Electromagnetic pulses from a cavity moving in vacuum : possible experiments

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ABSTRACT; Vacuum field fluctuations exert radiation pressure on mirrors in quantum vacuum. For a pair of mirrors this effect is well known as the Casimir force, that is an attractive force between two mirrors at rest in vacuum. When a single mirror is moving in vacuum, radiation pressure leads to a dissipative force which opposes itself to the mirrors motion. Accordingly the electromagnetic field does not remain in the vacuum state but photons are emitted by the mirror into vacuum. This motion-induced radiation and the associated radiation reaction force are dissipative effects related to motion in quantum vacuum, although this motion has no further reference than vacuum itself.

This article describes the photon emission of a high-finesse cavity oscillating globally in quantum vacuum. Novel effects of the quantum radiation like pulse shaping and frequency up-conversion are predicted, which could be used to experimentally demonstrate motion-induced dissipative effects. Possible experimental realisations are discussed in the end of the paper.

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1 Introduction

Relativity of motion is one of the basic principles of physics since Galileo. In classical physics this principle applies without further considerations, as vacuum is considered to be completely empty. However, the face of the problem today is changed by quantum theory. Quantum vacuum is no longer empty. It contains irreducible field fluctuations which lead to mechanical effects for any scatterer in vacuum. In this paper I will discuss some observable effects of these fluctuations associated with the motion of mirrors in vacuum.

I will focus my attention on the vacuum fluctuations of the electromagnetic field. These fluctuations are characterised by a mean energy of $\frac{1}{2}\hbar\omega$ per field mode of frequency ω. On a microscopic scale, their coupling to electrons in atoms lead to phenomena like spontaneous emission and the Lamb shift of energy levels for a single atom, or van der Waals forces between two atoms or molecules. But vacuum fluctuations have also observable effects on macroscopic objects, for the following reason. While the mean value of the electromagnetic field vanishes in the vacuum state, the mean value of the squared field is not zero. As a consequence, vacuum fluctuations have a non-vanishing radiation pressure and thus exert mechanical action on scatterers. For macroscopic objects, the most famous effect induced by vacuum fluctuations is the Casimir force, an attractive force arising between two mirrors at rest in vacuum [1]. But vacuum fluctuations produce also mechanical effects related to the motion of scatterers in vacuum. For a single mirror moving in vacuum a dissipative force may arise, opposing itself to the mirror's motion [2]. In fact, even when the mirror is at rest in vacuum, it experiences a fluctuating force due to the radiation pressure of field fluctuations [3, 4]. However, as the radiation pressure is the same on both sides of the mirror at rest, no mean force appears in this case. The dependence of the dissipative force is directly connected to the spectral properties of the fluctuating force through the fluctuations-dissipation relations [5] as will be shown in the following.

2 The dissipative force

Let me begin with the simple model of a perfect mirror in a two-dimensional space-time. In order to motivate the origin of the dissipative effect of quantum vacuum, I will first consider a mirror in a thermal field. As is well known, in a thermal field, the dissipative force $F_{\text{diss}}(t)$ is proportional to the mirror's velocity $q'(t)$

$$F_{\text{diss}}(t) = -\frac{\hbar\theta^2}{6\pi c^2} q'(t) \tag{1}$$

The force may equivalently be written in the frequency domain

$$F_{\text{diss}}[\omega] = \frac{\hbar\theta^2}{6\pi c^2} i\omega q[\omega] \tag{2}$$

where $F_{\text{diss}}[\omega]$ and $q[\omega]$ are the Fourier transform of the force and mirror's displacement. In both formulas, θ is the field temperature expressed in frequency units

$$\theta = \frac{2\pi k_{\mathrm{B}} T_{\mathrm{field}}}{\hbar} \tag{3}$$

$\hbar$, k_{B} and c are the Planck constant, the Boltzmann constant and the speed of light respectively. This force is a classical expression which tends towards zero when temperature goes to zero. In fact, it neglects the effect of vacuum fluctuations.

When this effect is taken into account, the linear susceptibility is found to scale as the third power of frequency at the limit of zero temperature

$$F_{\mathrm{diss}}[\omega] = \frac{\hbar}{6\pi c^2} i\omega^3 q[\omega] \tag{4}$$

This result, which could be expected from mere dimensional arguments, implies that the force is proportional to the third order time derivative of the mirror's position

$$F_{\mathrm{diss}}(t) = \frac{\hbar}{6\pi c^2} q'''(t) \tag{5}$$

The linear susceptibilities (2,4) are directly connected to the spectral properties of the fluctuating force exerted upon a mirror at rest through the fluctuations-dissipation relations [5]. At an arbitrary temperature the dissipative force is just the sum of the 2 contributions (2) and (4). These expressions can also be generalized to the case of a real mirror with frequency-dependent reflection and transmission amplitudes [6].

The dissipative force arising for a mirror moving in quantum vacuum has interesting consequences with respect to the problem of relativity of motion. In contrast to the dissipative force experienced by a mirror in a thermal field, the dissipative force in quantum vacuum vanishes for a motion with uniform velocity. This is a direct consequence of the Lorentz invariance of quantum vacuum. It also vanishes for a motion with uniform acceleration. The appearance of vacuum in an accelerated frame is a much debated question [7]. For the present problem of motion of a mirror in vacuum there is a clear answer at our disposal. No dissipative force arises for a motion with uniform acceleration and this fact may be explained as a consequence of the conformal invariance of electromagnetic vacuum.

However, a dissipative force arises for a mirror moving in vacuum with a non-uniform acceleration. For example, an oscillating mirror will find its motion damped out through the coupling to vacuum fluctuations towards a motion with uniform velocity. As the mirror is moving without any further reference than vacuum fluctuations themselves, one may say that vacuum fluctuations act as a sort of reference with respect to which motion takes place. In other words, this implies that quantum vacuum may be considered as defining privileged reference frames for motion.

3 Observation of dissipative effects?

¿From a fundamental point of view, it would be very interesting to get experimental evidence of the disspative effects related to motion in vacuum. So far, these effects have not yet been observed for macroscopic objects like mirrors. As a matter of fact, the orders of magnitude are exceedingly small for the fluctuating force as well as for the dissipative force. This raises the question which will be discussed in this paper: how can one increase the order of magnitude of dissipative effects of vacuum fluctuations on mirrors and eventually render those effects observable ?

A first idea is to observe changes in the field rather than in the mechanical forces. Indeed, due to energy conservation, photons are emitted into vacuum when the mirror's motion is damped. In other words, the dissipated energy is transformed into radiation emitted by the mirror. Let us consider a mirror oscillating in vacuum at a frequency Ω with an amplitude q_0 as shown in figure 1. The number of emitted photons N

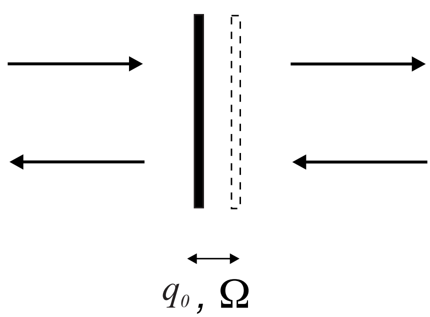

Figure 1: Single mirror oscillating in vacuum. The arrows represent the vacuum field which, in a monodimensional space, may be considered as two counterpropagating fields.

during the measurement time T can be calculated to be

$$N = \frac{\Omega^3 q_0^2 T}{3\pi c^2} = \frac{\Omega T}{3\pi} \frac{v^2}{c^2}$$
$$v = q_0 \Omega \qquad (6)$$

Ω^3 characterizes the already discussed motional susceptibility and v is the mirror's maximal velocity. Since N scales as the square of the ratio between the mirrors mechanical velocity and the speed of light, it remains very small for any possible macroscopic motion. If we consider the velocity of a macroscopic object to be bound by the sound velocity in typical materials (e.g. quartz), one obtains at most one emitted photon per 10^{10} oscillation periods.

A second idea for improving the orders of magnitude of this motion-induced radiation is to study a cavity oscillating in vacuum instead of a single mirror [8]. In this configuration one may profit from the resonant amplification of radiation inside the cavity. The resonant enhancement is determined by the cavity finesse $\mathcal{F}$ which gives the number of roundtrips of the field before it leaves the cavity and depends on the mirrors amplitude reflection coefficients r chosen equal here for simplicity

$$\mathcal{F} = \frac{\pi}{1 - r^2} \qquad (7)$$

Hence the cavity has to be treated as an open system with mirrors having reflection coefficients smaller than unity so that the field can leave the cavity by transmission through the mirrors. This distinguishes the present calculations from the numerous works devoted to photon production between a pair of perfectly reflecting mirrors [2, 10] in which case the amount of radiation emitted outside the cavity cannot be evaluated.

Motion-induced radiation is effectively enhanced if the photons radiated by the oscillating mirrors are emitted at a cavity resonance frequency. In the following I will concentrate on a motion, where the cavity oscillates as a whole in vacuum as shown in Figure 2. This means that both mirror oscillate with respect to quantum vacuum and not with respect to each other. For this situation, motion-induced radiation is amplified inside the cavity when the mechanical oscillation frequency Ω is an odd multiple of the fundamental cavity resonance frequency π/τ

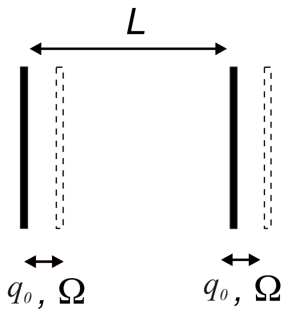

Figure 2: Cavity oscillating globally in vacuum with an amplitude q_0 at a frequency Ω with a constant mechanical length L.

[8]

$$\Omega = \frac{3\pi}{\tau}, \frac{5\pi}{\tau}, \frac{7\pi}{\tau}, \ldots$$
$$\tau = \frac{L}{c} \tag{8}$$

τ is the time of flight of photons between the two mirrors separated by a distance L. At perfectly tuned resonance, the number of photons emitted by the cavity is the product of motion-induced radiation emitted by a single oscillating mirror (6) by the cavity finesse $\mathcal{F}$

$$N = \mathcal{F}\, \frac{\Omega T}{3\pi} \frac{v^2}{c^2} \tag{9}$$

Since the cavity finesse can be a very large number, up to $10^9 - 10^{12}$ for instance for microwave cavities, this increases considerably the order of magnitude of motion-induced radiation.

In addition, for a high finesse cavity, the system shows novel physical signatures [11] which might become very important in an experimental observation in order to discriminate motion-induced radiation from potential stray effects. These signatures will be discussed in the following two sections.

4 Pulse shaping

For a single reflection, the field scattered by the oscillating mirror undergoes a phase shift of the order of $\frac{v}{c}$ compared to the incoming field. This phase shift is always small for a macroscopic mirror. However, inside the cavity the field is scattered a great many times by the moving mirrors leading to an accumulation of the dissipative effects of vacuum on the moving mirrors. For the cavity oscillating globally in vacuum, the phase shifts over a great number of reflections accumulate in an optimal way. One may then introduce effective parameters to characterize the system. The first one is an effective phase velocity $\mathcal{F}v$, the second one an effective phase shift η, which is the ratio of the effective velocity to the velocity of light and which characterizes the efficiency of the multiple scattering

$$\eta = \mathcal{F}\frac{v}{c} \tag{10}$$

While the single scattering parameter $\frac{v}{c}$ is necessarily very small for macroscopic motions, this is not the case for the multiple scattering parameter η thanks to the multiplication by the cavity finesse.

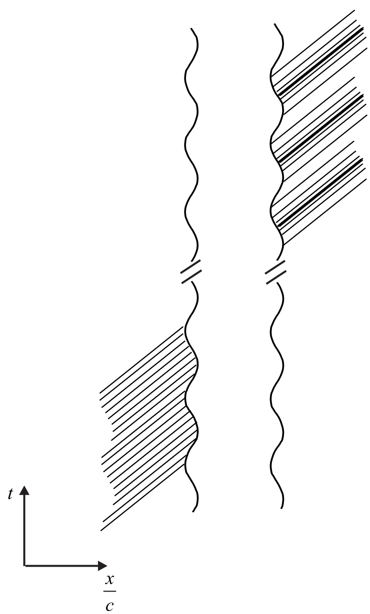

Figure 3: Space-time diagram of the multiple scattering process for a cavity oscillating as a whole in vacuum.

The multiple scattering process is represented schematically on the space-time diagram of Figure 3. On this diagram light rays are presented by lines making a 45-degree angle with the space and the time axis. The moving mirrors correspond to the sinusoidal lines. The scale of the mirrors oscillations is largely exaggerated.

The important point is that multiple scattering gives rise to periodic orbits. An incoming light ray is attracted to the neighboring stable orbit while it is repelled from the neighboring unstable orbit. When considering a fixed number of scattering processes one obtains the input-output transformation shown in Figure 3. An ensemble of equally spaced light rays entering the cavity will leave the cavity with a different temporal distribution, where the field is concentrated into short time intervals. This process leads to the formation of regularly spaced field pulses bouncing back and forth the cavity. At each scattering on one of the mirrors, there is a small probability for a photon for escaping the cavity and therefore being detected outside the cavity. This probability is given by the inverse of the cavity finesse. Based on this qualitative argument one may calculate precisely the energy density emitted into vacuum by the oscillating cavity as a function of time. The result is shown in Figure 4 where is plotted the energy density for three different values of η. This

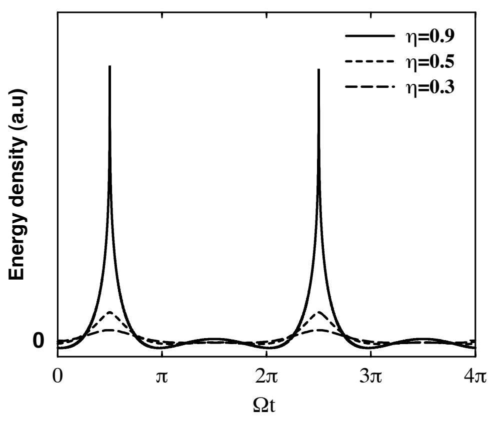

Figure 4: Energy density emitted by the cavity as a function of time for different effective phase shifts η. With increasing values of η the energy starts to concentrate in pulses emitted periodically by the cavity.

plot is based on an analytical solution of the multiple scattering problem in terms of homographic mappings of phase exponentials [11]. This approach remains valid in the case of interest $\eta \sim 1$ whereas an approach linearizing the fields in the phase shifts would be restricted to $\eta \ll 1$.

5 Frequency up-conversion

Another interesting feature is the frequency spectrum of the emitted radiation shown in Figure 5. Radiation is emitted at the resonance

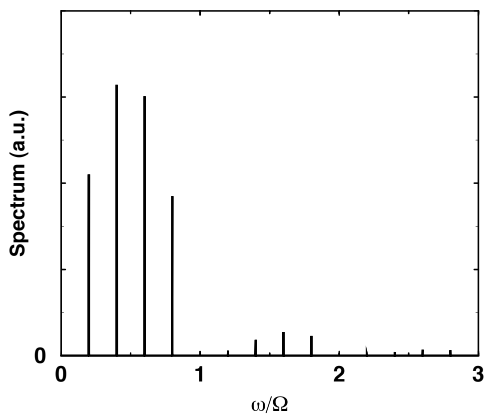

Figure 5: Spectrum of the radiation emitted by the cavity for $\eta = 0.9$. The peaks correspond to cavity resonance frequencies. The spectrum is plotted for a cavity oscillating globally at a mechanical frequency $\Omega = 5\pi c/L$. Photons are created at frequencies higher than the mechanical oscillation frequency through frequency up-conversion in the opto-mechanical coupling between vacuum fluctuations and the mirrors motion. Furthermore the radiation spectrum vanishes for frequencies equal to a multiple integer of the mechanical excitation frequency.

frequencies of the cavity corresponding to the peaks in the spectrum. The spectrum shown here is plotted for a cavity oscillating at a frequency

$$\Omega = \frac{5\pi}{\tau} \tag{11}$$

This means that the cavity performs five oscillations during one roundtrip of the field inside the cavity. Photons are emitted at multiple integers of the fundamental cavity frequency, that is at specific rational multiples of the mechanical excitation frequency Ω

$$\begin{aligned}\omega &= \frac{\pi}{\tau}, \frac{2\pi}{\tau}, \frac{3\pi}{\tau}, \frac{4\pi}{\tau}, \frac{6\pi}{\tau}, \ldots \\ &= \frac{\Omega}{5}, \frac{2\Omega}{5}, \frac{3\Omega}{5}, \frac{4\Omega}{5}, \frac{6\Omega}{5}, \ldots \end{aligned} \tag{12}$$

A striking feature is that no radiation is emitted at multiple integers of Ω. There the spectrum vanishes rigorously. This constitutes an interesting property in so far as it is very different from the expected behavior of any pick-up or spurious field which would be strong at multiples integers of the mechanical oscillation frequency. In addition, photons are emitted not only for frequencies lower but also for frequencies higher than the oscillation frequency Ω which means that a process of frequency up-conversion takes place in the system.

6 Orders of magnitude

Clearly, the specific temporal and spectral signatures of the emission may help to discriminate motion-induced radiation from potential stray effects in an experimental observation.

To be more specific about the orders of magnitude, let me recall that I have assumed the input fields to be in the vacuum state. This assumption requires the number of thermal photons per mode to be smaller than 1 in the frequency range of interest

$$\hbar\omega \ll k_{\mathrm{B}}T \tag{13}$$

Low temperature requirements thus point to experiments using small mechanical structures with optical resonance frequencies as well as mechanical oscillation frequencies in the GHz range. This corresponds to an operation temperature

$$T \sim 10\mathrm{mK} \tag{14}$$

At such a temperature, the finesse of a superconducting cavity [17] can reach $10^9 - 10^{12}$. A peak velocity ranging from $v \sim 2.5 \cdot 10^{-3}$m/s for a finesse of 10^{12} to $v \sim 0.25$m/s for a finesse of 10^9 would then be sufficient to obtain an effective phase shift η close to unity ($\eta \sim 0.9$). Under these conditions, the radiated flux of motion-induced photons created outside the cavity ranges from 0.001 photon/second for a finesse of 10^{12} to 1 photon per second for a finesse of 10^9. Inside the cavity, the stationnary photon number is in both case of the order of 1. This clearly illustrates that a very high finesse cavity needs much longer detection times as the mirrors transmission to the outside is so small. Nevertheless photon fluxes of these orders of magnitude are measurable by efficient photon-counting detection available in the GHz range. Alternatively, the field produced inside the cavity could be probed with the help of Rydberg atoms [17].

It is important to emphasize that the peak velocity considered here is only a small fraction of the typical sound velocity in materials so that fundamental breaking limits do not oppose to these numbers. This velocity corresponds to a small amplitude q_0

$$q_0 \sim \frac{v}{\Omega} \sim 10^{-11}\mathrm{m} \tag{15}$$

but to a very large acceleration a

$$a \sim \Omega v \sim 10^9 \mathrm{m/s}^2 \tag{16}$$

The observation of motional radiation in vacuum seems to be achievable by an experiment of this kind. The difficulty remains to find means for exerting a very large force to excite the motion of the cavity while keeping the optical part of the experiment at a very low temperature and unaffected by the stray fields induced by the excitation. In the following section several excitation mechanisms and experimental possibilities will be discussed in this respect.

7 Discussion of experimental possibilities

I will now discuss to which extent and under which conditions it is possible to reach the orders of magnitude presented before. I will concentrate on two excitation mechanisms for the oscillation of the cavity, the first one being through radiation pressure, the second one using the piezoelectric effect. However before studying the excitation mechanisms, let me specify the mirrors suspension system and possible mechanical parameters.

7.1 Mounting system

In order to have an efficient coupling between the mirrors' mechanical motion and the modes of the electromagnetic vacuum one has to suppose

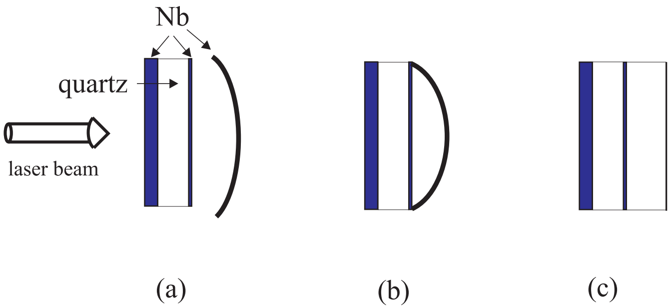

Figure 6: Niobium layer on massive support, producing either open (a) or closed (b,c) cavities.

a mirror of macrsocopic size. In the follwoing I will assume as an example a mirror of surface $A = 25\text{mm}^2$. A typical material for superconducting cavities is Niobium which has a density of $\rho = 10\text{g/cm}^3$. One needs a 1μm thick layer of Niobium in order to achieve a finesse of 10^9. The mirrors volume is then $2.5{\cdot}10^{-4}\text{cm}^3$, leading to a mass of 2.5 mg. The Niobium layer would probably to be fixed to a massive support, capable to evacuate the excitation heating (see different possibilities on Figure 6). This support should not vibrate due to radiation pressure and should thus have a very different mechanical behavior than Niobium, as for example quartz.

7.2 Excitation of the mirrors motion through radiation pressure

For simplicity I consider the excitation of one single mirror, while the other one will remain motionless. Photons belonging to translational cavity modes can nevertheless be distinguished by the their frequency $(2n+1)\pi/\tau$ for odd modes.

In order to move a mirror of mass m with an acceleration a, a force $F = ma$ is needed. The force exerted by radiation pressure of a laser beam is given by

$$F = 2h\nu N/c \tag{17}$$

where N is the photon flux per second and ν the photonic frequency in units of Hz. The laser power needed is then

$$P = Fc = 2h\nu N \tag{18}$$

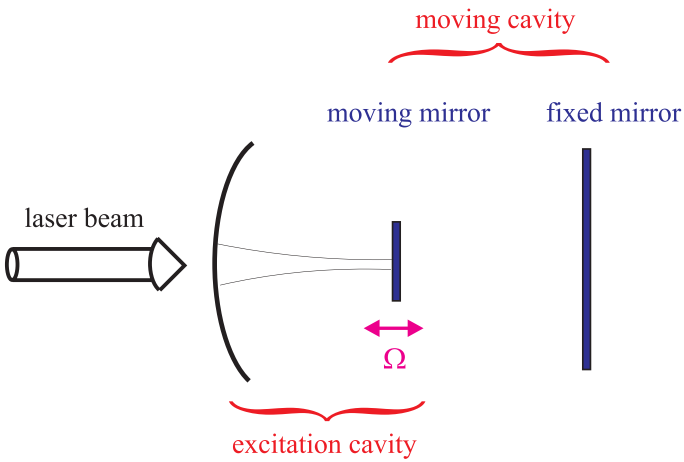

Figure 7: Schematic set-up of the excitation of the mirror's mechanical motion through radiation pressure using a second cavity.

The central idea is to use a second high-finesse cavity next to the moving one in order to enhance the radiation pressure for the mirror's excitation (see Figure 7). The excitation can be performed with microwave photons having the same frequency as the mechanical oscillation or with optical photons where the signal should be modulated at Ω. I will suppose a mirror with a mass of $m \sim$2.5mg oscillating at a frequency of $\Omega = 1$GHz. Depending on two different regimes for the cavity finesse which can be realized experimentally, that is $\mathcal{F} = 10^9$ and $\mathcal{F} = 10^{12}$, I consider two different maximum velocities such that the effective phase shift η values about 0.9 as considered before for the radiated energy density and the radiation spectrum.

For a cavity finesse of the order of $\mathcal{F} = 10^9$ a mirror's maximum speed of about v=25cm/s gives an effective phase shift of 0.9. At 1GHz this mechanical velocity corresponds to an acceleration of $a = 10^9$m/s^2 which means that a force of about $2{\cdot}10^3$N would have to be applied to the mirror in order to excite a sufficient mechanical motion. Using an excitation microwave cavity with a finesse of the order of $\mathcal{F} = 10^9$, a laser power of the excitation beam of about 300 W would be needed.

For very high finesse cavities reaching values of $\mathcal{F} = 10^{12}$, the mirrors maximum speed needs only to be about $v = 2.5 \times 10^{-4}$m/s, amounting to an acceleration of $a = 10^6$m/s^2 and to an excitation force of about $F = 2$N. Using again a preamplifying high finesse cavity this lowers the power of the excitation laser beam to 300mW, which is very easily achievable.

A possible problem with this mechanism might be that the moving mirror is also part of the excitation cavity, which is thus not continuously resonant. This should have an influence insofar as the cavity finesse is very large and the resonance peak therefore very narrow. On the other hand, the mirrors motion does not need to be continuously excited. A second problem is that it might be extremely difficult to separate the excitation field from motion-induced radiation as both lie in the same frequency range.

Both problems might be circumvented by using the radiation pressure of optical photons and to modulate the signal at Ω in order to excite the mirror's motion. These photons would not be detected by a microwave detector. However in the optical domain the preamplifying excitation cavity can only have a finesse up to about 10^6. Under these conditions, for a moving cavity of finesse $\mathcal{F} = 10^9$ a laser input power of 300kW would be needed to excite the mirror's oscillation. For a moving cavity with a very high finesse of $\mathcal{F} = 10^{12}$ and a mirror maximum speed of $v = 2.5 \times 10^{-4}$m/s, a laser input power of only about 10W would be sufficient.

One may resume this discussion of the mechanical excitation through radiation pressure in the following way. While an excitation using microwave photons is more easily achievable concerning orders of magnitude of the laser power it will produce considerable spurious effects against which the motion-induced radiation would have to be distinguished, making the specific signatures of motion-induced radiation extremely important. On the other hand, an excitation through optical radiation pressure implies higher laser powers which are however perfectly achievable, especially in the case where the finesse of the moving cavity reaches 10^{12}. A great advantage of this method would be the absence of spurious signals coming from the mechanical excitation.

7.3 Excitation of the mirrors motion with a piezoelectric crystal

Considering the mirrors mounting system as shown in Figure 6 it is also natural to think eventually of using the piezoelectric effect to excite the mechanical motion. One might benefit from a high mechanical quality factor of the support to excite the motion of the Niobium layer. Indeed mechanical quality factors of quartz for hypersonic acoustic waves ($\Omega \sim 10$MHz) have been reported to be of the order of $10^6 - 10^8$ in the low temperature domain. However to my knowledge, it remains uncertain if the quality factors hold in the GHz domain. On the other hand several papers investigate the piezoelectric constant of quartz at Gigacycle frequencies called hypersonic waves [12, 13, 14] . There the excitation has been measured up to frequencies of 10GHz.

The efficiency of the conversion of electromagnetic energy into acoustic energy for the piezoelectric effect is given by [13]

$$P_{\rm out}^{\rm ac} = P_{\rm in}^{\rm em} \frac{C^2 Q \lambda q}{2\pi V} \tag{19}$$
$$C^2 = \frac{4\pi d_{11}^2 c_{11}}{\epsilon}$$

C is the piezoelectric coupling factor which depends on the piezoelectric constant in a specific direction, here d_{11}, and on the elastic constant c_{11}. Q is the mechanical quality factor, λ the wavelength of the electromagnetic wave, q the crystal's thickness, V an appropriately chosen effective volume of the interaction and ϵ the dielectric constant of the medium. Because of the the dependence on the Q value, the sensitivity might in practice be considerably improved by using superconductive cavities at low temperatures.

There exist two possibilities to couple an electromagnetic wave to a piezoelectric crystal, either with a reentrant cavity or a direct coupling via an impedance adaptation system [15]. The reentrant cavity allows to couple to a crystal with a large surface, that is a diameter of about $\phi \sim 5$mm, but the coupling efficiency is only of the order of $\eta = -30dB$. The impedance adaptation system allows for a better coupling of about -15dB, but only to small surfaces corresponding to a diameter of the order of $\phi \sim 0.1$mm. Such a surface seems to small in order to have sufficient coupling between the mechanical motion and the electromagnetic vacuum field modes. For this reason I will not consider this possibility in the following although the coupling is more efficient.

If one excites mechanical motion via piezoelectric effect on one end of a crystal, electromagnetic energy will be emitted through the inverse effect on the crystal's other end. This might cause a difficulty for the detection of motion-induced radiation. This difficulty can be circumvented either by an efficient screening with a metallic mirror, or by replacing the crystal with aluminium onto which one poses a quarter-wave thick layer of ZnO [15].

The orders of magnitude for the amplitude of the moving surface which can be reached with the reentrant cavity are exposed in the following. Suppose an electromagnetic source of $P_{\rm in} = 1$W. The acoustic power is then given by

$$P_{\rm ac} = \frac{P_{\rm in}}{10^{-\eta/10}} \tag{20}$$

This power is converted into mechanical motion of the surface with an amplitude q_0 at a frequency Ω

$$P_{\rm ac} = \frac{1}{2} Z \Omega^2 q_0^2 A \tag{21}$$

Z is the acoustic impedance and A the moving surface. As before I suppose a mirror of surface $A = 25$ mm^2. The coupling efficiency is of about -30dB. The acoustic input power is then

$$P_{\rm ac} = \frac{1000\text{mW}}{10^{-30/10}} = 1\text{mW} \tag{22}$$

For an oscillation frequency of 1GHz and with an acoustic impedance for Aluminium of $Z = 4 \times 10^7$kg/m^2/s one finds

$$q_0 = \sqrt{\frac{2P_{\rm ac}}{Z\Omega^2 A}} \sim 10^{-12}\text{m} \tag{23}$$

In order to reach an effective phase shift of the order of $\eta \sim 0.9$, where the effects of pulse shaping and frequency up-conversion appear, a minimum amplitude q_0 of the mechanical motion is needed. As shown before, for a cavity finesse $\mathcal{F} = 10^9$ this amplitude corresponds to

$$q_0 = 10^{-11}\text{m} \tag{24}$$

while for a cavity finesse of $\mathcal{F} = 10^{12}$ the required amplitude is

$$q_0 = 10^{-14}\text{m} \tag{25}$$

With the conservative cavity finesse, an excitation using the piezoelectric effect is therefore within reach using the reentrant cavity system.

For a moving cavity with an even better finesse of 10^{12}, the oscillation amplitude which may be reached using the piezoelectric effect is even three orders of magnitude larger than what is needed. The piezoelectric effect therefore seems a possible alternative maybe better suited and more easily implemented experimentally than excitation through radiation pressure.

8 Consistency check : condition for superconduction

An experiment designed to observe the dissipative effects of quantum vacuum will necessarily employ a superconducting cavity in order to achieve a cavity finesse of about $\mathcal{F} \sim 10^9 - 10^{12}$. This fact gives a physical limiting condition for the excitation of the mirrors mechanical motion. No matter how motion will be excited, the maximum field strength at the surface of the superconductor is limited to [16]

$$E_{\max} = 25\text{MV/m} \tag{26}$$

For higher field strengths Cooper pairs are broken up and the superconductor becomes normally conducting. As consistency check we may therefore calculate the maximum velocity which can be excited by such a field.

The power corresponding to the maximum field strength is

$$P_{\max} = \varepsilon_0 c E_{\max}^2 A \tag{27}$$

where A is the mirrors surface in units of m. If a field of such a power acts on a surface it can produce a change of impulsion Δp on the surface during a time interval Δt. The change of impulsion per unit time corresponds to the maximum force $F_{\max}$ tolerated

$$\Delta p = 2\frac{P_{\max}\Delta t}{c} = F_{\max}\Delta t \tag{28}$$

We can then deduce the expression of the maximum force and thus, by using explicitly an oscillating motion, the maximum velocity of the mirror

$$\begin{aligned} F_{\max} &= \varepsilon_0 E_{\max}^2 A \\ &= ma = m\Omega^2 q_0 \end{aligned} \tag{29}$$

leading to a maximum velocity of

$$v_{\max} = \Omega q_0 = \frac{\varepsilon_0 E_{\max}^2 A}{m\Omega} \tag{30}$$

With $\varepsilon_0 = 8.85 \times 10^{-12}$As/Vm and supposing as before a mirror of mass $m \sim 2.5$mg and an oscillation frequency of 1GHz, the numerical value for the maximum velocity is found to be

$$v_{\max} \sim 2 \times 10^{-4}\text{m/s} \tag{31}$$

which is in accordance with the maximum velocity for a cavity finesse of 10^{12}. There is therefore no contradiction between using a superconducting cavity and the sufficiently strong excitation of the mirror's mechanical motion in order to observe motion-induced radiation with its particular signatures.

9 Conclusion

To resume the possibilities of observing expermentally motion-induced radiation emitted by an oscillating high finesse cavity, the emitted photons may be detected outside the cavity by performing sensitive photon-counting detection of the radiated flux. Inside the cavity the state of the field could be probed with the help of Rydberg atoms [17]. However, to excite the motion, a huge force would have to be applied onto the mirrors which might create spurious signals against which one would have to distinguish experimentally the motion-induced photons. This is why the particular signatures of the effect are extremely important.

The challenge of this experiment does indeed not come from one particular constraint, but from the fact that all of the above discussed conditions will have to be fulfilled simultaneously. The present paper shows that there is no fundamental objection to the realisation of such an experiment, but that in the contrary it is possible, although difficult, to meet the ensemble of necessary conditions in an experiment. However, as dissipative effects of vacuum fluctuations are a fundamental phenomenon related to important conceptual questions in physics, their observation would be worth the effort.

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