Variation of the speed of light due to non-minimal coupling between electromagnetism and gravity

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ABSTRACT. We consider an Einstein-Maxwell action modified by the addition of three terms coupling the electromagnetic strength to the curvature tensor. The corresponding generalized Maxwell equations imply a variation of the speed of light in a vacuum. We determine this variation in Friedmann-Robertson-Walker spacetimes. We show that light propagates at a speed greater than c when a simple condition is satisfied.

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1 Introduction

Theories with a varying speed of light have been recently proposed to solve the initial value problems of standard cosmology (see, e.g., [1]–[6] and Refs. therein). However, these theories break local Lorentz invariance and suffer from quite unclear physical interpretations. So, it seems to us that it is necessary to explore less extremist ideas.

The present paper is based on the observation that the invariant speed c involved in the local Lorentz transformations must be carefully distinguished from the speed of light considered as a signal propagating in a vacuum. In fact, it is possible both to maintain the basic principles of metric theories of gravity and to obtain a variable speed of light by modifying Maxwell's equations [7]. For this reason, we suggest to call c the "spacetime structure constant" instead of the "speed of light", which is a very misleading terminology.

In what follows, we consider the modified Einstein-Maxwell equations which arise from an electromagnetic field non-minimally coupled to the gravitational field. Several kinds of non-minimal couplings may be proposed [8], some of them violating gauge invariance [9]. In order to avoid such a radical consequence, we define the action as

$$\mathcal{I} = \int \left[-\frac{c^3}{16\pi G} (R + 2\Lambda) + L_{EM} - j^\mu A_\mu + L_{matter} \right] \sqrt{-g} d^4x \,, \qquad (1)$$

where G is the Newtonian gravitational constant, Λ is the cosmological constant, R is the scalar curvature, j^μ is the current density vector, and A_μ is the vector potential. The non-minimally coupled electromagnetic field Lagrangian is defined as

$$L_{EM} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{4} \xi R F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \eta R_{\mu\nu} F^{\mu\rho} F^{\nu}_{.\rho} + \frac{1}{4} \zeta R_{\mu\nu\rho\sigma} F^{\mu\nu} F^{\rho\sigma} \,, \qquad (2)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic strength, $R_{\mu\nu\rho\sigma}$ is the curvature tensor, $R_{\mu\nu}$ is the Ricci tensor, ξ, η, and ζ are constants having dimensions [length]^2. The field variables are the metric $g_{\mu\nu}$, the vector potential A_μ and the variables describing matter in L_{matter}.

It has been previously noted that the generalized Maxwell equations deduced from Eqs. (1)–(2) imply a variable speed of light [10, 11, 12], except in the case where $\eta = \zeta = 0$ [13]. However, as far as we know, the formula giving the speed of light in the Friedmann-Robertson-Walker (FRW) cosmological models has not been found in the general case. The main purpose of the present paper is to yield this formula. A detailed analysis of its cosmological implications will be developed elsewhere.

The plan of the paper is as follows. In Sect. 2, we recall why a varying speed of light is compatible with the general axioms underlying special and general relativity. Section 3 is devoted to arguments in favour of the non-minimal coupling (NMC) examined here. In Sect. 4, we derive the equations of motion for the electromagnetic strength $F_{\mu\nu}$ in any space-time. In Sect. 5, we form the equations satisfied by the wave vector in the limit of the geometric optics approximation, with a special emphasis on the case where the Einstein tensor has the structure corresponding to a perfect fluid. In Sect. 6, we outline the theory of light rays in the FRW background. In Sect. 7, we get the explicit expression of the speed of light as a function of the energy and pressure content of the Universe when Einstein equations are satisfied. Finally, we give some concluding remarks in Sect. 8.

Conventions and notations.– The signature of the Lorentzian metric g is $(+---)$. Greek indices run from 0 to 3. We put $x^0 = ct$, t being a coordinate time. Given a vector or a tensor T, we often use the notations $T_{,\alpha} = \partial_\alpha T$ and $T_{;\alpha} = \nabla_\alpha T$. The curvature tensor $R_{\mu\nu\rho\sigma}$ is defined by $w_{\mu;\rho;\sigma} - w_{\mu;\sigma;\rho} = -R_{\mu\nu\rho\sigma} w^\nu$. For the Ricci tensor, we take $R_{\mu\nu} = R^{\lambda}_{.\mu\lambda\nu}$. Given two 4-vectors v and w, we often use the notation $(v{\cdot}w) = g_{\mu\nu} v^\mu w^\nu$. When there is no ambiguity, we write $l^2 = (l \cdot l) = g_{\mu\nu} l^\mu l^\nu$. We put $\kappa = 8\pi G/c^4$.

2 Variable speed of light and relativity

Let us give some additional arguments which justify the necessity to distinguish the spacetime structure constant c from the speed of light. Our analysis is inspired by [14].

The widely held idea that c must be identified with the speed of light in relativistic theories is supported by the fact that the statement of invariance of the speed of light played a crucial role in the original Einstein's paper [15]. However, this earliest approach is not the more logical one and can be criticized for several reasons.

1) If we state that the invariant quantity c is linked to a property of electromagnetic radiation, it is hard to understand why all interactions are governed, at least locally, by special relativity.

2) The invariance of the laws of physics under the Poincaré group allows the existence of zero-mass particles but does not imply that any empirical interaction must be mediated by a zero-mass particle. As a consequence, if once a day a non-zero mass is found for the photon, only the usual presentations of SR based on the invariance of the speed of light will be disproved.

3) Shortly after the Einstein's paper, it was pointed out in [16, 17] that it was not necessary to assume the constancy of the light velocity in order to derive the Lorentz transformations. Since these pioneering works, several other derivations of the correct transformation equations have been performed without imposing the existence of an invariant speed (see, e.g., [18] and Refs. therein). In [14], it was proved by elementary considerations that if *i)* the principle of relativity is valid, *ii)* spacetime is a four-dimensional manifold, *iii)* spacetime is homogeneous, *iv)* space is isotropic, *v)* the inertial transformations constitute a group and *vi)* causality is preserved, then the tranformation equations may be

written as follows in the two-dimensional case:

$$x' = \frac{x - vt}{\sqrt{1 - \chi v^2}}\,, \quad t' = \frac{t - \chi v x}{\sqrt{1 - \chi v^2}}\,, \tag{3}$$

where χ is a constant which must be ≥ 0 ($\chi < 0$ would imply violations of causality). The Galileo transformations are recovered for $\chi = 0$, while the Lorentz transformations are obtained for $\chi > 0$. Clearly, Eq. (3) implies that the quantity $c = 1/\sqrt{\chi}$ is both an invariant and a limiting speed.

The above-mentioned axioms are universal principles which do not require any definite, well-developed theory of some physical interaction. The existence of such axioms proves that the constant c appearing in the Lorentz transformations is not essentially the speed of light in a vacuum, but is in fact a structural constant which characterizes the four-dimensional continuum constituting the arena of physical events.

It follows from this discussion that a variable speed of light cannot be forbidden by special or general relativity.

3 Arguments in favour of non-minimal coupling

Several arguments may be given in favour of the non-minimal coupling defined by Eqs. (1)-(2).

1) It may be argued from a theorem due to Horndeski [19, 20] that the most general electromagnetic equations which are *i)* derivable from a variational principle, *ii)* at most of second-order in the derivatives of both $g_{\mu\nu}$ and A_μ, *iii)* consistent with the charge conservation, and *iv)* compatible with Maxwell's equations in flat space-time are given by a Lagrangian defined by Eq. (2) with

$$\eta = -2\xi\,, \quad \zeta = \xi\,, \tag{4}$$

ξ being arbitrary. This beautiful theorem proves that a particular NMC is inevitable if one considers the generalization of Maxwell's equations compatible with the currently accepted principles of electromagnetism. The interest of the Horndeski Lagrangian is enhanced by the fact that it can be recovered from the Gauss-Bonnet action in a five-dimensional space with a Kaluza-Klein metric (see [21], [22] and Refs. therein).

2) It may also be argued that couplings to the curvature are induced by vacuum polarization in quantum electrodynamics(QED) [10, 23]. Indeed, vacuum polarization confers a size to the photon of the order of

the Compton wavelength of the electron. So the motion of the photon is influenced by a tidal gravitational effect depending on the curvature. Working in the one-loop approximation, Drummond and Hathrell found an effective Lagrangian for QED given by Eq. (2) with

$$\xi = -\frac{\alpha}{36\pi}\lambda\!\!\!^{-}_c{}^2 \,, \quad \eta = \frac{13\alpha}{180\pi}\lambda\!\!\!^{-}_c{}^2 \,, \quad \zeta = -\frac{\alpha}{90\pi}\lambda\!\!\!^{-}_c{}^2 \,, \tag{5}$$

where α is the fine-structure constant and $\lambda\!\!\!^{-}_c$ is the Compton wavelength of the electron defined as $\lambda\!\!\!^{-}_c = \hbar/m_e c$ [24].

We think that these remarkable results show that the NMC introduced by Eqs. (1)–(2) deserves to be studied in detail.

4 Equations of generalized electromagnetism

Varying the action $\mathcal{I}$ with respect to the vector potential A_μ leads to generalized Maxwell equations. It is easy to form these equations by using the following lemma. Let L be a scalar Lagrangian such that

$$L = \frac{1}{4}E^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} - j^\mu A_\mu \equiv E^{\mu\nu\rho\sigma}A_{\mu;\nu}A_{\rho;\sigma} - j^\mu A_\mu \,, \tag{6}$$

where $E^{\mu\nu\rho\sigma}$ is a 4-rank tensor which involves neither the vector potential A_μ nor its derivatives of any order and which satisfies the properties of symmetry and antisymmetry

$$E^{\mu\nu\rho\sigma} = E^{\rho\sigma\mu\nu} \,, \tag{7}$$

$$E^{\mu\nu\rho\sigma} = -E^{\nu\mu\rho\sigma} = -E^{\mu\nu\sigma\rho} \,. \tag{8}$$

It is easily seen that the corresponding Euler-Lagrange equations may be written as

$$\nabla_\nu (E^{\mu\nu\rho\sigma}F_{\rho\sigma}) = j^\mu \,. \tag{9}$$

Applying this lemma to the Lagrangian L_{EM} defined by Eq. (2) and using Eq. (9), we get the equations of motion

$$\widetilde{F}^{\mu\nu}{}_{;\nu} = -j^\mu \,, \tag{10}$$

where $\widetilde{F}^{\mu\nu}$ is the 2-rank tensor

$$\widetilde{F}^{\mu\nu} = (1 - \xi R)\, F^{\mu\nu} - \eta \left(R^\mu_\lambda F^{\lambda\nu} - R^\nu_\lambda F^{\lambda\mu} \right) - \zeta R^{\mu\nu\rho\sigma} F_{\rho\sigma} \,. \tag{11}$$

Of course, Eq. (10) must be complemented by the equations

$$F_{\mu\nu;\rho} + F_{\rho\mu;\nu} + F_{\nu\rho;\mu} = 0\,. \tag{12}$$

Since the tensor $\widetilde{F}^{\mu\nu}$ defined by Eq. (11) is obviously antisymmetric, the charge conservation equation $\nabla_\alpha j^\alpha = 0$ is a condition of integrability of Eqs. (11)-(12). So, charge conservation is embodied in the NMC theory deduced from the action $\mathcal{I}$.

It follows from Eq. (10) that $\widetilde{F}^{\mu\nu}$ may be considered as the electromagnetic excitation [25]. Thus, the non-minimally coupled electromagnetic excitation in a vacuum must be distinguished from the electromagnetic strength $F^{\mu\nu}$. Using the identities

$$R^{\mu\nu\rho\sigma}{}_{;\nu}F_{\rho\sigma} \equiv 2R^{\mu}_{\rho;\sigma}F^{\rho\sigma}\,,$$

it is easily seen that Eq. (10) may be written as

$$(1-\xi R)F^{\mu\nu}{}_{;\nu} - \eta(R^{\mu}_{\lambda}F^{\lambda\nu}{}_{;\nu} - R^{\nu}_{\lambda}F^{\lambda\mu}{}_{;\nu}) - \zeta R^{\mu\nu}{}_{\cdot\,\cdot\,\rho\sigma}F^{\rho\sigma}{}_{;\nu}$$
$$-\frac{1}{2}(2\xi+\eta)R_{,\nu}F^{\mu\nu} - (\eta+2\zeta)R^{\mu}_{\rho;\sigma}F^{\rho\sigma} = -j^{\mu}\,. \tag{13}$$

Introducing now the Weyl tensor $C^{\mu\nu\rho\sigma}$ defined by

$$C^{\mu\nu\rho\sigma} = R^{\mu\nu\rho\sigma} - \frac{1}{2}\left(R^{\mu\rho}g^{\nu\sigma} + R^{\nu\sigma}g^{\mu\rho} - R^{\mu\sigma}g^{\nu\rho} - R^{\nu\rho}g^{\mu\sigma}\right)$$
$$+\frac{1}{6}R\left(g^{\mu\rho}g^{\nu\sigma} - g^{\mu\sigma}g^{\nu\rho}\right)\,, \tag{14}$$

Eq. (13) becomes

$$\left[1-\frac{1}{3}(3\xi-\zeta)R\right]F^{\mu\nu}{}_{;\nu} - (\eta+\zeta)(R^{\mu}_{\lambda}F^{\lambda\nu}{}_{;\nu} - R^{\nu}_{\lambda}F^{\lambda\mu}{}_{;\nu}) - \zeta C^{\mu\nu}{}_{\cdot\,\cdot\,\rho\sigma}F^{\rho\sigma}{}_{;\nu}$$
$$-\frac{1}{2}(2\xi+\eta)R_{,\nu}F^{\mu\nu} - (\eta+2\zeta)R^{\mu}_{\rho;\sigma}F^{\rho\sigma} = -j^{\mu}\,. \tag{15}$$

This last form of the equations of motion will be very useful to study the propagation of light in a gravitational wave or in the FRW background.

5 Geometric optics approximation

In order to determine the speed of light, we shall work in the limit of the geometric optics approximation. Treating the vector potential A_μ as the real part of a complex vector, we suppose that there exist solutions to Eq. (15) which admit a development of the form

$$A_\mu(x,\varepsilon) = \Re\left\{[a_\mu(x) + O(\epsilon)]\exp\left(\frac{i}{\varepsilon}\widehat{S}(x)\right)\right\}, \tag{16}$$

where a_μ is a slowly varying, complex vector amplitude, $\widehat{S}(x)$ is a real function and ε is a dimensionless parameter which tends to zero as the typical wavelength of the electromagnetic signal becomes shorter and shorter. Let us define the wave vector l_μ as

$$l_\mu = \frac{1}{\varepsilon}\widehat{S}_{,\mu}\,. \tag{17}$$

We have

$$F_{\mu\nu} = \Re\left\{i(l_\mu a_\nu - l_\nu a_\mu)\exp\left(\frac{i}{\varepsilon}\widehat{S}(x)\right)\right\} + \cdots\,. \tag{18}$$

Inserting Eq. (18) into Eq. (15), and then retaining only the leading terms of order ε^{-2}, we obtain the equations constraining the wave vector l^μ in the form:

$$\begin{aligned}\left\{\left[1-\left(\xi-\frac{1}{3}\zeta\right)R\right]l^2-(\eta+\zeta)Ric(l,l)\right\}a^\mu-(\eta+\zeta)R^\mu_\lambda\,l^2\,a^\lambda \\ -\left\{\left[1-\left(\xi-\frac{1}{3}\zeta\right)R\right](a\cdot l)-(\eta+\zeta)Ric(a,l)\right\}l^\mu \\ +(\eta+\zeta)R^\mu_\lambda\,(a\cdot l)\,l^\lambda-2\zeta C^{\mu\nu\rho\sigma}l_\nu a_\rho l_\sigma=0\,,\end{aligned} \tag{19}$$

where we use the notation $Ric(v,w) = R_{\mu\nu}v^\mu w^\nu$.

Equation (19) shows that the wave vector is generally not a null vector and that l^2 will depend on the polarization vector $f^\mu = a^\mu/a$, a being the scalar amplitude defined by $a = \sqrt{|\,a^\mu\bar{a}_\mu\,|}$. Thus, light rays are not null geodesics and a gravitational field has properties of birefringence [10, 12, 26]. Moreover, Eq. (19) remain invariant under scaling of the wave vector l_μ. As a consequence, the photon trajectories are frequency independent: the gravitational field is not dispersive.

Let us now restrict our attention to the case where the Ricci tensor is of the form corresponding to a perfect fluid in general relativity. This means that there exists a unit timelike vector u^μ such that $R_{\mu\nu}$ may be written in the form

$$R_{\mu\nu} = \frac{1}{3}(4U - R)u_\mu u_\nu - \frac{1}{3}(U - R)g_{\mu\nu}\,, \tag{20}$$

where U is a scalar function. This scalar function is such that

$$U = Ric(u, u)\,. \tag{21}$$

Then, defining $\mathcal{A}$ and $\mathcal{B}$ as

$$\mathcal{A} = -\frac{1}{3}(3\xi + 2\eta + \zeta)R + \frac{2}{3}(\eta + \zeta)U\,, \quad \mathcal{B} = \frac{1}{3}(\eta + \zeta)(4U - R)\,, \tag{22}$$

Equation (19) reduces to

$$\begin{aligned}&\left[(1+\mathcal{A})l^2 - \mathcal{B}(u\cdot l)^2\right] a^\mu - \left[(1+\mathcal{A})(a\cdot l) - \mathcal{B}(u\cdot a)(u\cdot l)\right] l^\mu \\ &\quad -\mathcal{B}\left[(u\cdot a)l^2 - (u\cdot l)(a\cdot l)\right] u^\mu - 2\zeta C^{\mu\nu\rho\sigma} l_\nu a_\rho l_\sigma = 0\,.\end{aligned} \tag{23}$$

Contracting Eq. (23) by u_μ yields the relation

$$(1 + \mathcal{A} - \mathcal{B})\left[(u\cdot a)l^2 - (a\cdot l)(u\cdot l)\right] = 2\zeta C(u, l, a, l)\,, \tag{24}$$

where

$$C(u, l, a, l) = C^{\mu\nu\rho\sigma} u_\mu l_\nu a_\rho l_\sigma\,. \tag{25}$$

Eliminating $(u\cdot a)l^2 - (a\cdot l)(u\cdot l)$ between Eq. (23) and Eq. (24), we find

$$\begin{aligned}&\left[(1+\mathcal{A})l^2 - \mathcal{B}(u\cdot l)^2\right] a^\mu - \left[(1+\mathcal{A})(a\cdot l) - \mathcal{B}(u\cdot a)(u\cdot l)\right] l^\mu \\ &\quad -2\zeta \frac{\mathcal{B}}{1+\mathcal{A}-\mathcal{B}}\, C(u, l, a, l)u^\mu - 2\zeta C^{\mu\nu\rho\sigma} l_\nu a_\rho l_\sigma = 0\,.\end{aligned} \tag{26}$$

It is easily seen that Eq. (26) are equivalent to Eq. (23) if the inequalities

$$1 + \mathcal{A} - \mathcal{B} \neq 0\,, \quad 1 + \mathcal{A} \neq 0 \tag{27}$$

are satisfied.

6 Application to FRW cosmological models

In what follows, we assume that the field $F_{\mu\nu}$ is a test field propagating in a Friedmann-Robertson-Walker (FRW) universe with a metric

$$ds^2 = (dx^0)^2 - a^2(x^0)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{28}$$

where $a(x^0)$ is the scale factor and $k = 0, 1, -1$ for flat, closed and open models, respectively. In these models, the Ricci tensor may be written in the form given by Eq. (20), where u^μ is the unit 4-velocity of a comoving observer (observer moving with the average flow of cosmic energy). Moreover, the metric g is conformally flat, which means that

$$C^{\mu\nu\rho\sigma} = 0 \tag{29}$$

throughout spacetime. As a consequence, Eq. (26) reduces to

$$\left[(1+\mathcal{A})l^2 - \mathcal{B}(u\cdot l)^2\right] a^\mu - \left[(1+\mathcal{A})(a\cdot l) - \mathcal{B}(u\cdot a)(u\cdot l)\right] l^\mu = 0\,. \tag{30}$$

Of course, we suppose that inequalities (27) hold.

If it is assumed that $(1+\mathcal{A})l^2 - \mathcal{B}(u\cdot l)^2 \neq 0$, Eq. (30) yields

$$a^\mu = \frac{(1+\mathcal{A})(a\cdot l) - \mathcal{B}(u\cdot a)(u\cdot l)}{(1+\mathcal{A})l^2 - \mathcal{B}(u\cdot l)^2} l^\mu\,,$$

which implies $F_{\mu\nu} = 0$ (see Eq. (18)). As a consequence, the wave vector must fulfill the condition

$$(1+\mathcal{A})l^2 - \mathcal{B}(u\cdot l)^2 = 0\,. \tag{31}$$

Two cases are to be envisaged.

1. *General case:* $\mathcal{B} \neq 0$.– It results from Eq. (31) that $l^2 \neq 0$ and $(u\cdot l) \neq 0$ (indeed, $l^2 = 0$ would imply $(u\cdot l) = 0$, which is impossible if $l \neq 0$). Therefore, the phase velocity of light is not equal to the fundamental constant c if $\eta + \zeta \neq 0$ and $4U - R \neq 0$. Since $l^2 \neq 0$, it is possible to choose the gauge so that the Lorentz condition

$$(a\cdot l) = 0 \tag{32}$$

is satisfied. With this choice, Eq. (30) gives

$$(u\cdot a) = 0\,. \tag{33}$$

The corresponding polarization vector is orthogonal to the unit 4-velocity u^μ and to the wave vector l_μ.

2. *Case where* $\mathcal{B} = 0$.– The wave vector l is then a null vector, as in the usual geometric optics approximation. It is worthy of note that Eq. (32) is now a consequence of Eq. (30) and does not result from a special choice of gauge. Nevertheless, the gauge may be chosen so that Eq. (33) is satisfied.

It follows from Eq. (31) that the phase speed of light with respect to a comoving observer has the same value c_l in all directions and for all polarizations. Since the gravitational field is not dispersive, c_l is also the group speed of light. So we shall simply call c_l the speed of light with respect to a comoving observer without any further specification. Using the general theory of geometric optics exposed in [27], we deduce from Eq. (31) that the ratio c_l/c is determined by

$$\frac{c_l^2}{c^2} = \frac{1+\mathcal{A}}{1+\mathcal{A}-\mathcal{B}} = 1 + \frac{(\eta+\zeta)(4U-R)}{3-(3\xi+\eta)R-2(\eta+\zeta)U}\,, \tag{34}$$

where $\mathcal{A}$ and $\mathcal{B}$ are defined by Eq. (22). This formula shows that the speed of light in a FRW background generically differs from c except in the case where $\eta + \zeta = 0$.

We can suppose that at present time $1+\mathcal{A} \approx 1$ and $1+\mathcal{A}-\mathcal{B} \approx 1$. So, we shall henceforth restrict our attention to the part of the history of the Universe such that the two conditions

$$1+\mathcal{A} > 0\,, \quad 1+\mathcal{A}-\mathcal{B} > 0 \tag{35}$$

are satisfied. Indeed, it is clear that the violation of at least one of these conditions can only occur in a domain of spacetime where the curvature is so great that the present theory is probably no longer realistic. Inequalities (35) are sufficient to ensure that the quantity c_l determined by Eq. (34) is real. Moreover, they imply the following equivalence:

$$c_l > c \iff (\eta+\zeta)(4U-R) > 0\,. \tag{36}$$

Equation (34) shows that the vacuum acts as a medium moving with the unit 4-velocity u^μ and having a refractive index n given by

$$n = \frac{c}{c_l} = \sqrt{1 - \frac{\mathcal{B}}{1+\mathcal{A}}}\,. \tag{37}$$

Using a well-known result of the geometric optics approximation [27], we can state that the light rays are null geodesics with respect to the associated metric tensor $\overline{g}$ defined by

$$\overline{g}_{\mu\nu} = g_{\mu\nu} - \left(1 - \frac{1}{n^2}\right) u_\mu u_\nu = g_{\mu\nu} + \frac{\mathcal{B}}{1 + \mathcal{A} - \mathcal{B}} u_\mu u_\nu \,. \tag{38}$$

However, taking into account Eqs. (28) and (37), it is easily seen that the conformal metric $d\tilde{s}^2 = n^2 d\overline{s}^2$ is a FRW metric with the scale factor $\tilde{a} = n\, a$. So, we can enunciate the following theorem:

Theorem 1.– *In a FRW background, the light rays are null geodesics of the new FRW metric defined as*

$$d\tilde{s}^2 = (dx^0)^2 - \tilde{a}^2(x^0) \left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \right] , \tag{39}$$

where

$$\tilde{a}(x^0) = \frac{c}{c_l} a(x^0) = \sqrt{1 - \frac{\mathcal{B}}{1 + \mathcal{A}}}\, a(x^0) \,, \tag{40}$$

with

$$\mathcal{A} = 2(3\xi + \eta)\frac{\ddot{a}}{a} + 2(3\xi + 2\eta + \zeta) \left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) , \tag{41}$$

$$\mathcal{B} = 2(\eta + \zeta) \left(-\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \right) , \tag{42}$$

$\dot{a}$ *denoting the derivative* da/dx^0.

It follows from this theorem that all geometrical properties of light rays can be obtained by substituting $\tilde{a}(x^0)$ for $a(x^0)$ into the usual definitions and relations (luminosity distances, number counts, gravitational lensing,...). In particular, the red-shift of an extragalactic comoving object will be given by

$$1 + z = \frac{a_0}{a_e}\frac{n_0}{n_e} = \frac{a_0}{a_e}\frac{(c_l)_e}{(c_l)_0} , \tag{43}$$

where the subscripts e and 0 stand for the time of emission and for the present time, respectively. As a consequence, a non-minimal coupling between the electromagnetic field and gravity is able to affect our informations concerning the evolution of the Universe.

7 Speed of light and energy content of FRW models

In order to connect the speed of light with the energy content of the Universe, we now suppose that the metric satisfies Einstein equations. In the FRW models, the r.h.s. of Einstein equations may always be considered as the energy-momentum tensor $T_{\mu\nu}$ of a perfect fluid having an energy density μ and a pressure p, μ and p only depending on the cosmic time. So, we have

$$R_{\mu\nu} = \kappa \left[(\mu + p) u_\mu u_\nu - \frac{1}{2}(\mu - p) g_{\mu\nu} \right] - \Lambda g_{\mu\nu} \,. \tag{44}$$

As a consequence, R and U are given by

$$R = -\kappa(\mu - 3p) - 4\Lambda \,, \quad U = \frac{1}{2}\kappa(\mu + 3p) - \Lambda \,. \tag{45}$$

Substituting for R and U from Eq. (45) into Eq. (34), we obtain the second theorem of this paper.

Theorem 2.– *Given a FRW model, let μ, p and c_l respectively denote the total energy density, the pressure and the speed of light with respect to a comoving observer. Then*

– *if* $\eta + \zeta = 0$,

$$c_l = c \,; \tag{46}$$

– *if* $\eta + \zeta \neq 0$,

$$c_l = c \sqrt{1 + \frac{\mu + p}{\mu_m + \frac{1}{3}(\sigma - 2)\mu - \sigma p}} \,, \tag{47}$$

where σ is defined as

$$\sigma = \frac{3\xi + 2\eta + \zeta}{\eta + \zeta} \tag{48}$$

and

$$\mu_m = \frac{1}{(\eta + \zeta)\kappa} + \frac{2}{3}(2\sigma - 1)\frac{\Lambda}{\kappa} \,. \tag{49}$$

From Eq. (45), it is easily seen that Eq. (36) may be written as

$$c_l > c \iff (\eta + \zeta)(\mu + p) > 0 \,. \tag{50}$$

As a consequence, the speed of light is greater than c for any reasonable equation of state if and only if the condition $\eta + \zeta > 0$ is satisfied.

Neglecting the pressure, Eq. (47) yields

$$\frac{c_l}{c} = 1 + \frac{\mu}{2\mu_m} + O(\frac{\mu^2}{\mu_m^2})\,. \tag{51}$$

Thus, up to the first order in μ/μ_m, the variation of the speed of light when p is negligible is entirely governed by the value of μ_m.

Using the values of ξ, η and ζ given by Eq. (5), it may be seen that $\eta + \zeta > 0$. So, the speed of light obtained from the Drummond-Hathrell Lagrangian is greater than c as long as $\mu + p > 0$. Neglecting the contribution of the cosmological constant in Eq. (49), we find $\mu_m/c^2 = 2.5 \times 10^{51}$ g.cm^{-3}. Then, taking $\mu_0/c^2 \approx 2.5 \times 10^{-30}$ g.cm^{-3} and neglecting the pressure, we see that at the present time

$$\left(\frac{c_l}{c}\right)_0 - 1 \,\approx\, 5 \times 10^{-82}. \tag{52}$$

As a consequence, the difference between c_l and c predicted in the one-loop approximation of QED cannot be detected by local experiments.

Finally, let us note that the speed of light $c_l = c$ in a de Sitter spacetime whatever the parameters ξ, η, and ζ, since one has $\mu + p = 0$ in this case.

8 Conclusion

In this paper, we have outlined the theory of light rays propagating in a FRW background according to the NMC between electromagnetism and gravity defined by Eqs. (1)–(2). We have obtained the general expression of the speed of light c_l as a function of the energy density and of the pressure of the Universe. We have found that light propagates at a speed greater than c if and only if $\eta + \zeta > 0$, provided that $\mu + p > 0$. This conclusion generalizes a result previously obtained by Drummond and Hathrell in the framework of QED.

An application of these results to the horizon problem in cosmology is in preparation.

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