On the potential of the Coulomb field and the gauge potentials
Dialogue with a standard physicist

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RÉSUMÉ. Les points de vue adoptés par la quasi totalité des physiciens sur l'électrodynamique en mécanique quantique (théorie quantique des champs, formulation abstraite des théories de jauge) sont comparés à ceux, directement liés à la géométrie de l'espace-temps, qu'on peut déduire des travaux de l'Ecole Louis de Broglie et ou de l'utilisation des algèbres réelles dans le formalisme d'Hestenes.

ABSTRACT. The points of view assumed by the quasi totality of the physicists upon quantum electrodynamics (quantum fields theory, abstract formulation of the gauge theories) are compared with the ones, directly related to the geometry of space-time, that may be deduced from the works of the Louis de Broglie's school or the use of the real algebras in the Hestenes formalism.

1 Introduction

Notations. Let $M = R^{1,3}$ the Minkowski spacetime and $\{e_\mu\}$ a galelean frame of M be. We will write for simplicity $a.b = a^\mu b_\mu$ (and $a^2 = a.a$) for the scalar product of two vectors $a, b \in M$, $a \wedge b$ for their Grassmann product (whose components are $a^\mu b^\nu - a^\nu b^\mu$) or (simple) bivector, $\partial = e^\mu \partial_\mu$ for the gradient operator of M.

Electromagnetism may be presented into two aspects quite different but both closely and very simply related to the geometry of the Minkowski spacetime M.

(1) *The Maxwell-Lorentz electromagnetism.* We have schown in [1, 2002] that this part of electromagnetism may be built on the following simple consiserations taken as a fundamental principle, directly related to the pseudo-euclidean structure of M:

One associates with a punctual charge q -or, in quantum theory, the *eventuality of a punctual presence* of a charge q- a point P of M and a unit timelike vector u, $(u^2 = 1)$. Then one considers the spacetime vector

$$A(X) = q\frac{u}{\overrightarrow{PX}.u} = \frac{q}{r}u \in M \tag{1}$$

as the potential created at all point X of M such that the vector

$$\overrightarrow{PX} = r(u+n), \quad n, u \in M, \quad n.u = 0, \quad n^2 = -u^2 = -1 \tag{2}$$

is isotropic, and X is in the future of P.

We have schown that the possibility to join together different charges -or *different eventualities* of presence of *a same* charge- in a conserved charge current $j \in M$ (with $\partial.j = 0$) leads, *via* the Lorentz integral formula of the retarded potentials, to the Maxwell-Lorentz laws

$$\partial^2 A = 4\pi j, \ \partial.A = 0 \tag{3}$$

That explains why these laws, which were established on grounds of classical, deterministic and macroscopic phenomenas (free electrons moving in moving wires), have been then revealed as applicable to the fields created by the microscopic, probabilistic, quantal charge currents of electrons bound in atoms.

(2) *The potentials of the gauge theories.* They are also directly related to the geometry of M by means of the orthogonal properties of this space.

G. Lochak has given in 1956 a geometrical interpretation of the $U(1)$ gauge of the Dirac electron theory [2]: it is the set of the rotations upon itself of a *real* plane $\pi(x)$ of M, "the spin plane", orthogonal at each point x of M to the timelike direction of the Dirac charge probability current $j = e\rho v$, $(\rho > 0, v \in M, v^2 = 1)$.

The plane $\pi(x)$ may be defined by two orthonormal vectors n_1, n_2, orthogonal to v, and is colinear to the direction of the intrinsic angular momentum i.e. the bivector spin $(\hbar c/2) n_1 \wedge n_2$. A change of gauge is defined by a rotation of the vectors n_1, n_2 in the plane $\pi(x)$, through an angle χ, which changes ω into $\omega - \partial\chi$ and the gauge invariance is achieved by the addition to A of the spacetime vector $-(\hbar c/2e)\partial\chi$.

This interpretation of the $U(1)$ gauge was independently and indisputably confirmed as a necessity, by means of real algebraic ways, ten years later by D. Hestenes in 1967 [3]. We have shown in 1995 [1, 1997] that the electroweak $SU(2)\times U(1)$ gauge corresponds, for $SU(2)$, to the rotations in the *real* three-space orthogonal to the direction of the time-like current associated with the doublet left-electron, left-neutrino, and, for $U(1)$, to the rotations in the "spin planes" of the electron and the neutrino, in a way which allows the direct product of these rotations.

We have also shown that what we call energy appears as the product of physical constants by the infinitesimal rotations upon themselves of these three and two dimensional spaces.

In particular we had interpreted in [1, 1971] the energy-momentum spacetime vector $p = T(v)/\rho$, where T is the Tetrode energy-momentum tensor, of the Dirac electron, as including the infinitesimal rotation of the spin plane $\pi(x)$ upon itself, i.e. the spacetime vector $\omega \in M$, in such a way that

$$p = \frac{\hbar c}{2}\omega - eA, \quad \omega_\mu = (\partial_\mu n_1).n_2 = -(\partial_\mu n_2).n_1 \tag{4}$$

where $A \in M$ is the exterior potential.

The entity $(\hbar c/2)\omega$, divided by the charge electron e, appears in addition to a potential $-A_\mu$, and for this reason deserves to be called a gauge potential.

The term of gauge potential was assigned by F.Gliozzi [4] to exactly the same geometrical object $(\hbar c/2e)\omega$, corresponding to the rotation upon itself of a spacelike plane $\pi(x)$ and introduced, quite independently, in his article, not in the electron but in the strings theory.

But rather, this entity is here to be associated with the energy of the electron: for exemple in the Darwin solutions, where $A = -(eZ/r)e_0$, the orthogonal projection of $(\hbar c/2)\omega = p + eA$ upon e_0 is the constant E corresponding to the energy of the state of the electron.

Gliozzi (deceased about ten years ago) associated with the $U(1)$ gauge topological defects, as it is made by many authors (see for example [5]). But in contrast with the contain of these works, these defects are to be related to the rotations upon itself of a *real* plane $\pi(x)$ of spacetime, instead of the abstract group $U(1)$. Here, these defects seem to correspond to the variations of p, i.e. to the emissions or absorptions of photons.

Despite the independant works of Lochak (almost an half century old!), of Hestenes (and his followers) and the remarkable article of Gliozzi, the quasi totality of the physicists continue to interprete the gauges as internal properties of particles, related to the abstraction of the complex algebras, instead of the reality of the geometry of spacetime. Certainly the reasons of this incomprehension deserves to be discussed. It will be the subject of the present paper.

2 The Maxwell-Lorentz electromagnetism and the Quantum Field Theory

The Quantum Field Theory (QFT) has been constructed "*by exact analogy with the ordinary quantum theory*" ([6], p. 56). Indeed, in the starting commutation relations of the theory ([6], Eq. (6), p. 56) $\hbar$ is associated with the number i, in the product $i\hbar$, in the same way that it appears in the Dirac electron equation.

But, from the first glance, this construction may be considered as suspicious to an user of real algebras. The i associated with $\hbar$ in the Dirac equation *is not at all* the auxiliary and undefined number $\sqrt{-1}$ allowing the representation of the real potentials A by means of half sums of complex numbers and their conjugates ([6], Eq. (3), p. 56). The Dirac i is in fact the bivector $e_1 \wedge e_2$, generator of the rotations in the (x^1, x^2) plane of the galilean frame $\{e_\mu\}$ [3,1967]. This bivector becomes, after the Lorentz rotation which transforms e_0 into v, the bivector $n_1 \wedge n_2$ defining the intrinsic spin $(\hbar c/2)n_1 \wedge n_2$. Applied to the potentials the analogy is meaningless.

This fact escapes the attention of the standard physicists because in quasi totality they do not know that, in the *real* Clifford algebra $Cl(p, n-p)$ associated with a euclidean space $R^{p,n-p}$, there are different elements of the Grassmann algebra $\wedge R^n$ whose square is equal to -1, and forget that the objects of physics are elements of the Grassmann algebra of M. For example the i of the bivector of M which is the electromagnetic field, $F = \partial \wedge A = \overrightarrow{E} + i\overrightarrow{H}$ is the multivector of rank four $e_0 \wedge e_1 \wedge e_2 \wedge e_3$, different from the bivector $e_1 \wedge e_2$. The square of both is -1 in $Cl(1, 3)$.

Nevertheless, because it is used generally in conformity with the Maxwell-Lorentz laws, QFT may lead to right results. However in all the problems I have met: spontaneous emission (see [1, 1993]), quantum transitions, photoeffect [7], even in the calculation of the Lamb shift (at least in its unrenormalized part) [8],[9], the use of QFT may be advantageously replaced by the consideration of the Maxwell-Lorentz fields created by the transition Dirac charge currents. The reason of the presence of $\hbar$ in these field lies in the presence in their sources the charge currents, not in constitutive laws of the fields.

However *there is at least a case where the use of QFT leads to a major absurdity.*

The quantization of the number π! ([1,1990]). For the inattentive readers I emphasize that I am not the author of the stupidity I am going to describe. I do not know the name of its author but I know the ones of some of its users: for example Kroll and Lamb [8], French and Weisskopf [9], i.e. the first calculators of what is just considered as an outstanding confirmation of QFT, the Lamb shift. It is visible in the lecture of Eqs. (16), (17), (21), and Eq. (60) union of Eqs. (26) and (27) of [8]. It escapes the attention of the readers because in this article one writes $c = \hbar = 1$ (a detestable simplification!). But in [6], Heitler leaves apparent these constants and (catastrophe!) the non-sense may be detected by anyone carries out the detail of the calculation.

One has to consider a potential of the form e/r and to be in conformity with the constitutive laws of QFT the introduction of $\hbar$ inside $1/r$ becomes a necessity.

The Eq. (4'), p. 341 of [6] is the following

$$\frac{(\psi_0^*(\mathbf{r})\psi_n(\mathbf{r}))(\psi_n^*(\mathbf{r}')\psi_0(\mathbf{r}'))}{|\mathbf{r}-\mathbf{r}'|}$$

$$= \frac{1}{2\pi^2\hbar c}\int \frac{d^3k}{k^2}(\psi_0^*(\mathbf{r})e^{i(\mathbf{k}.\mathbf{r})/\hbar c}\psi_n(\mathbf{r}))(\psi_n^*(\mathbf{r}')e^{-i(\mathbf{k}.\mathbf{r}')/\hbar c}\psi_0(\mathbf{r}')) \quad (?!) \tag{5}$$

One observes that $\hbar$, which is absent from the left hand side of the equation, appears in the right hand side. By in what way? Let $R = |\mathbf{R}| = |\mathbf{r}-\mathbf{r}'|$ be.

One has used the "Fourier transform" of $1/R$ ([8], Eq. (16)):

$$\frac{1}{R} = \frac{1}{2\pi^2}\int \frac{d^3K}{K^2}e^{i(\mathbf{K}.\mathbf{R})} \tag{6}$$

where $d^3K/K^2 = d\Omega dK$, $d\Omega = \sin\theta d\theta d\varphi$. But this formula is deduced from the relations

$$\frac{\pi}{2} = \int_0^\infty \frac{\sin x}{x}dx, \quad \frac{\sin KR}{KR} = \frac{1}{2}\int_0^\pi e^{iKR\cos\theta}\sin\theta d\theta = \frac{1}{4\pi}\int e^{i(\mathbf{K}.\mathbf{R})}d\Omega$$

And for introducing $\hbar$ what has been done in reality? One has written

$$\frac{1}{R} = \frac{2}{\pi}\times(\frac{\pi}{2}\times\frac{1}{R}) \quad \text{(exact!)} \tag{7}$$

The Planck constant is here! I repeat Planck's constant is here, inside $\pi/2$ (not inside $2/\pi$!) !! Indeed one can write

$$\frac{\pi}{2} = \int_0^\infty \frac{\sin KR}{K} dK = \int_0^\infty \frac{\sin(kR/\hbar c)}{k} dk \ \ !!! \tag{8}$$

by means of the change of variable $K = k/\hbar c$ and so that $d\Omega dK = (d^3k/k^2)/\hbar c$!

Qu'on mette, tant mieux ou tant pis,

h barre dans un sur r.

Mais qu'on l'introduise dans Pi

Est une idée qui me sidère.

Quantum mechanics contain many mysteries. But one of the most incomprehensible for me is the fact that the authors of this so admirable work which is the Lamb shift calculation, have, innocently or knowingly, used a so coarse expedient.

Nevertheless nothing is wrong in the above calculation, ... except then the possibility to consider the constitutive laws of QFT as corresponding to a physical reality.

3 Dialogue on the gauge theories with a standard physicist

About the other aspect of electrodynamics which is related to the gauge theories, I am going to describe the (standard) conversation I had with at least half a dozen of standard physicists (some of them considered as eminent).

me: "I would like to speak about some geometrical aspects of the $SU(2) \times U(1)$ gauge of the electroweak theory."

him: "Ah, very well."

me: "To be sure that we will be in agreement, I begin with some recalls on the $U(1)$ gauge of the Dirac electron theory. I consider the bivector spin $(\hbar c/2)n_1 \wedge n_2$ ..."

him: "The bivector what? For me the spin is an operator."

me: "Oh, excuse me. I mean the bivector, i.e. the antisymmetric tensor of rank two, which represents the intrinsic angular momentum of the Dirac particle, and which is a bivector of the Minkowski spacetime M as all angular momentum. n_1, n_2 are orthonormal spacelike vectors, orthogonal to the timelike Dirac current j, you know?"

him: "Of course j is defined in all the treatises. Please continue to more serious things."

me: "The tolal angular momentum is $x \wedge p + (\hbar c/2) n_1 \wedge n_2$ (see [3, 1973]), where p is the spacetime energy-momentum at the point x of M. A first remarkable feature is that p contains the spacetime vector $(\hbar c/2)\omega$, where ω represents the infinitesimal rotation upon itself of the plane $\pi(x)$ orthogonal to j defined by n_1, n_2."

him (inattentive): "Ah."

me: "I attain now the $U(1)$ gauge. A second remarkable feature is the fact that a change of gauge, i.e. the change of the wave function, the Dirac spinor Ψ, into $\Psi \exp(i\chi/2)$, corresponds to the rotation through an angle χ, in the plane $\pi(x)$, of the vectors n_1, n_2. Thus, three apparently very different data of the theory, the spin, the energy-momentum and the $U(1)$ gauge are unified into the movement of a spacetike plane $\pi(x)$, in such a way ..."

him (furious): "IMPOSSIBLE. What you say about the pseudo movement of your pseudo plane $\pi(x)$ is IMPOSSIBLE. The Dirac spinor Ψ contains a representation of the Lorentz group. You agree?"

me: "Of course. Even, Ψ has been completely explicited by Lochak [2], and quite independently by Hestenes [3, 1967], in the form $\Psi = \sqrt{\rho}\exp(\underline{i}\beta/2)R$. $\rho > 0$ is the invariant probability density, the scalar β is the Yvon-Takabayasi "angle" (you know?), R is a Lorentz rotation. In [2], R is expressed by means of the Dirac matrices γ_μ and $\underline{i} = \gamma_0\gamma_1\gamma_2\gamma_3$. In [3, 1967], Ψ is a biquaternion Q element of the even Clifford subalgebra $Cl^+(1,3)$ associated with M and

$$\underline{i} = e_0 e_1 e_2 e_3 = e_0 \wedge e_1 \wedge e_2 \wedge e_3$$

The formalisms used are not the same but may be unified by the identification $\gamma_\mu = e_\mu$. The rotation R allows one to define $j = e\rho v$, $(v^2 = 1)$, in such a way that $v = Re_0R^{-1}$, but *also* that $n_k = Re_kR^{-1}$, $k = 1, 2, 3$, which give at the point x the Takabayasi moving frame (you know?) $\{v, n_k\}$. And thus R transforms the generator $e_1 \wedge e_2$ of the rotations in the (x^1, x^2) plane into the generator $n_1 \wedge n_2$ of the rotations in the plane $\pi(x)$."

him (with rage): "I don't care of your pseudo algebra $Cl(1,3)$, you have not heard me. One can write $i\Psi = \Psi i$, YOU AGREE? So it is IMPOSSIBLE that the $U(1)$ gauge may be the set of the rotations in a plane of spacetime. BY THE FACT OF THE ABOVE COMMUTATION RELATION, $U(1)$ is to be considered in a DIRECT PRODUCT with the representation of the Lorentz group included in the use of Ψ, and NOT AS A SUB-GROUP OF THE ORTHOGONAL GROUP $O(1,3)$ associated with the Minkowski spacetime. The $U(1)$ gauge NEEDS to be considered as an internal ABSTRACT property, and so do also the other gauges of the theory of particles."

me: "But Lochak wrote R in the form $R = R_0 \exp(\gamma_1\gamma_2\varphi/2)$, where φ is the proper rotation Euler angle, and $\gamma_1\gamma_2$ (written $e_1e_2 = e_1 \wedge e_2$ in $Cl(1,3)$) is the corresponding generator of this rotation. $\gamma_1\gamma_2$ is in fact your above i. One can already find $\exp(\gamma_1\gamma_2\mu\varphi)$, ($\mu = m \pm 1/2$), in the biquaternionic form given to Ψ by Sommerfeld during the years 1930 (see [10], p. 275. *Nota*: It is a pity that Sommerfeld is not still alive!) to the solutions of the Kepler-Problem. Gliozzi ..."

I stop my speech. I have heard the banging of a door behind me. I am alone in front of the blackboard.

I will not repeat here into details, as I have done in [1], 1997, 2001, on grounds of the works of Hestenes 1967 and more recently Lasenby *et al* [11] in what way the reasoning of the standard phycistics is wrong. Readers may consult the above articles and e-mail me for additional explanations.

I simply recall that the Pauli and the Dirac spinors ξ and Ψ are the forms *dislocated* and *truncated* into two and four "complex components" of the Hamilton quaternion $q \in Cl^+(3,0)$ and the Hestenes biquaternion $Q \in Cl^+(1,3)$ which are to be considered as whole entities. The number $\sqrt{-1}$ in these components is in reality k, the third of the three "imaginary numbers" of Hamilton, which are the bivectors (with a change of sign)

$$i = e_2 \wedge e_3, \quad j = e_3 \wedge e_1, \quad k = e_1 \wedge e_2 \tag{9}$$

When one changes, *by an action on the components*, Ψ into $\sqrt{-1}\Psi = \Psi\sqrt{-1}$, one changes in reality Q into Qk, i.e. one multiplies Q *on the right* by the generator k of the rotations in the (x^1, x^2) *real* plane of spacetime (this point is mentioned for the first time in [3], 1967).

4 Conclusion

Classical and quantal electromagnetisms related to the Maxwell-Lorentz theory may be considered as identical. They are both based on entities, as electrons, free or bound in atoms, protons inside nucleus, which are punctual charges, sources of potentials of the forms q/r. The only difference lies in the fact that, when the charges are joined together in a current, in the classical case the charges are distinct and in the quantal one the current is a probability current associated with an unic charge.

Electrodynamics associated with the gauge theories are of a quite different nature, but may be related more to a a theory of material particles, by means of their momentum-energy tensors, than a theory of the pure electromagnetism field.

Photons seem belong to this second part of electomagnetism, but the mystery of the exchange of their energy with particles is not yet solved to my knowledge. On grounds of an idea of Gliozzi, they could be related to topological defects of a global geometrical nature.

In all cases the geometry of spacetime: isotropic propagation of the electromagnetic action for the Maxwell-Lorentz theory, rotations upon themselves of spacetime frames for the gauge theories, perhaps global geometry for the photons, is to be considered.

Unfortunately, the geometrical aspects of the gauge theories may be accessible only to physicists, as those of the Louis de Broglie's school, which have studied entities independent of all galilean frame and their relations, or used the real Clifford algebras introduced in physics by D. Hestenes.

The reasons of this ignorance are visible on the first pages of the treatises. Not one word on the Grassmann algebra $\wedge R^n$ associated with the vector space R^n, even though this algebra is the first to be associated with R^n. Not one word on the real Clifford algebra $Cl(p, n-p)$, associated with an euclidean space $E = R^{p,n-p}$, and acting on the elements of $\wedge E$, which is the direct continuation of the quaternionic algebras of Hamilton and Clifford, with the so simple definition of the Clifford product of two vectors

$$ab = a.b + a \wedge b, \quad a, b \in E \tag{10}$$

. Note that if $E = R^{2,0}$, ab defines the elements of the field C.

Instead, one finds a certainly powerfull but blind formalism based on the representations on complex spaces, in which i is not a geometrical object, but the undefined "number" $\sqrt{-1}$. Indeed, this formalism has been the base of the foundation of quantum mechanics and all their present developments. But *it is now an obstacle* to the union of entities, apparently very different, but closely related between them when their links with the geometry of spacetime are explicited. *Certainly it is a* *major obstacle to the unification of quantum mechanics with the general relativity.* It is sufficiently coherent to avoid algebraic errors, but the danger that it leads to fallacious constructions, as the one of QFT when this theory is applied to the potential q/r, is not excluded.

In a conceptual point of wiew, imagine some intelligence, completety outside of the world, having only the knowledge of the continuity of its own existence. One can imagine that it would invent the set N, the ring Z, the field Q, and to reach this continuity, the field R, then the direct products R^2, R^3, ...To prolonge R by a field in R^2, then in R^3, it would invent the euclidean spaces $R^{2,0}$ and $R^{3,0}$ to obtain the field C as $Cl^+(2,0)$ and the field of the Hamilton quaternions H as $Cl^+(3,0)$ (Note that H may be also considered as $Cl(0,2)$). No possibility to obtain then a field. But H may be prolonged by the ring of the Clifford biquaternions $Cl(3,0)$, and our intelligence, inventing $R^{1,3}$, identifying (see [3, 1966]) the bivectors $e_k \wedge e_0$ of this new space to vectors of $R^{3,0}$, and $Cl(3,0)$ to the ring $Cl^+(1,3)$ of the Hestenes biquaternions, could prolonge this ring by the ring $Cl(1,3)$ which contains all the geometrical objects of the spacetime.

If Kant would live to-day he could exclaim "The spacetime of the special relativity is a certitude *algebrically* apodictic".

(Note that the same processus allows one to introduce $R^{1,4}$, the space of the fourth dimension, in such a way that $Cl(1,3)$ may be identify to $Cl^+(1,4)$).

But, *in supplement to the spacetime $R^{1,3}$, there is another marvellous algebraic gift of Nature*, i.e. the fact that the brik by which are composed the physical objects situed inside this spacetime, is the Dirac spinor, *alias* the biquarternion considered, not as a *complex abstract* entity, but as a *real geometrical* object, element of $Cl^+(1,3)$.

Kant could say "Two objects fill up my soul with an unceasingly renovated admiration: inside me, the biquaternion; outside me, the biquaternion."

It is not sure that the present incomprehension of the real algebras will be soon dissipated. But perhaps in some decennies, in the same way that J. Dieudonné (one of the first founders of the Bourbaki school) has evoked the tragedy of Grassmann [12], an eminent physicist will describe the tragedy of Hestenes.

I apologize for the (pseudo) poetical or philosophical comments included in the present paper.

Note. *Electrodynamics, electromagnetism, and the gauge potentials*

Electrodynamics is the study of the behaviour of charged particles endowed with a mass and eventually a spin in a given electromagnetic field or potential. For example, the equations of Lorentz, Eq. (12), and Dirac of the electron are to be related to this discipline.

Electromagnetism is the study of the properties of the fields or potentials created by charged particles without the direct intervention of their mass and spin, as in the Maxwell-Lorentz theory.

Since the check (see [6], Sect. I-4) of the attempt of interpreting the mass m of the electron as being of an electromagnetic nature, these two disciplines are to be distinguished, but they may be mixed. We give an illustration of the fact that the gauge potentials belong to the first discipline, and also that they are mixed with the second one. It is relating with the role of the movement of the "spin plane" $\pi(x)$ of the electron in an exterior potential A which may depend on the Maxwell-Lorentz theory.

We have schown in [1], 1992, that, in the case where one can consider that the direction of $\pi(x)$ is fixed, orthogonal, say, to e_3, the angle β null, the potential A depending only of x^0, x^3 and so that $A^1 = A^2 = 0$, the part of the intrinsic Dirac equation ([1], 1971) which does not contain the density ρ (see [1], 1984) is exactly

$$\frac{\hbar c}{2}\omega - eA = mc^2 v \tag{11}$$

In this case the gauge potential $(\hbar c/2e)\omega$ is a gradient, and we have schown that if one takes the spacetime curl $\partial\wedge$ of this equation one obtains, with the elimination of $\hbar$, the classical Lorentz equation

$$\frac{e}{c}F.V = m\frac{dV}{d\tau}, \quad V = cv = \frac{dx}{d\tau}, \quad F = \partial \wedge A \tag{12}$$

where x describes one of the current lines C of the solution of the Dirac equation (in this particular case), whose proper time parameter is τ.

The disparition of $\hbar$ in (12), perhaps also the fact that no topological defect is in this case to be associated with the variation of $\pi(x)$, may explain that no photon is then to be considered.

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