the case.

We now consider the figure of the earth. The flattening of the earth according to relational mechanics is given by [17, Sections 8.5 and 9.5.1]

$$f \equiv \frac{R_> - R_<}{R_<} \approx \frac{5\alpha}{8} \frac{\omega_{eU}^2}{H_o^2} \frac{\rho_{go}}{\rho_{ge}} \ . \tag{9}$$

As the values of Hubble's constant and of the average matter density of the universe are not yet known with great precision, it is not possible to give an exact value for the ratio above. But the order of magnitude is compatible with the observed value of 0.004. We can also utilize that this is the observed value of f and in this way (together with the known value of the angular rotation of the earth relative to the distant galaxies and together with the known matter density of the earth) derive the value of $5\alpha\rho_{go}/8H_o^2$.

But here what we want to emphasize are the Machian aspects of this result. The first one is that the angular rotation ω_{eU} which appears in relational mechanics is the angular rotation of the earth relative to the distant universe (that is, relative to the frame of distant galaxies). It is no longer to be understood as the angular rotation of the earth relative to empty space. According to relational mechanics there will be the same flattening of the earth no matter if the earth rotates relative to an arbitrary reference frame while the distant universe remains stationary in this frame, or if the distant universe rotates in the opposite direction relative to this frame of reference while the earth remains stationary in this frame, provided the quantitative relative rotation between the earth and the distant universe is the same in both cases. The flattening of the earth cannot be considered anymore as a proof of the real or absolute rotation of the earth. Relational mechanics is not the only theory which implements this effect. The same can be said of another reformulation of mechanics due to Barbour and Bertotti, [51], [52] and [53]. Their approach involves relational quantities, intrinsic derivatives and the relative configuration space of the universe. They follow now more closely the approach of general relativity, see [54]. For a discussion of other approaches to implement Mach's principle with many references, see [17, Chapt. 11].

The second Machian aspect is that this flattening depends on the ratio of densities of the distant universe and of the earth. We can increase the flattening decreasing the density of the earth or increasing the density of the distant universe (supposing in both cases a constant angular rotation ω_{eU}). When the gravitational mass density of the distant universe goes to zero, the same happens with flattening of the earth. This is completely reasonable because in this case there would be only the earth in the universe and it is then meaningless to speak of its rotation (it would be rotating relative to what?), consequently its flattening should disappear. This happens only in relational mechanics, but not in classical mechanics. Only ratios of known quantities are important here. Gravitational mass or gravitational matter density are not absolute quantities in relational mechanics. The last aspect to be considered here is the ratio of the angular rotation of the earth and Hubble's constant. If we double the rotation of the earth relative to the distant universe, the flattening increases four times as it is proportional to the square of the angular rotation of the earth. To say that the rotation of the earth increased we must compare it with something else (for instance, with a clock). The same result should appear if the earth did not change its rate of rotation (relative to an arbitrary standard), but all other motions in the universe became slowed by a factor of 2 (relative to the same arbitrary standard). This means that Hubble's constant must somehow be like an average frequency of oscillation and/or rotation of the matter in the universe; or the angular rotation of the characteristic cosmological gravitational mass M_{go} relative to the very distant universe; or ... If we decrease by two all of these frequencies (except the frequency of rotation of the earth relative to the distant universe), Hubble's constant is divided by 2 compared with its present value and the flattening increases four times, as in the previous situation. This happens in relational mechanics but not in classical mechanics. Doubling all frequencies (including ω_{eU} and H_o) does not change f. All of these are physically reasonable results.

At present there are no conflicts of relational mechanics with known observations. Possible experimental tests to be performed in the future were presented in [17, Section 10.4]. They include a controlled change in the effective inertial mass of a body inside a massive spherical shell, the detection of anisotropic effective inertial masses of bodies surrounded by anisotropic distribution of distant masses, detection of geodetic and motional precession of gyroscopes (see also [55] and [56]), and to test if there is or not an exponential decay in gravitation. It should be remarked that according to Weber's electrodynamics the effective inertial mass of a charged body should change if it is placed inside a stationary and uniformly charged spherical shell, but nothing of this should happen according to Maxwell's equations or to Lorentz's force. This effect is analogous to an electrical Mach's principle and was quantitatively predicted in [57] and [58]. To our knowledge the first experiments to test the existence of this effect were performed by Mikhailov, [59] and [60]. The magnitude and sign of the effect he detected coincided with those predicted by Weber's electrodynamics. According to Costa de Beauregard and Lochak, if Mikhailov's experiment be confirmed by independent researches it may become a "landmark", [61].

6 Applications to Other Situations

We now consider the application of the PPP to situations involving different physical concepts. We do not know how to implement the principle in these new situations. But we wish to show the consequences of the principle in order to motivate the search for a way to implement it.

We first analyse electrostatics. Consider two charges q_1 and q_2 of the same sign repelling one another. We can keep them separated at a constant distance d applying an external force, for instance, placing a dielectric spring of elastic constant k and relaxed length ℓ_o between them. By equating the coulombian force with the elastic force $k(d-\ell_o)$ we obtain that the fractional displacement f of the spring is given by

$$f \equiv \frac{d-\ell_o}{\ell_o} = \frac{q_1 q_2}{4\pi\varepsilon_o d^2 \ell_o k} \ . \tag{10}$$

Here $\varepsilon_o = 8.85 \times 10^{-12}\ C^2 s^2/kgm^3$ is called the vacuum permittivity. Doubling the value of the two charges increases f four times. The fractional displacement should also increase four times according to the PPP by keeping q_1 and q_2 unaltered but halving all other charges in the universe (that is, the charges of all atoms and molecules of the spring, of the earth and of all other bodies of the universe, except q_1 and q_2). However, this consequence is not implemented in present theories, indicating that they must be incomplete. The influence may be completely local (halving all the charges of the spring and distant galaxies changes only the elastic constant to $k/4$, without affecting ε_o), completely cosmological (halving all the charges of the spring and of all astronomical bodies does not change k but only the vacuum permittivity to $\varepsilon_o/4$), or a mixture of both effects (halving all the charges of the spring and of all astronomical bodies affects the elastic constant and the vacuum permittivity, their new values becoming $k/2$ and $\varepsilon_o/2$, for instance).

Suppose now we remove the spring, releasing the charges. They will then be accelerated in opposite directions. According to classical mechanics the value of the initial acceleration of q_1 relative to absolute space or to an inertial frame of reference is given by: $a_1 = q_1 q_2/4\pi\varepsilon_o d^2 m_{i1}$, where m_{i1} is the inertial mass of body 1. Doubling q_1 and q_2 relative to an arbitrary standard increases the acceleration of q_1 four times. The same should happen by maintaining q_1 and q_2 unaltered but diving by two (compared to the same arbitrary standard) all other charges in the universe. But the increase of a_1 in this second situation is not represented by this law, as no other charges are involved on it. This means that electrical charges are absolute concepts in classical physics. The value of a_1 depends on $q_1 q_2$ and not on the ratio of these charges to other known charges in the universe. In order to implement the PPP it would be necessary to find other charges built in (or hidden in) the product $\varepsilon_o m_{i1}$. Maybe these are the microscopic charges composing the inertial masses m_{i1} and m_{i2} (that is, the charges composing the atoms or molecules of bodies 1 and 2), or the charges composing the stars and galaxies of the distant universe. Anyhow the PPP has not yet been implemented in this case.

According to relational mechanics the value of the acceleration of q_1 relative to the universal frame of reference is given by, [17, Section 8.5, Eqs. (8.42) to (8.44)]:

$$a_{1U} = \frac{q_1 q_2}{4\pi\varepsilon_o d^2 m_{g1}} \ . \tag{11}$$

Here m_{g1} is the gravitational mass of body 1. In relational mechanics there appears only gravitational masses, being inertial mass a derived concept which arises only when we compare relational mechanics with classical mechanics. This acceleration increases four times by doubling q_1 and q_2. The same must happen by keeping q_1 and q_2 unaltered but halving all other charges in the universe (that is, halving the charges of all atoms and molecules of distant galaxies, and halving the microscopic charges composing the molecules of bodies 1 and 2). Again the effect may be totally cosmological (affecting only the vacuum permittivity), totally local (affecting only the gravitational masses m_{g1} and m_{g2}) or a mixture of both effects (affecting the vacuum permittivity and both gravitational masses).

One example of how the gravitational mass of a body may depend on its microscopic constituent charges was given in [62] and in [63]. The Newtonian gravitational force between two bodies of gravitational masses m_{g1} and m_{g2} was derived as a residual electromagnetic force arising from the interaction between the neutral oscillating dipoles composing body 1 and the neutral oscillating dipoles belonging to body 2, where each dipole consisted of a negative charge oscillating around a positive one. The gravitational mass of each body was then found proportional to the number of oscillating dipoles composing it and to q^2/ε_o, where q represents the positive (or negative) charge of each neutral dipole. With this model it is possible to implement the PPP for charges.

Another situation is Ampère's force between electrical circuits carrying currents I_1 and I_2, proportional to $I_1 I_2$. As the currents are proportional to the drifting velocities of the electrons, we can increase the force four times by doubling these drifting velocities. The consequences of this effect can be seen statically (an increase in the tension of a spring holding the two circuits at a constant distance) or dynamically (an increase in the acceleration of the two circuits when the spring is released). The same consequences must happen by keeping I_1 and I_2 unaltered but making all other bodies in the universe move with half their present value velocities. As the modern theories do not implement this property, they must be incomplete.

Consider now the equation of state of an ideal gas, $PV = Nk_BT$. This equation does not satisfy the PPP. The equation of an ideal gas satisfying this principle should take the form $(P/P_o)(V/V_o) = a(N/N_o)(T/T_o)$, where a is a dimensionless number and P_o, V_o, N_o, T_o are local and/or cosmological pressures, volumes, number of particles and temperature. When the theory leading to this new equation is found, it will be possible to relate Boltzmann's constant to the properties (like pressure, density and temperature) of the local or cosmological environment. For instance, relational mechanics showed that the universal constant of gravitation G is proportional to H_o^2/ρ_{go}. This shows that it is not a constant anymore, but a function of the properties of the distant universe. Something analogous should hold for Boltzmann's constant. The new equation describing the behaviour of an ideal gas will be different to the present one. But not only that, we will also gain a new understanding of the law, perceiving new connections of the local properties of a gas with the distant universe. For instance, if a fixed number of atoms is enclosed in a fixed volume and we increase its temperature four times, the pressure of the gas according to the new law will also increase four times, as it happens according to the present law. But when the new law is obtained it will be possible to show that we can also increase the gas pressure four times (as indicated, for instance, by a manometer) by keeping its temperature, volume and number of particles constant, while simultaneously dividing by four the temperature of the distant bodies in the universe (stars and galaxies). The same can be said as regards the volume and number of particles in the gas or in the distant universe. That is, if some effect is measured locally when we change the pressure, volume, number of atoms or temperature of the gas; it will be possible to show when we have a complete theory that the same effect will also happen when the opposite change is perfomed in the distant cosmos.

The same can be said of almost all relations in physics. Other universal constants like light velocity in vacuum, Planck's constant etc. must all be functions of properties of the distant universe (macroscopic relations) or of the local particles (microscopic relations). In this regard we can see that this principle has some relations with Dirac's great cosmological numbers (or variation of the universal constants), [64]. By observing several dimensionless numbers of the order of 10^{39}, like the ratio of the electric to the gravitational force between an electron and a proton, or the ratio of Hubble's time to a unit of time fixed by the constants of atomic theory, Dirac supposed that they should be related to one another. In his words, "such a coincidence we may presume is due to some deep connexion in Nature between cosmology and atomic theory." His new principle of cosmology was stated as follows: "Any two of the very large dimensionless numbers occuring in Nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity."

The principle of physical proportions presented here may help to elucidate the connexion between the atom and the cosmos perceived by Dirac, as it offers directions of where to find this deeper understanding of nature.

Acknowledgments:

The author wishes to thank the Alexander von Humboldt Foundation, Germany, for a research fellowship during which this work was completed. He thanks also the Institute and Foundation Louis de Broglie for financial support to participate in the Congress on Electromagnetism (Peyresq, France, September 2002). Discussions with A. Ghosh, C. S. Unnikrishnan, A. R. Prasanna, N. Graneau, O. E. Rössler and F. A. G. Redondo were highly appreciated.

References

[1] Euclid. *The Thirteen Books of The Elements*, volume 1-3, Books I-XIII. Dover, New York, 1956. Translated with introduction and commentary by Sir Thomas L. Heath.

[2] R. de A. Martins. The origin of dimensional analysis. *Journal of the Franklin Institute*, 311:331–337, 1981.

[3] F. A. G. Redondo. *Historia del Análisis Dimensional*. PhD thesis, Universidad Politécnica de Madrid, Madrid, 2000.

[4] J. B. J. Fourier. Analytical Theory of Heat. In *Great Books of the Western World, Vol. 45*, pages 161–251, Chicago, 1952. Encyclopaedia Britannica.

[5] J. Palacios. *Análisis Dimensional*. Espasa-Calpe S. A., Madrid, 2nd edition, 1964. English translation: Dimensional Analysis, McMillan, London, 1964; French translation: Analyse Dimensionnelle, Gauthier Villars, Paris, 1964.

[6] R. C. Tolman. The principle of similitude. *Physical Review*, 3:244–255, 1914.

[7] E. Buckingham. On physically similar systems; illustrations of the use of dimensional equations. *Physical Review*, 4:345–376, 1914.

[8] P. W. Bridgman. Tolman's principle of similitude. *Physical Review*, 8:423–431, 1916.

[9] A. O'Rahilly. *Electromagnetic Theory — A Critical Examination of Fundamentals*. Dover, New York, 1965.

[10] I. Newton. *Mathematical Principles of Natural Philosophy*. University of California Press, Berkeley, 1934. Cajori edition. Original publication in Latin in 1687.

[11] E. Mach. *The Science of Mechanics — A Critical and Historical Account of Its Development*. Open Court, La Salle, 1960.

[12] J. B. Barbour. *Absolute or Relative Motion? - A study from a Machian point of view of the discovery and the structure of dynamical theories*, volume 1: *The Discovery of Dynamics*. Cambridge University Press, Cambridge, 1989.

[13] A. Ghosh. *Origin of Inertia: Extended Mach's Principle and Cosmological Consequences*. Apeiron, Montreal, 2000.

[14] A. K. T. Assis. Applications of the principle of physical proportions to gravitation. In K. Rudnicki, editor, *Gravitation, Electromagnetism and Cosmology - Toward a New Synthesis*, pages 1–7, Montreal, 2001. Apeiron.

[15] A. K. T. Assis. Comparação entre a mecânica relacional e a relatividade geral de Einstein. In O. Pessoa Jr., editor, *Fundamentos da Física 2 - Simpósio David Bohm*, pages 27–38, São Paulo, 2001. Editora Livraria da Física.

[16] A. K. T. Assis. On Mach's principle. *Foundations of Physics Letters*, 2:301–318, 1989.

[17] A. K. T. Assis. *Relational Mechanics*. Apeiron, Montreal, 1999. ISBN: 0-9683689-2-1.

[18] R. J. Boscovich. On space and time as they are recognized by us. In O. E. Rössler, editor, *Endophysics - The World as an Interface*, pages 107–112. World Scientific, Singapore, 1998. English translation by O. E. Rössler from the original Latin version of 1755.

[19] Archimedes. *The Works of Archimedes*. Dover, New York, 1912. Edited in modern notation with introductory chapters by T. L. Heath with a supplement *The Method of Archimedes* recently discovered by Heiberg.

[20] Archimedes. *The Works of Archimedes including* The Method, volume 11 of *The Great Books of the Western World*. Encyclopaedia Britannica, Chicago, 1952. Translated by T. L. Heath.

[21] S. Stevin. The Inclined Plane. In W. F. Magie, editor, *A Source Book in Physics*, pages 22–27, New York, 1935. McGraw-Hill.

[22] K. R. Symon. *Mechanics*. Addison-Wesley, Reading, third edition, 1971.

[23] R. Hooke. Law of elastic force. In W. F. Magie, editor, *A Source Book in Physics*, pages 93–95, New York, 1935. McGraw-Hill.

[24] P. Graneau. The riddle of inertia. *Electronics and Wireless World*, 96:60–62, 1990.

[25] P. Graneau. Far-action versus contact action. *Speculations in Science and Technology*, 13:191–201, 1990.

[26] P. Graneau. Interconnecting action-at-a-distance. *Physics Essays*, 3:340–343, 1990.

[27] P. Graneau. Some cosmological consequences of Mach's principle. *Hadronic Journal Supplement*, 5:335–349, 1990.

[28] P. Graneau. Has the mystery of inertia been solved? In U. Bartocci and J. P. Wesley, editors, *Proceedings of the Conference on Foundations of Mathematics and Physics*, pages 129–136, Blumberg, Germany, 1990. Benjamin Wesley Publisher.

[29] J. P. Wesley. Weber electrodynamics, Part III. Mechanics, gravitation. *Foundations of Physics Letters*, 3:581–605, 1990.

[30] J. P. Wesley. *Selected Topics in Advanced Fundamental Physics.* Benjamin Wesley Publisher, Blumberg, 1991.

[31] P. Graneau and N. Graneau. *Newton Versus Einstein – How Matter Interacts with Matter.* Carlton Press, New York, 1993.

[32] A. Zylbersztajn. Newton's absolute space, Mach's principle and the possible reality of fictitious forces. *European Journal of Physics*, 15:1–8, 1994.

[33] T. E. Phipps, Jr. Clock rates in a machian universe. *Toth-Maatian Review*, 13:5910–5917, 1996.

[34] J. Guala-Valverde. *Inercia y Gravitacion.* Fundacion Julio Palacios, Neuquen, Argentina, 1999. In collaboration with J. Tramaglia and R. Rapacioli.

[35] W. Weber. *Wilhelm Weber's Werke,* W. Voigt, E. Riecke, H. Weber, F. Merkel and O. Fischer (Eds.), volume 1 to 6. Springer, Berlin, 1892-1894.

[36] J. P. Wesley. Weber electrodynamics extended to include radiation. *Speculations in Science and Technology*, 10:47–61, 1987.

[37] J. P. Wesley. Weber electrodynamics, Part I. General theory, steady current effects. *Foundations of Physics Letters*, 3:443–469, 1990.

[38] J. P. Wesley. Weber electrodynamics, Part II. Unipolar induction, Z-antenna. *Foundations of Physics Letters*, 3:471–490, 1990.

[39] T. E. Phipps Jr. Toward modernization of Weber's force law. *Physics Essays*, 3:414–420, 1990.

[40] T. E. Phipps Jr. Weber-type laws of action-at-a-distance in modern physics. *Apeiron*, 8:8–14, 1990.

[41] T. E. Phipps Jr. Derivation of a modernized Weber force law. *Physics Essays*, 5:425–428, 1992.

[42] G. Galeczki. The ultimate speed and Weber's potential. *Physics Essays*, 6:448–450, 1993.

[43] A. K. T. Assis. *Weber's Electrodynamics.* Kluwer Academic Publishers, Dordrecht, 1994. ISBN: 0-7923-3137-0.

[44] G. Galeczki and P. Marquardt. Über die longitudinalen Ampèreschen Kräfte und Webers Elektrodynamik. *Fusion*, 16:15–17, 1995.

[45] E. T. Kinzer and J. Fukai. Weber's force and Maxwell's equations. *Foundations of Physics Letters*, 9:457–461, 1996.

[46] J. Fukai and E. T. Kinzer. Compatibility of Weber's force with Maxwell's equations. *Galilean Electrodynamics*, 8:53–55, 1997.

[47] A. K. T. Assis, W. A. Rodrigues Jr., and A. J. Mania. The electric field outside a stationary resistive wire carrying a constant current. *Foundations of Physics*, 29:729–753, 1999.

[48] A. G. Kelly. Experiments on unipolar induction. *Physics Essays*, 12:372–382, 1999.

[49] M. d. A. Bueno and A. K. T. Assis. *Inductance and Force Calculations in Electrical Circuits*. Nova Science Publishers, Huntington, New York, 2001. ISBN: 1-56072-917-1.

[50] J. Guala-Valverde, P. Mazzoni, and R. Achilles. The homopolar motor: A true relativistic engine. *American Journal of Physics*, 70:1052–1055, 2002.

[51] J. B. Barbour. Relative-distance Machian theories. *Nature*, 249:328–329, 1974. Misprints corrected in Nature, vol. 250, p. 606 (1974).

[52] J. B. Barbour and B. Bertotti. Gravity and inertia in a Machian framework. *Nuovo Cimento B*, 38:1–27, 1977.

[53] J. B. Barbour and B. Bertotti. Mach's principle and the structure of dynamical theories. *Proceedings of the Physical Society of London A*, 382:295–306, 1982.

[54] J. B. Barbour and H. Pfister (editors). *Mach's Principle: From Newton's Bucket to Quantum Gravity*. Birkhäuser, Boston, 1995.

[55] P. Eby. Gyro precession and Mach's principle. *General Relativity and Gravitation*, 11:111–117, 1979.

[56] P. Eby. Machian mechanics with isotropic inertia. *General Relativity and Gravitation*, 29:621–635, 1997.

[57] A. K. T. Assis. Centrifugal electrical force. *Communications in Theoretical Physics*, 18:475–478, 1992.

[58] A. K. T. Assis. Changing the inertial mass of a charged particle. *Journal of the Physical Society of Japan*, 62:1418–1422, 1993.

[59] V. F. Mikhailov. The action of an electrostatic potential on the electron mass. *Annales de la Fondation Louis de Broglie*, 24:161–169, 1999.

[60] V. F. Mikhailov. Influence of an electrostatic potential on the inertial electron mass. *Annales de la Fondation Louis de Broglie*, 26:33–38, 2001.

[61] O. Costa de Beauregard and G. Lochak. Comment on Mikhailov's article. *Annales de la Fondation Louis de Broglie*, 24:159–160, 1999.

[62] A. K. T. Assis. Deriving gravitation from electromagnetism. *Canadian Journal of Physics*, 70:330–340, 1992.

[63] A. K. T. Assis. Gravitation as a fourth order electromagnetic effect. In T. W. Barrett and D. M. Grimes, editors, *Advanced Electromagnetism: Foundations, Theory and Applications*, pages 314–331, Singapore, 1995. World Scientific.

[64] P. A. M. Dirac. A new basis for cosmology. *Proceedings of the Royal Society of London A*, 165:199–208, 1938.

(Manuscrit reçu le 30 octobre 2002)