The symmetry of the motion, the mass and the quantum state

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> It is the dissymmetry, which creates
> the phenomenon Pierre Curie [1]

ABSTRACT. The study of the quantum state is revisited to emphasize the possibility to interpret the wave function in Dirac's model as the mechanical action leading the electron along its trajectory. Indeed it is still very surprising that Sommerfeld and Dirac models lead to the same expression for the energy of the levels of the various quantum states, whereas the interpretation of the regular doublets escapes to the corpuscular traditional approach of Sommerfeld. In this respect it is first underlined that the symmetry of the motion of rotation reveals a corresponding intrinsic axis on the electron. Furthermore the relativity of motion requires that the same causes must be responsible for it in the space of the electron as well in that of the proton. This leads to suppose that the moments are due to exchange of matter between the electron and the proton. This approach of the motion implies exchanges of matter in a volume and mechanical action acting into three orthogonal directions. Furthermore the exchanges of matter lead to consider the mass as variable, with absorption and ejection, giving the clue to explain the doublets and the moments with half-integer quantum numbers of the angular momentum.

1   Introduction

The experimental study of the spectral lines emitted by an atom reveals that they are classified in series. Some lines of these series are double, called regular doublets. The traditional example is that of the D line of the sodium with the respective wavelengths $\lambda_1 = 5890$Å and $\lambda_2 = 5896$Å. The set of lines thus observed for various atoms forms the experimental base of the quantum state. To interpret the spectral lines Sommerfeld was brought to quantize, in the study of the motion of the electron around the proton, the angular and radial action [2]. Doing so leads to a great number of remarkable results but does not provide any answer as to the origin of the regular doublets and the existence of the half integer numbers [3]. Up to now only the introduction of the wave functions and the theoretical model of Dirac have allowed finding out the set of the quantum states and the energy levels associated with the regular doublets.

However, the study of the quantum states corresponding to the doublets shows that they correspond to a small difference of mass, but the connection with the intrinsic rotation is still to put in view. On the other hand, there remains a very surprising fact: these two theories lead to the same expression of the energy of the levels of the various quantum states whereas the interpretation of the regular doublets escapes the corpuscular traditional approach of Sommerfeld.

Now at its advantage one has to keep in mind that Sommerfeld'model with the concept of trajectory has a remarkable explanatory force that Dirac's model has not. For example it makes it possible to understand the attraction between atoms of which the most external electrons are in a state "s" [4]. The trajectory indeed gives an electric dipolar character to the atoms and thus allows for the understanding, for example, of attraction between alkaline atoms. On the other hand the trajectory of electron has allowed proposing an interpretation of the mechanism of conductivity and superconductivity in the superconducting oxides [5]. Furthermore the assumption of trajectory is suggested by the magnetic properties of the matter: indeed magnetism is before all a consequence of the motion of electric charges. These various remarks suggest that the equation of Dirac gives access to particular aspects of the trajectory. The purpose of this work is to emphasis these aspects of the quantum state. We have already tackled this purpose [6] and introduced the symmetry of the motion in a recent work [7].

In Sommerfeld's model the electron considered as a point cannot exhibit an intrinsic rotation axis. However we know that it gravitates around the proton in an orbital planar motion and that this rotation is well described by a central potential. Yet this symmetry of the potential does not correspond to

that of the plane motion of rotation. Take up then the analysis by Pierre Curie between the causes and the effects of the symmetry elements [1]. His words were as follows:

When certain causes produce certain effects, the elements of symmetry of the causes must be found in the effects produced.

When certain effects reveal certain dissymmetry, this dissymmetry must be found in the causes, which have produced them.

In this spirit the orbital motion of the electron is inseparable from its intrinsic motion of rotation. Thus to describe the motion of the electron, we must in addition to the potential, introduce the symmetry properties of the orbital motion into the volume of the electron. Thus we suppose that the orbital rotation is the result of the intrinsic rotation[1] to which corresponds one alone quantum of action. It is important to remark that the expression intrinsic rotation is still imperfect. Indeed intrinsic suggests a property which belongs to the object itself, but in fact all that we know is defined in comparison to another object or property. In particular the intrinsic rotation must be defined in comparison to the orbital rotation.

On the other hand in the study of the phenomena, the causes of the physical laws must be independent of the place of observation. Let us consider the volume of the electron; the question is: what variables are able to determine the quantization of the electron-proton system? When the radial speed of the electron varies, in the volume of the electron, only the mass variations can be associated with it. As a result it is necessary to assume that in fact the variations of the electron mass are at the origin of the variations of its radial speed. Therefore the mass of the electron is the variable producing the different moments. This approach of the interaction leads to consider the proton and the electron as fluid matter. It also leads to interpret the wave function as a wave of matter that is the amount of matter determining the mechanical action guiding the electron along its trajectory upon an element of length and time [8, 9].

This approach to motion consists in giving to the mechanical action a fundamental role. This role is natural since any momentum and any quantity of kinetic energy are always related to a space displacement and an interval of time. It was in fact the idea of Sommerfeld [2] to extend the assumption of Bohr concerning the angular momentum [10] to elliptic orbits. For Louis de

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[1] We will use the expression intrinsic rotation preferably to spin or eigenrotation to avoid any possible confusion and we will specify the meaning of this expression along this study.

Broglie the action also plays a fundamental role: it is "guided by the idea of a major identity between the principle of less action and that of Fermat" that he was led to propose the hypothesis of a wavelength associated with the momentum of the electron by the quantum of action "h" [11]. It was also the idea of Schrödinger who built the differential equation, whose solution is the wave function, by introducing a propagation velocity of surfaces of constant action [12]. We propose in this work to show how this concept of the action by exchange of matter between the proton and the electron makes it possible to understand the existence of half-integer angular momentum and to find the set of the energy levels in the approach of Sommerfeld.

2   The action and the exchange of matter

In classical mechanics the action is the product of the momentum by the element of length dl, or of the energy by the element of time dt. In the theory of Dirac the operators act on the wave function by first order derivation with respect to the variables of space and time. If these operators act on a function representative of the action, they give access to the various components of the momentum and energy.

Then let us take a different point of view from the classical interpretation, and suppose that the representative function of the action is precisely the wave function. Moreover we suppose that the action generated by the wave function takes place by exchanges of absorbed or rejected matter between the proton and the electron. The quantization of the wave function is then that of the action associated with the different degrees of freedom over one period. This assumption leads to suppose that the electron charge and the potential are made of elements extremely small as compared with electron dimensions, having a mass and that we call grains. Thus we suppose that exchanges of matter results from exchanges of grains between the electron and the proton.

To describe the motion of the electron around the proton we consider an atomic reference frame Ra, made of a system of orthogonal axes, the centre of gravity P of the proton being at its origin (figure 1). This centre P is also the centre of the potential, acting on the electron. The intensity of the potential at a point A is inversely proportional to the distance PA that separates it from P. As a result the density of matter allowing describing the potential is itself inversely proportional to this distance. Let then be γ the centre of gravity of the electron. As for the potential we suppose that in the volume of the electron, the density of matter allowing describing the electron charge is a function inversely proportional to the distance to the centre of gravity γ of the electron. The surface, which delimits in the volume of the proton from that of the electron, is consequently that which corresponds to the minimum of den-

sity. It is through this surface that the exchange of matter determines the action and the trajectory. During these exchanges the grains are suppose to move with the speed of the light in vacuum.

2.1   Plane motion and exchanges in volume

In Sommerfeld's model the motion is plane, there are only two independent degrees of freedom. On the other hand the exchanges of matter that determines the momentum and generates the orbital rotation are distributed in a volume. Therefore the action associated with rotation cannot be correctly described by the product of two vectors, the momentum and dl displacement, both being contained in the plane of the trajectory. These two vectors must necessarily have three components not in the same plane. This approach allows describing the motion with a distribution of the mass along the different directions changing step by step.

2.2   The mass

With a fluid model of electron, one has to consider the mass determining the mechanical action as a function of the occupied quantum state. This hypothesis is suggested by the emission or the absorption of photon during the electronic transitions. Furthermore it seems that several experiments confirm this approach [13, 14] corresponding to a current thinking of different authors [15-20].

Considering the equivalence between mass and energy, it is possible to describe this aspect of the mass by supposing that the constituting grains are in a ceaseless motion, continuously exchanged with those of the proton their speed being that of the light. The proton being supposed at rest one can consider two parts: one disordered the inert mass, the other the active mass. This last corresponds to the kinetic energy of the state and determines the motion. The grains of these two masses are exchanged along the trajectory with those of the proton. The state will be stable if the exchange takes place during the period of the motion, therefore the importance of the mechanical action. The mass m_0 call the rest mass, is the mass that the electron would have before the loss as photon of an amount of energy equal to that of its state. It also represents the amount of inert mass of the proton, considered at rest, of the occupied volume by the electron. As a result there is a simple relation between the mass at rest m_0, the inert mass m_i and the active mass m_a:

$$m_i + m_a = m_0 \qquad (1)$$

Let W be the energy of the motion we have:

$$m_i = W\,c^{-2} \quad \text{with} \quad E = W - m_0\,c^{-2} \quad \text{thus} \quad m_a = -E\,c^{-2} \tag{2}$$

The active mass is the kinetic energy. In this approach the mass is a stock of disordered energy, and becomes energy insofar a part of the grains leads to a motion.

2.3 The intrinsic rotation

Consider the reference frame Px, Py, Pz, where P is the centre of the potential. To introduce the symmetry of the intrinsic rotation we suppose the corresponding axis parallel to Pz. Let G be the plane of gravitation of the electron containing the axis Px. The action associated with the motion has two of its components parallel to the equatorial plane E containing the axes Px and Py and one parallel to the axis Pz that is normal to E (figure 1). The G plane of gravitation cuts the equatorial plane along the axis Px. B is one of the two points common to the trajectory and E plane.

The space we are studying the properties is defined with those of the proton and the electron. In the extent where the electron can be considered as a point, the space of the motion is that of the proton. The proton being much heavier than the electron, it is the motion of the electron in the proton at rest that one has to describe. The motion being generated by the intrinsic rotation, there is a cause to choose the z axis parallel to the axis of intrinsic rotation. However in the space of the proton, one has to consider the angle between the G plane of gravitation and the E equatorial plane perpendicular to the axis of intrinsic rotation. For the perpendicular coordinates to this axis they must reflect the rotation speed, as a result there is a connection between those two variables and thus just one independent variable. This variable is the angle φ of the polar coordinates, the speed of which being that of the intrinsic rotation. Finally the third space coordinate is the radial distance to the centre of the potential of the proton which defined its intensity, that is the density of grains per unite of volume.

The exchanges of matter take place in all the directions. Therefore the symmetry properties of the motion lead to distinguish those producing a moment in a direction parallel to the intrinsic rotation axis from those producing a moment in a direction perpendicular to this axis. Moreover one has to distinguish the exchanges producing by absorption or reject one or the other of these moments.

Indeed there are through the surface of separation between the proton and the electron, two fluxes of grains inverse one of the other, and each one con-

tributes to the orbital motion whereas keeping in the electron the total energy equal to m_0c^2. In other words into the proton but outside from this surface nothing allows to suspect the motion of the electron.
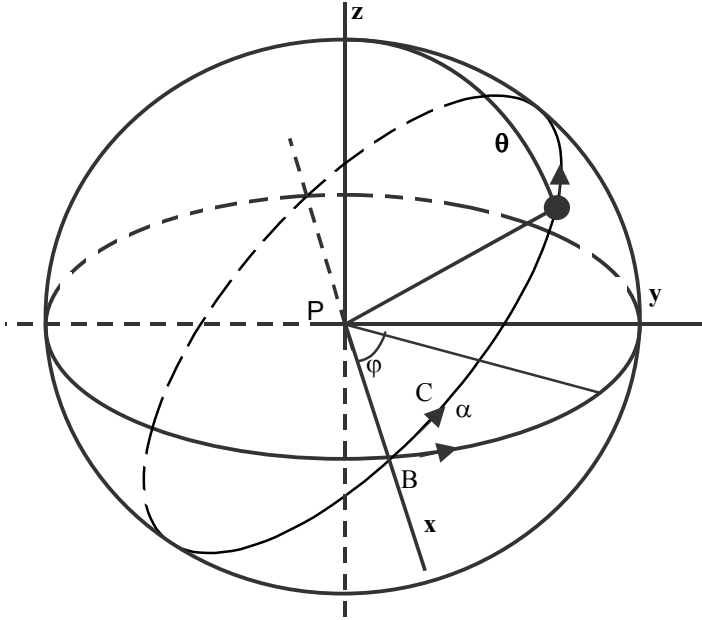

Figure 1. The motion of the electron

To describe the two fluxes we suppose that the inert mass of the electron absorbs the grains of the incoming into flux characteristic of the orbital rotation in the proton when others are rejected making the outgoing flux characteristic of the intrinsic rotation of the electron. This mechanism allows sharing the motion into two contributions: one for the incoming flux the other for the getting out flux. The volume of the electron being small in comparison to the proton, if there is rotation, considering the electron like a point, the grains which determine this rotation can be divided into two parts defining the two fluxes. The same for the parallel motion to the axis of intrinsic rotation, the two fluxes contribute to the motion. To keep the symmetry of the intrinsic rotation we suppose that upon one period, there is equipartition of the energy and therefore of the mass between the variables defining it. *This implies, for*

the intrinsic rotation, that there are as many exchanges contributing to the motion of rotation than that contributing to the translation motion.

We know that the space and the time are linked through the mass and the energy. This comes from the fact that at the differential scale, each interval of time is links to the space through the energy and the momentum. As a result the corresponding quantum can just appears with algebra having four dimensions making sure to introduce the properties linking the space and the time. This is done with the algebra introduced by Dirac, specially imposing the linearization between the differential operators of space and time.

2.4 The components of the wave function

The wave function is supposed to be mechanical action, by exchanges of absorbed or rejected matter on a short interval of time and space, whose derivatives give the different moments associated to the different amounts of energy. In Dirac's model the wave function has four components linearly independent. The analysis of the properties in connection with the exchanges of matter allows shedding light upon this aspect of the quantum state.

The need of independent components to describe the motion is the result of the grain which does not act all in the same way. They can produce a rotation in one or the other direction and the same for the motion parallel to the axis of intrinsic rotation. Thus there is a cause for each flux, on a short interval of space and time, to separate with different components the amount of matter giving rotations and translations of opposite directions. This to avoid canceling the contributions of opposite directions. As a result there are, for each flux, for the inert and active masses, two components of opposite direction that means four relations. This analysis implies for each relation an amount of matter that is of energy which balances the three components of the moments.

The problem is thus to find, for each flux, a system of four simultaneous differential equations where the four unknown are the four components of the wave function playing the role of generating function of the moments and energy. These equations are those of the equation of Dirac. To find these equations the method is to use a relation of the special relativity and to search the differential operators acting on the wave function that is the generating function of the different components of the motion.

3   The equation of Dirac

To find the equation of the quantum mechanics in special relativity the way is to start from the expressions of the energy and moment. The rest mass

of the electron being m_0, its speed being v and c that of the light in vacuum, write $\beta = v/c$, consider the electron of charge e in a scalar potential V and vector potential $\vec{A}$. In special relativity the corresponding energy W and the potential V obey to the following equation:

$$\frac{1}{c}W = \frac{m_0 c}{\sqrt{1-\beta^2}} + eV \tag{3}$$

its momentum $\vec{p}$ and the vector potential $\vec{A}$ obey to the following relation:

$$\vec{p} = \frac{m_0 \vec{v}}{\sqrt{1-\beta^2}} + \frac{e\vec{A}}{c} \tag{4}$$

The components of the moment $\vec{p}$ and the quantity $\frac{1}{c}W$ form the components of a space time vector. Considering the quantity:

$$\frac{1}{c}(W - eV) = \frac{m_0 c}{\sqrt{1-\beta^2}} = M \tag{5}$$

and the vector of the space genre:

$$\vec{p} - \frac{e\vec{A}}{c} = \frac{m_0 \vec{v}}{\sqrt{1-\beta^2}} = \vec{V} \tag{6}$$

The calculation of $M^2 - \vec{V}^2$ leads to the expression [3]:

$$\frac{1}{c^2}(W - eV)^2 - \left(\vec{p} - \frac{e\vec{A}}{c}\right)^2 - m_0^2 c^2 = 0 \tag{7}$$

For the hydrogen atom the expression (7), without vector potential, becomes:

$$\frac{1}{c^2}(W - eV)^2 - (p_x^2 + p_y^2 + p_z^2 + m_0^2 c^2) = 0 \tag{8}$$

Dirac's theory uses the equations (7) and (8) to determine the operators leading to the wave function having the good energy levels [21]. Then if h is Planck's constant the research of the wave function leads to introduce the following operators:

$$W = i\mathrm{h}\frac{1}{c}\frac{\partial}{\partial t} \ ; \ \ P_X = -i\mathrm{h}\frac{\partial}{\partial x}; \ \ P_y = -i\mathrm{h}\frac{\partial}{\partial y}; \ \ P_Z = -i\mathrm{h}\frac{\partial}{\partial z} \tag{9}$$

Introducing these operators in the equation (8) one obtains the operator:

$$F = \frac{1}{c^2}(W - eV)^2 - (P_x^2 + P_y^2 + P_z^2 + m_0^2 c^2) \tag{10}$$

To determine the equation to which the wave function must satisfy Dirac supposes that it must be linear in W leading to suppose the linearity in P_x, P_y and P_z according to the operator (10). This supposes that it can be write:

$$F = P \times Q \tag{11}$$

with $$P = [\ -\frac{1}{c}(W - eV) + \alpha_1 P_X + \alpha_2 P_y + \alpha_3 P_Z + \alpha_4 \, m_0 c \ ] \tag{12}$$

and $$Q = [\ \frac{1}{c}(W - eV) + \alpha_1 P_X + \alpha_2 P_y + \alpha_3 P_Z + \alpha_4 \, m_0 c \ ] \tag{13}$$

In these expressions the operators α_k, with $k = 1, 2, 3$ or 4, are matrices which to find out the form (8) obey to the following relations:

$$\alpha_\mu^2 = 1 \text{ et } \alpha_\mu \alpha_\nu + \alpha_\nu \alpha_\mu = 0 \text{ with } \mu \neq \nu \text{ and } \mu, \nu = 1, 2, 3, 4 \tag{14}$$

The equation :

$$Q\Psi = 0 \tag{15}$$

is Dirac's equation, it corresponds to positive energies.

The equation:

$$\mathbf{P}\Psi = 0 \tag{16}$$

corresponds to negative energies. As a result it seems to have no physical meaning. The hypothesis of exchanges of matter divided into two fluxes of opposite directions brings a simple answer. Indeed each flux is characterized with a direction of the speed of propagation of the grains, they determine the masses active and inert, thus the energy must be considered as positive or negative according to the flux. The classical equation corresponds (15) to the positive flux and the equation (16) to the negative flux. In these two equations the direction of the light speed must be considered as positive for the equation (15) and negative for the equation (16).

To the components of $\vec{V}$ we associate the quantity M that is $G = (\vec{V}, M)$. The four components of G form a space-time vector. The discussion on the wave function has led us to distinguish for each flux, four components of the energy each one associated to a moment having three components. Then consider:

$$M^e_i ; P^e_{xi} ; P^e_{yi} ; P^e_{zi} \quad \text{and} \quad M^s_i ; P^s_{xi} ; P^s_{yi} ; P^s_{zi}$$

$$\text{with} \quad i = 1, 2, 3, \text{ or } 4 \tag{17}$$

the different components for each flux.

We can consider the algebra of Dirac for the equation (15) as the search of the generating function of the different components of G for the incoming flux. To the components of $\vec{V}$ we now associate the quantity –M, that is $H = (\vec{V}, -M)$. The four components of H form a space-time vector and the scalar product H.G, leads equally to the expression (7). We can interpret the (16) equation as that of the outgoing flux homologous of the (15) for the incoming flux. The two G and H vector are thus in a particular correspondence in the way that they allow to consider the equation (7) as their scalar product. Thus the algebra of Dirac plays for the equation (16) the same role as for the equation (15) that is the search of the different components of H. Thus the equation of Dirac appears as the resolution of the vector moment and of the energy of the plane motion in several components in such a way to respect the symmetry of the orbital motion.

Let Ψ_e and Ψ_s be the respective wave functions of (15) and (16) for the incoming and out fluxes. The equation of Dirac is a group of four simultaneous

equations. It is the same for the (16) equation. Identify in each (15) and (16) equation, the different space and time components with the different results of the application of the operator (9) on the components Ψ_i of the wave function for the incoming and out fluxes. It is possible to express each vector G and H in sixteen components; they obey to the algebra of Dirac. Then if *G* and *H* are the representative matrixes of G and H, the scalar product G.H is equal to the product of the *H.G* that is:

$$\text{H.G} = \textbf{\textit{H.G}} \tag{18}$$

making sure that $\Psi_e \Psi_s = 1$, imposing that the wave function is normalized with $\Psi_s = \Psi^*_e$. Taking for Ψ_s the conjugate complex of Ψ_e it is important to notice that one preserves the same direction of rotation since the fluxes are inverse one of the other. In this algebra the expressions $(\textbf{Q}\Psi_e)$ and $(\textbf{P}\Psi_s)$ lead in a simple way to the components of the vectors G and H. It is just sufficient to determine the wave function for the (13) equation then the (12) equation is also determined.

3.1 *The solutions of the equation of Dirac.*

According to (13) the equation of Dirac of the positive flux can be written:

$$\textbf{Q}\Psi = [\frac{1}{c}(W - eV) + \alpha_1 \textbf{\textit{P}}_X + \alpha_2 \textbf{\textit{P}}_Y + \alpha_3 \textbf{\textit{P}}_Z + \alpha_4 m_0 c]\Psi \tag{19}$$

Each one of the four components of the wave function Ψ is the product of same function of the time Ψ_t by a function of the space different for each component. Take for the temporal function $\Psi_t = \exp(i\hbar W)$. Introduce the four components of $\Psi_t \Psi_i$ of Ψ with i = 1, 2, 3 or 4, following Darwin [22] and de Broglie [3], the equation of the wave function can be writes:

$$i\hbar^{-1}[(W + eV)/c + m_0 c]\,\Psi_1 = (P_X + iP_Y)\Psi_4 + P_Z\Psi_3 \tag{D_1}$$

$$i\hbar^{-1}[(W + eV)/c + m_0 c]\,\Psi_2 = (P_X - iP_Y)\,\Psi_3 - P_Z\,\Psi_4 \tag{D_2}$$

$$i\hbar^{-1}[(W + eV)/c - m_0 c]\Psi_3 = (P_X + iP_Y)\Psi_2 + P_Z\Psi_1 \tag{D_3}$$

$$i\hbar^{-1}[(W + eV)/c - m_0 c]\Psi_4 = (P_X - iP_Y)\Psi_1 - P_Z\Psi_2 \tag{D_4}$$

These four equations have the dissymmetry of the intrinsic rotation underlined with the discussion about the components of the wave function. The

solutions of these equations are the product of a radial function with a spherical harmonic. Indeed as far the electron can be assimilated to a point, the proportion between the inert and active masses is determined by the value of the potential at the considered point. This proportion determines the intensity of the moment. The sharing of the moment between its different components is obtained with the spherical harmonics and the radial functions. These functions for the different quantum states are recalled in the appendix.

The quantum number n ℓ and r have their classical meaning: principal, orbital and radial of Sommerfeld and Schrödinger models. There are two types of solutions which are better defined with the quantum numbers k and p. One has for the solutions of type I: $k = -\ell - 1$ and $p = r$; for the solutions of type II: $k = \ell$ and $p = r + 1$. As a result between the quantum numbers it comes:

$$n = \left| k \right| + p \ = \ \ell + r + 1 \tag{20}$$

Thus there is a floating unite resulting from the intrinsic rotation. It is attached to it either through the angular characteristics with the spherical harmonics or through the mass with the radial functions by the degree of the polynomials F and G functions (see appendix).

3.2   The doublets and the half integer moments

The doublets correspond to two distinct levels appearing without magnetic field. Thus it is the origin of these two levels that one has to understand. This property leads Uhlenbeck and Goudsmit to suppose the existence of the intrinsic rotation [23, 24] having an angular momentum supposed to be added or subtracted to the orbital moment. In the absence of the hypothesis of the variation of the mass it was difficult to understand how the doublets are formed.

The existence of the doublets arrive indeed from the two possible direction of the orbital rotation in comparison with the intrinsic rotation taken as positive direction, that is definitely the hypothesis of Uhlenbeck and Goudsmit. Furthermore for each direction of the orbital rotation, the direction of the intrinsic rotation can be the positive or negative direction of the z axis. As a result for any type of solutions, the quantum states are in even number. But the most interesting point arrives from the direction of the rotation corresponding to one of the two types of solutions. Indeed we have seen that there are two fluxes of grains piloting the electron along its orbit. The outgoing flux is characteristic of the electron and thus of the intrinsic rotation to the

orbital motion; as for the getting on flux is characteristic of the proton. These two contributions have to be added or subtracted according they have the same or opposite direction. As a result choosing the positive direction for the intrinsic direction, the orbital angular momentum has an algebraic sign. Thus for the angular momentum, one has to subtract or add the contribution of the intrinsic rotation to that of the orbital rotation.

Consider the succession of the quantum states.

1°)   When the motion is generated with just one quantum of action, the intrinsic rotation determines in only one way the connection between the time and the space through the energy and the moment. The total angular momentum cannot be higher to that of the corresponding number of quanta. Therefore the two fluxes determining the rotation are of opposite direction, the orbital angular momentum is negative. We have the two states "1s" which are of the type I.

2°)   When there is a second quantum of action, it can give a radial moment to the electron, we have the two states "2s".

3°)   The second quantum of action can be obtained with the increase of the mass, the alone kinetic energy of rotation being that of intrinsic rotation. In this way the angular momentum increases of one unite without to modify the number of quantum states. The total angular momentum increases of one unite, as a result the orbital and intrinsic rotations have the same directions. We have the two $2p_{1/2}$ quantum states, they correspond to the type II. The second quantum of action modifies the radial properties of the wave function. The increase of the action comes from that of the period associated to a decrease of the kinetic energy therefore of the speed.

4°)   The orbital and intrinsic rotations having opposite directions, the second quantum of action can be obtained with the increase in absolute value of the orbital angular momentum which has a negative sign, we have the four $2p_{3/2}$ quantum states, they correspond to the type I. For these states the variations of the inert mass and of the energy are analogous to that of the $2p_{1/2}$ but in this case there is augmentation of the maximum of the equatorial angular momentum.

5°)   The others increases of the action lead to the same consequences and explain the succession of the different quantum states.

4   The magnetic states and the factor of Landé

Consider H a magnetic field generated by a solenoid. The modification of density of grains which leads are described with just one angular variable, that of the orthogonal planes to the H field. The measurements of magnetic

moments consist to determine the asymptotic value of these moments when it is possible to consider that they are parallel with the magnetic field. In such condition the magnetic field modified the equatorial action of rotation acting on the electron. Consider the case of small field, we can suppose that the angular moment is not modified by the field.

The period of the motion of the electron determines the intensity of the electrical current generating the orbital magnetic moment. In classical mechanics the period of the motion in a central field is given by the relation:

$$2mS = T_M M \qquad (22)$$

where m is the mass of the mobile, M its orbital angular moment, S the area of the trajectory and T_M the period of the motion. To utilise this relation in the calculation of the magnetic moment one have to take for M the absolute value of the total angular moment M_{To}.

Let then ℓ be the number of quanta of the angular momentum to add to the intrinsic rotation. For the type II, to the orbital moment $M_{Or} = k = \ell$ generated by the number $k = \ell$ of additional quanta one has to add the contribution of the intrinsic rotation, it comes:

$$M_{To} = (k + \tfrac{1}{2}) = (\ell + \tfrac{1}{2}) \qquad (23)$$

For the type I with $k = -\ell - 1$, we have $M_{Or} = -k = -(\ell + 1)$. To the angular moment M_{Or} one has to add the contribution of the intrinsic rotation, it comes:

$$M_{To} = -(k + \tfrac{1}{2}) = (\ell + \tfrac{1}{2}) \qquad (24)$$

The period T_M of the motion is different from T_F that which would correspond to the orbital action kh replacing M by M_{Or} in the relation (22). Thus the factor g of Landé which is introduced in the calculation of μ_e the magnetic moment of a quantum state is a consequence of the difference between T_M and T_F. Indeed consider an electron with a classical orbit. For an action kh its angular moment is $M_{Or} = k$. Let u be the projection of M_{To} along the direction of magnetic field H.

The magnetic moment μ_e is given by the relation:

$$\mu_e = u\mu_B \qquad \text{with} \qquad \mu_B = \hbar\frac{e}{2mc} \qquad (25)$$

To establish this relation one uses the expression of the angular moment of a loop of current:

$$\mu_e = IS/c \qquad (26)$$

where I is the intensity circulating in the loop, S the area of the loop and c the speed of the light. For an electron on its orbit with the period of revolution T_F we have $I = e/T_F$. On the other hand the area law expressed on a period leads to replace M par M_{To} giving the relation $2mS = T_M M_{To}$. For $M_{To} = $ we find the relation (25) if $T_F = T_M$. But we have just seen that it is not like this. The period T_F is in fact fictive and taking into account the sign of k, we have the relation:

$$\pm 2mS = T_F k\ = T_M M_{To} = (k + \tfrac{1}{2}) \qquad (27)$$

That is: $T_M = gT_F$ with $g = \dfrac{k}{k + \dfrac{1}{2}}$ $\qquad (28)$

As a result the magnetic moment corresponding to u is:

$$\mu_e = gu\mu_B \qquad (29)$$

One can find the relation (28) in the second part of the original work of Dirac "*The quantum Theory of the Electron*" [25], that we already have found [6, 9] in a different context.

All the magnetic properties: g factor and the number of magnetic states can be found by the calculation [3] and are experimentally verified with the interpretation of the measurements of the magnetic moments [26] and Curie constants [27]. All the previous attempts to explain the doublets ignoring the role of the mass and its repercussions on the magnetic states could be just partial and therefore partially correct. It is the case of that of Uhlenbeck and Goudsmit which has the merit to consider the intrinsic rotation even before the hypothesis of the wave function [23, 24]. It is also the case of our previous attempts [6, 7] already based on the exchanges of matter with variations of the mass; they were useful steps leading to this explanation introducing the symme try of the problem. For the study of the magnetism it was important to

understand this origin of the doublets for the different levels connected to them pre-exist to the application of the magnetic field [26].

5   Conclusion

In this study of the quantification, the orbital motion is supposed the result of the intrinsic rotation of the electron and of exchanges of matter between the electron and the proton. These exchanges take place in all the directions of the space. From this fact the orbital rotation must be regarded as the result of a mechanical action taking place into three orthogonal directions: one is the intrinsic axis of rotation of the electron the two others are perpendicular to it. The electron being supposed small in comparison of the proton, to keep the symmetry of the intrinsic rotation, the half integer values observed in measurement of magnetic moments are attributed to the equipartition of the energy.

In this approach Dirac's equation corresponds to exchanges of matter giving the incoming flux in the volume of the electron, the other aspect of the exchanges the outgoing flux allows to interpret the equation with negative energies. The components of the wave function determine the proportions of the mass generating the different components of the moment. With this conception two very close masses can generate distinct quantum states. It is this possibility associated to the two directions of the orbital rotation compared to that of the intrinsic rotation which originate the doublets. With this conception of the mass the model of Sommerfeld keeps it value. On the other hand the understanding of all the observed states relies on that of the approach of Dirac.

This conception of the interactions answered for the electromagnetism to an important difficulty of the physique where the notion of force allows understanding a great number of observed phenomena but leaves a manifest vacuum on the way by which the forces take place. One can hopes that similar essays will do progress the understanding of the gravitation.

Appendix

The two types of solutions of the equation of Dirac according to [3], n ℓ and r are the quantum numbers: principal, orbital and radial.

Type I $k = -\ell - 1;$ $p = r;$ $n = \ell + r + 1;$ $-\ell \leq m \leq \ell + 1$

$$\psi_1 = i \, F_+ \, Y_{\ell+1}^m \; ; \qquad\qquad\qquad \psi_2 = -i \, F_+ \, Y_{\ell+1}^{m-1}$$

$$\psi_3 = (\ell - m + 1)G_+ Y_\ell^m \; ; \qquad\qquad \psi_4 = (\ell + m)G_+ \, Y_\ell^{m-1}$$

Type II $k = \ell; \qquad p = r + 1; \qquad n = \ell + r + 1; \qquad -(\ell-1) \le m \le \ell$

$$\psi_1 = - i(\ell - m) \, F_- Y_{\ell-1}^m \; ; \qquad\qquad \psi_2 = i(\ell + m - 1) \, F_- \, Y_{\ell-1}^{m-1}$$

$$\psi_3 = G_- \, Y_\ell^m \; ; \qquad\qquad \psi_4 = - \, G_- \, Y_\ell^{m-1}$$

With $k = -\ell-1$, $F = F_+$ et $G = G_+$ for the type I, $k = \ell$, $F = F_-$ and $G = G_-$ for the type II. The radial functions are solutions of the equations:

$$\hbar^{-1}[(W + eV)/c + m_0 c]F + \frac{dG}{dr} + \frac{k+1}{r} G = 0$$

$$-\hbar^{-1}[(W + eV)/c - m_0 c]G + \frac{dF}{dr} - \frac{k-1}{r} F = 0$$

$$F = \exp{-ABr}[a_0 r^\gamma + a_1 r^{\gamma+1} \; \ldots \; a_p r^{\gamma+p}]$$

$$G = \exp{-ABr}[b_0 r^\gamma + b_1 r^{\gamma+1} \; \ldots \; b_p r^{\gamma+p}]$$

with $A^2 = \hbar^{-1}(m_0 c + W c^{-1})$ and $B^2 = \hbar^{-1}(m_0 c - W c^{-1})$

and $\gamma = -1 + \sqrt{k^2 - \alpha^2}$ where α is the fine structure constant: $\alpha = \dfrac{e^2}{\hbar c}$

The energy of the level is given by the relation:

$$E_{n,k} = \left[1 + \frac{\alpha^2}{\left[p + \sqrt{k^2 - \alpha^2} \right]^2} \right]^{-1/2} \tag{21}$$

Recall that the functions ψ_1 and ψ_2 are the fine components and he functions ψ_3 ψ_4 the gross components.

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