Localizability and quantum behaviour

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ABSTRACT. We go over various view-points about wave-particle duality and criticize Schrödinger-type solution of this problem. In this framework we show that Barut's quantum theory of single events has some problems and that the non-linearization of the Schrödinger equation is of no help.
PACS number: 03.65.+, 03.65.Ca, 03.65.Bz

1 Introduction

While some experiments indicate that an electron can simultaneously be present at different points of space, other experiments show that it is a localized object. The same holds true for protons, neutrons, et al. Thus, it seems that every microscopic object has a dual character: an extended wave-like behaviour and a localized point-like behaviour. This type of duality is called wave-particle duality. Wave character and particle character are distinguished in classical physics and they can be easily described. But, here the wave-particle duality indicates that a single entity (e.g. electron) is showing both types of behaviour, and this is what makes it incomprehensible. Some physicist are simply content with the successful results of quantum theory and do not have any appetite for these kinds of speculations. But, as de Broglie pointed out, a problem which has been and still is one of the mysteries of science is the wave-particle duality [1].

In this paper we first go over various view-points about this matter, and then criticize one of the view-points which has been and still is one of the popular ones. Our paper is organized as follows: in section II we mention various views about the wave-particle duality. In section III we show the Barut's theory of single events, which was aimed at resolving the problems of Schrödinger view, has some difficulties. In section IV we show that the non-linearization of the Schrödinger equation is of no help for giving a proper wave packet which could represent a particle.

2 Various view-points concerning wave-particle duality

So far there have been five view-points concerning the dual character of micro physical entities.

Some physicists believe that primacy belongs to particles, and that waves are only mathematical tools that determine the probability of finding particles at a definite point [2]. To justify this claim, they refer to the role of Lagrangian and Hamiltonian in classical mechanics. There, these functions are mathematical tools for the determination of the evolution of classical systems. Here, too, ψ is a mathematical tool for the determination of the evolution of classical systems. The main difficulty with this view is the question of explaining the interference with the help of a merely mathematical entity.

Bohr appealed to the principle of complementarity to handle the problem of duality. According to this principle, a quantum system shows particle behaviour or wave behaviour depending upon the experimental arrangement. In any specific experiment, one sees either the particle character or the wave character of the micro physical entity, but it never shows both behaviours in the same experiment. For example, light shows wave character in the two-slit experiment and particle character in the photo-electric effect. This seems to avoid the confrontation of two incompatible pictures. But, in fact, it ignores the problem rather than solving it.

Some physicists consider the micro physical entity to be neither a wave nor a particle. Rather, it is something that unifies the two concepts. For example, if we apply the formalism of quantum mechanics to classical fields that have wave character, a discreteness appears in their structure that leads to a particle interpretation. It appears that quantum fields provide a framework for understanding the wave-particle duality in micro-physics. Thus, some physicists give primacy to quantum fields [3]. In our view the wave-particle duality is not eliminated completely in quantum field theory. There the particle number operator and the field amplitude do not commute and so we cannot specify the particle number and the field amplitude at the same time.

Some physicists believe that reality has both a wave aspect and a particle aspect. In fact, with every particle a wave is associated which guides it. The pilot wave theory of de Broglie-Bohm belongs to this category [4]. Although this theory resolves some problems inherent in the standard interpretation (e.g. the so-called measurement problem), but it has other problems (e.g. there is no reciprocal action of the particle on the wave). Of course some of these problems seem to be solvable [5].

Some physicists give primacy to the wave aspect. The reality is that of waves, but they have particle manifestations. The particle is identified with that part of the wave where the energy is concentrated. This was the view that Schrödinger defended. He represented micro physical entities by wave packets. But, wave packets spread with time. Thus, this view seems untenable. De Broglie attempted to represent a particle by a singularity in the wave structure. This led to his so-called double-solution theory.

Here we try to criticize Schrödinger's view from a different perspective.

3 Quantum theory of single events

When Schrödinger introduced his idea of wave packets representing particles, he encountered the objection that his wave packets spread with time and that this is inconsistent with the idea of a localized particle. Nevertheless, this idea did not lose its appeal completely. Recently, Barut claimed that one can get solutions to the Schrödinger equation that involve locations with high energy density [6,7]. Furthermore, these locations do not change their shape with time. Usually, we expect this behaviour from the solutions of non-linear equations. In these equations, non-linear terms can cancel dispersive terms completely. Therefore, one can find solutions of these equations that save their shape with time. But, Barut obtained such solutions from the standard Schrödinger equation. He started with the equation:

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\nabla^2\psi-\frac{mc^2}{\hbar}\psi \tag{1}$$

which can be obtained from $\Box\ \varphi=0$, in the non-relativistic limit, by writing $\varphi=e^{\frac{-imc^2}{\hbar}}\psi$ and noticing that in this limit (where $v/c<<1$) $\frac{\partial^2\psi}{\partial t^2}<<\frac{mc^2}{\hbar}\frac{\partial\psi}{\partial t}$, Eq. (1) has a solution of the form:

$$\psi=F(\vec{x}-\vec{x}_0-\vec{v}t)\exp\left[\frac{i}{\hbar}\left(m\vec{v}.\vec{x}-\tfrac{1}{2}mv^2t\right)\right] \tag{2}$$

where F satisfies the following equation:

$$\nabla^2 F + \frac{m^2c^2}{\hbar^2} F = 0 \tag{3}$$

This equation has localized solutions having the extension $\frac{\hbar}{mc}$ [6]. In the presence of the potential $V(\vec{x})$, we consider the following solutions:

$$\psi(\vec{x},t;\vec{g}) = F(\vec{x} - \vec{g}t)\Psi(\vec{x},t) \tag{4}$$

where $\vec{g}(t)$ is a classical path involving hidden variables. According to Barut, both ψ and Ψ satisfy the Schrödinger equation and F satisfies (3). Thus, we must have [7]:

$$i\,\dot{\vec{g}}(t).\vec{\nabla}F = \frac{\hbar}{m}\vec{\nabla}F.\frac{\vec{\nabla}\Psi}{\Psi} \tag{5}$$

To solve this equation, Barut wrote $\Psi = G(\vec{x})e^{\frac{i}{\hbar}H(\vec{x},t)}$, where $G(\vec{x})$ satisfies the time-independent Schrödinger equation and $H(\vec{x},t)$ is an auxiliary phase [7]. But this hides the fact that (5) in general does not have a solution. To show this, we write $\Psi = \mathrm{R}e^{\frac{i}{\hbar}S}$. Then, from (5) we get:

$$\left[i\left(\dot{\vec{g}}(t) - \frac{\vec{\nabla}S}{m}\right) - \frac{\hbar}{m}\frac{\vec{\nabla}R}{R}\right].\vec{\nabla}F = 0 \tag{6}$$

Since R, S and $\vec{g}$ are all real function, we must have:

$$\dot{\vec{g}}(t) - \frac{\vec{\nabla}S}{m} = 0 \tag{7}$$

and

$$\frac{\vec{\nabla}R}{R}.\vec{\nabla}F = 0 \tag{8}$$

The first equation indicates that Barut's pseudo-particles have Bohmian paths. This is the desired result. Because it turns out that the assumption of such paths for microscopic particles is not inconsistent with any experiment, and it gives an acceptable picture of microscopic events (of course, no experiment verifies these paths). The second equation, however, puts much restriction on the form of *R* and therefore on the probability density $\rho = R^2$. This kind of restriction is not in accord with the present results. Because, it seems that with some preparation one can make the wave function of a system to have any desired form with arbitrary probability density.

4 Non-linear Schrödinger equation

One way to avoid the spreading of the solutions of the Schrödinger equation is to make this equation non-linear. But, a non-linear equation does not respect the superposition principle. Therefore, it does not lead to effects like interference. One can take care of this difficulty by non-linearizing the Schrödinger equation in such a way that non-linear effects would be important at the particle position and small at other locations. In this view most of the energy of the field is concentrated in the wave packet (i.e. at the particle position). On the other hand energy is the source of gravitational effects. Thus, one can hope that the introduction of gravitational effects leads to the desired solutions for the generalized Schrödinger equation. Recently, Rosen has sought such solutions for a Klein-Gorden field [8]. He has shown that if the wave packet is localized to within the Compton wavelength of the particle, then the mass of the particle should be around Planck's mass. Indeed, gravitational effects are so weak that they cannot produce such localization for smaller masses.

There is the possibility that by adding non-linear terms to the Schrödinger equation, one can non-linearize this equation in such a fashion that in spite of keeping quantum effects, energy is concentrated to the desired degree, to make the particle interpretation tenable. Here, we argue that the increase in the concentration of energy weakens the quantum effects. In order to understand more deeply the differences between the quantum and the classical behaviour, we look at Bohm's version of quantum mechanics. In this theory a non-classical potential, the so-called quantum potential, can be used as a criterion for the quantum behaviour of the system:

$$Q = -\frac{\hbar^2}{2m}\frac{\nabla^2 |\psi|}{|\psi|} = 0 \tag{9}$$

One can easily show that different criteria for the classical limit, like large quantum numbers, short de Broglie wavelengths, large masses, and vanishing of Planck's constant are all equivalent to $Q \to 0$. Thus, we use $Q = 0$ as a criterion for the absence of quantum behaviour.

To illustrate that the increase in the concentration of energy weakens the quantum behaviour, we consider a Klein-Gorden field, with an additional term in its Lagrangian:

$$L = \frac{1}{2}\partial_\mu \phi^* \partial^\mu \phi - \frac{m^2 c^2}{\hbar^2}\phi^* \phi - \frac{1}{2}\partial_\mu |\phi| \partial^\mu |\phi| \tag{10}$$

If, we write ϕ in the form of $\mathrm{R}e^{\frac{i}{\hbar}S}$, we get:

$$L = R^2\left[\partial_\mu S \partial^\mu S - m^2 c^2\right] \tag{11}$$

In such Lagrangian, R can have any form without spreading. In fact, one can choose a Dirac delta function for R. In this case, the particle (or the singularity) has a classical behaviour. Because for the field S we have:

$$\partial_\mu S \partial^\mu S = m^2 c^2 \tag{12}$$

which is the classical Hamilton-Jacobi equation for a particle without quantum potential. In the non-relativistic quantum mechanics we can choose a Lagrangian as follows:

$$L = R^2\left[\frac{\partial S}{\partial t} + \frac{\left(\vec{\nabla} S\right)^2}{2m} + V(\vec{x})\right] \tag{13}$$

which leads to a non-linear equation for $\mathrm{R}e^{\frac{i}{\hbar}S}$ [9]:

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2 \psi + V(\vec{x})\psi + \frac{\hbar^2}{2m}\frac{\nabla^2 |\psi|}{|\psi|}\psi \tag{14}$$

In fact, at one limit we have the linear Klein-Gorden or Schrödinger equation with all their quantum behaviour and with no proper concentration of energy, and at the other limit we have the classical Hamilton-Jacobi equation with the desired concentration of energy and with no quantum effects. The introduction of other non-linear terms leads to a result between these limiting cases, i.e., while we have some quantum effects, we also have wave packets with limited extension and proper characteristic.

5 Conclusion

It seems that there is a complementarity relation between quantum behaviour and localizability. When we try to obtain a proper concentration of energy in the field, we miss quantum behaviour. Non-linearization of the Schrödinger equation by the introduction of gravitational effects leads to a proper localization of energy in the limit of Planck's mass. But, we don't expect to have quantum behaviour in this limit. On the other hand, the non-linearization of the Schrödinger equation, by the introduction of a non-linear term, leads to the maximum concentration of energy in absence of quantum behaviour. Thus, it seems that the view-point of Schrödinger, in spite of its attractions, can not resolve the dilemma of wave-particle duality.

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Last revision Saturday 25 January 2003.