

## Alternative conventions and geometry for Special Relativity

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**ABSTRACT.** This paper argues that Einstein's conventionalist definition of time is sufficient for, but not necessary to the geometric modelling of Special Relativity. A different convention allows that any time interval  $t$ , can be measured by  $dc$ , the distance travelled from an origin by the spherical wave-front of a light pulse,  $c$ . This means that the relationships represented by the hyperbolic geometry of Minkowski can also be represented by circular function geometry (CFG), where the spherical surface of  $c$  provides both a fourth set  $t$ , of frame-dependent co-ordinate points and a parameter  $s$ , for measuring intervals that are invariant between reference frames. However, sine values under the circle range from 1-0, rather than 1- $\infty$ . This does not allow that for a reference frame velocity  $\approx c$ , any interval length  $\approx \infty$ . Furthermore, since CFG does not subdivide space-time into past and future zones, it excludes the possibility of backwards time travel for signal velocities  $> c$ .

**RÉSUMÉ.** Cet article argue que la définition conventionalist d'Einstein du temps est suffisante pour, mais non nécessaire de modeler géométrique de la relativité spéciale. Une convention différente permet ce quand l'intervalle  $t$ , peut être mesuré par  $dc$ , la distance a voyagé d'une origine par le front des ondes sphérique d'une pulsation lumineuse,  $c$ . Ceci signifie que les rapports représentés par la géométrie hyperbolique de Minkowski peuvent également être représentés par la géométrie circulaire de fonction (CFG), où la surface sphérique de  $c$  fournit un quatrième ensemble  $t$ , des points de coordination et un paramètre  $s$ , pour les intervalles de mesure qui sont invariables entre les référentiels. Cependant, les valeurs de sinus sous le cercle s'étendent de 1-0, plutôt que de 1- $\infty$ . Ceci ne permet pas à cela pour une vitesse référentiel  $\approx c$ , d'aucune longueur d'intervalle  $\approx \infty$ . En outre, puisque CFG ne subdivide pas l'espace-temps en zones passées et futures, il exclut la possibilité de vers l'arrière voyage de temps pour des vitesses de signal  $> c$ .

## 1 Introduction : The conventionalist definition of time

This paper argues that if physicists choose a convention different to the one Einstein adopted for indicating time units, then they can model the four co-ordinate manifold of Einstein's Special Relativity (SR) with a Euclidean geometry that is governed by the functions of the circle. It will be demonstrated that the circular function model provides exactly the same numerical solutions to SR problems as does the established Minkowski geometry, a non Euclidean form governed by the functions of the hyperbola. The one significant difference between the models is that the Minkowski one uses the length of the directrix under the hyperbola to indicate the relative length of time intervals, whilst the Euclidean one uses the height of sine under the circle. Hence, in Minkowski modelling the relative length of a time interval ranges from one to infinity, whilst in the Euclidean model, the length only ranges from one to zero under the circle. It will be suggested that this difference proves highly instructive when investigating the challenging notions of 'infinite' energy and time 'travel', which are problematic for present relativity theory.

Minkowski's geometric approach was based on Einstein's innovative strategy for defining space and time. Einstein had managed to circumvent notoriously difficult philosophical issues by electing to define space and time solely in terms of the established conventions for measuring each.<sup>1</sup> The presence of rigid measuring sticks would indicate intervals of space, and as for time:

'...we understand by the "time" of an event the reading (position of the hands) of that one of these clocks which is in the immediate vicinity (in space) of the event.'<sup>2</sup>

Philosophers have termed this pragmatic approach to space and time 'conventionalist'.<sup>3</sup> In the case of time, this has proved a successful strategy because it postulates that in physics, what is meant by a 'time' interval is in fact, a clock interval. Clocks of course, are *indicators for the concept of time*, not detectors or measurers of a physical "state" of time. Thus, Einstein's approach effectively bypasses philosophical concerns about the unobservable "presence" of time by focussing attention solely on the observable behaviour of clocks.

However, Einstein's conventionalist definition of time still leaves much to be desired. Any cursory observation of the kind of pocket watch he describes above finds the 'position of the hands' to be cyclical and reversible about the same starting point. Yet, the convention of western science is to

see time as a linearly asymmetric ordering. This is evidenced in Minkowski geometry, which portrays any sequence of clock readings as forming part of a linearly asymmetric axis,  $t$ . The reasons for re-arranging a cyclical sequence of clock readings into an asymmetric ‘time’ ordering need to be explained by reference to a process omitted from Einstein’s definition – that of record-keeping.

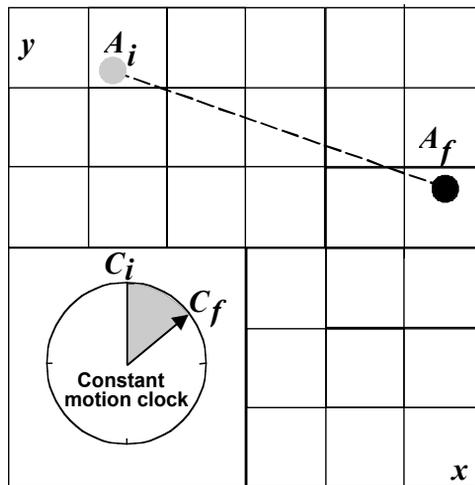
Recording is implicit in the act of observing, and an observer or observational device that cannot record data is of little use to science. The key characteristic of recording is that every registry of data is additive, never subtractive. Hence, a ‘later’ record must always accumulate more data than an ‘earlier’ one, in just the way that a given natural number set must by definition, always include all its predecessor numbers among its elements. The temporal ordering of physical events is typical of this characterisation, requiring *by convention* that clock readings  $t$  are arranged  $\{t_1, t_2, t_3, \dots\}$  in the linear, asymmetric order of the set of counting numbers  $N$ , so forming a set  $T$  of clock readings or ‘times’. The conventions for this ordering of  $T$  date back at least as far as Aristotle (‘Time is the number of motion in respect of before and after’)<sup>4</sup> and were established independently of any later theories concerning irreversible processes and so forth.

Hence, the raw observation of the movement of hands about a clock face is not in itself sufficient to explain why their positions indicate *time* readings. Underlying this observation is the more fundamental understanding that records always show the journey followed by the clock hands to be one of constantly increasing length. An observable process that perfectly demonstrates such a constant asymmetric journey is the propagation of light from a point origin in space. Light propagation  $c$ , is of course, the third essential observable measure for Einstein’s SR theory. Yet, rather than adopt intervals of light propagation to indicate time, for understandable practical reasons, Einstein and Minkowski chose to retain ‘seconds’ of the arc, the established measures of the pocket watch.

Nevertheless, Minkowski immediately realised that the journey of a light pulse provided a fundamental ‘natural’ measure for his geometric manifold, which could be viewed either in terms of a ‘dynamic’ temporal period, such as one second of light travel, or in terms of a ‘static’ distance, 300,000 kilometres of light travel.<sup>5</sup> The fact that these measures were completely interchangeable demonstrated Minkowski’s most celebrated claim - that this geometric manifold was in fact unitary, being the union of two sets of different interval types, metre and second, which by tradition indicated space and time respectively.<sup>6</sup> Only hyperbolic geometry appeared to provide the way of uniting these different measures of the same relativistic interval, and in a

form that still preserved the traditional conceptual distinctions, as seen in the differences between the ‘timelike’ region within the ‘light cone’ and ‘space-like’ region outside of it.

Yet if it is true that the measures of light metre and light second are completely interchangeable, then in principle, it should be perfectly acceptable to use only *one* of these measures to model the space-time continuum. This principle is easily demonstrated by considering the mapping of events that do not even involve relativity, such as those illustrated in Figure 1, which is a scale map of two objects on a table, a stopwatch with hands  $C$ , and a ball  $A$ , rolling across the surface. The map is created by observers plotting what they record to be the successive coincidences  $\{C_1, A_1\}, \{C_2, A_2\} \dots$  of the position of the second hand about the clock face, with the position of the ball on the tabletop. Since the map accurately plots all displacements, it gives observers the same freedom of choice about whether they wish to measure the intervals travelled by the hand of  $C$  as ‘seconds’, or as ‘millimetres’.



**Figure 1.** Comparison of motion of clock hands with the motion of a ball  $A$ .

The main reason for preferring ‘seconds’ is that the number of millimetres travelled by a hand may vary according to the size of the clock. However, if observers elected to standardise the size of all stop watches, then it would be perfectly acceptable to substitute a standard number of millimetres in place of the standard second. So the fact that some observers could choose to characterise the intervals  $\{C_1, A_1\} - \{C_2, A_2\}$  over which these events are recorded

as ‘periods of duration’ does not in any way prevent other observers from electing to measure the same interval in terms of the displacement they actually *observe*, as distinct from the ‘time elapse’ they ‘sense’.

To subscribers of this alternative convention, the map itself simply shows two spatial displacements  $dA$  and  $dC$ . Comparing the lengths of these two displacements provides a perfectly satisfactory measure of their relative motion, without reference to seconds. So under this new convention, time is always measured in units of spatial displacement, and the velocity  $v_A$  of the moving ball can be redefined as the ratio of its displacement  $dA$ , from an initial position  $A_i$  to a final one  $A_f$ , relative to the displacement  $dC$ , over the interval  $i-f$  by the designated clock motion,  $C$ :

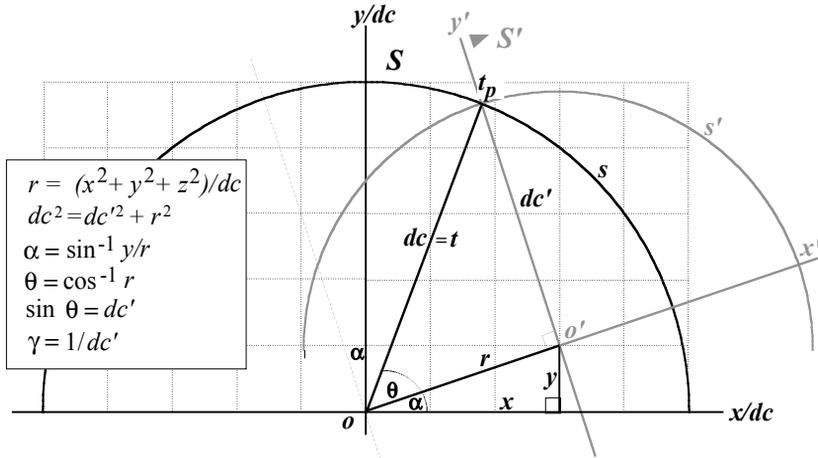
$$v_A = (A_f - A_i) : (C_f - C_i) = dA/dC \quad (1)$$

Moving on to situations involving SR, if light propagation is selected as the standard clock, then its unique properties as the natural constant of SR allow for the development of a further new rule for mapping events. Since SR postulates that nothing travels at a velocity greater than  $c$ , then:

$$\begin{aligned} \text{For all } v, v_c \geq v, \text{ hence for } d_A > 0, dc > 0, \\ \text{and where } dc = 0, \text{ then } d_A = 0 \end{aligned} \quad (2)$$

This choice of convention then permits observers to substitute the time ‘measured in light seconds’ axis  $ct$  of the Minkowski ‘light cone’, with a ‘time measured in distances travelled by a light pulse’ parameter,  $s$ . As is illustrated in Figure 2, rather than being a flat plane like the  $ct$  one of Minkowski space,  $s$  is the curved surface of a ‘light sphere’ centred on the spatial origin of the light pulse, with  $dc$  being the radius of  $s$ . Events occurring within this sphere may then analysed using circular function geometry, which reveals a Euclidean analogue of the hyperbolic function space-time (HFS) of Minkowski, herein called “CFS”.

## 2 Circular Function Analysis of Special Relativity



**Figure 2**

Figure 2 could be seen as a two-dimensional map made just like its Minkowski analogue, where observers in the  $S$  frame of reference retrospectively plot data retrieved from a series of observational devices such as movie cameras, placed at regular distances throughout the  $S$  frame. The circle centred on the origin  $o$ , of the  $S$  coordinate system,  $o, x, y$ , represents their observations of the wave-front of a light flash  $c$ , expanding equidistantly from  $o$ . This light flash provides the necessary readings by which to measure all the intervals traditionally regarded as being spatial or temporal. Since  $c$  is a natural constant, observers who retrieve data from detectors that are regularly spaced throughout their reference frame will obtain records of a regular sequence of light travel intervals,  $dc_1, dc_2, dc_3 \dots$ , the unit size of which are determined solely by the chosen spacing of their observational devices.  $S$  observers may then use any sequence of these ‘snapshot’ records of the propagating wave-front  $s$  to relate the coincidences of all ‘point’ particles observed at ‘point’ places in  $S$ , with what they judge to be the simultaneous position of  $s$ .

In Figure 2,  $S'$  is a translating, inertial frame of reference, separated from the  $x, y$  plane of  $S$  by a negligible “point” distance in the  $z$  direction. Over the interval of 1 lm,  $S'$  translates 0.63 lm in a  $+xy$  direction. Assume that the wave-front  $c$  is a light flash triggered by the close coincidence of the origins

$o$  and  $o'$ . Then circular function geometry allows observers in either reference frame to measure the relative differences in interval measurement between  $S$  and  $S'$ , by comparing their measurements of the distance travelled  $dc$ , along the  $S'$  spatial axis that is perpendicular to its translating trajectory. In this case, the perpendicular axis is  $y'$  and so observers at  $S$  rotate their own  $y$  axis by the angle  $\alpha$  to find the point  $t_p$  at which the wave-front  $c$  meets  $y'$  across a given translation interval  $r$ , where:

$$\alpha = \sin^{-1}y/r \quad (3)$$

In CFS, it is the set of all these coordinate points  $t_p$  for translating frames that forms the surface of the spherical parameter  $s$ , and it is the length of  $dc$  that gives the observer's constant, frame-dependent, space-time interval  $t$ :

$$dc = t = 1 \quad (4)$$

The interval separating  $t_p$  from  $o'$  gives the  $S'$  frame's simultaneous measurement  $dc'$  of the observer's interval  $dc$ . The angle  $\theta_r$ , relating the juncture of  $dc$  and  $dc'$  is given by:

$$\theta_r = \cos^{-1}r \quad (5)$$

Following the first postulate of SR that the velocity of light propagation is the same for observers in all inertial reference frames, the judgement of the length of  $dc'$  will remain frame invariant, with  $dc'$  being another function of the circle:

$$dc' = \sin \theta_r. \quad (6)$$

and

$$dc'^2 = dc^2 - (dx^2 + dy^2 + dz^2) \quad (7)$$

The length of  $dc'$  then provides the frame invariant metric  $ds$  for CFS, analogous to that of the Minkowski metric,  $ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2$ , with the CFS metric being:

$$ds^2 = dc'^2 = dc^2 - (dx^2 + dy^2 + dz^2) \quad (8)$$

The equivalence found in Minkowski geometry between  $ds$ , the velocity parameter  $\beta = [1 - (v/c)^2]^{1/2}$  and the proper time  $d\tau = dt/(1/\beta)$  is also found in CFS:

$$dc' = ds = d\tau = \beta \quad (9)$$

In CFS, the observer's plotting of successive  $t_p$  points on the axes of translating inertial frames reveals a straight line  $ot_p$ , related to the origin of the  $S$  co-ordinate system by the angle  $\theta_r$ . These lines bear some similarities to the sloping world lines of Minkowski geometry, since at any point  $\Delta r$  along the trajectory of a translating inertial frame, the ratio  $dc:dc'$  can be easily measured using the familiar rules of circular function geometry:

$$\cos^2 \theta_r + \sin^2 \theta_r = 1 = dc \quad (10)$$

Let these useful lines be designated in CFS as "clock lines". By combining rotation of axes with measurement of the invariant interval  $\sin \theta_r$  under a clock line, observers can then deduce the Lorentz transformations of their coordinates to the simultaneous space of translating inertial frames such as  $S'$ . Observers in  $S$  use the length of  $dc'$  to determine the simultaneous space of  $S'$ , which appears as another light sphere of relatively smaller dimension, shown in Figure 2 by the grey circle centred on  $o'$ . Thus, in CFS the Lorentz transformation of coordinates between reference frames appears as a displacement and shrinking of Euclidean spheres, rather than the tilting and stretching of non Euclidean light cones. In CFS, the Lorentz transformation factor  $\gamma$  is given by:

$$\gamma = 1/\sin \theta_r = 1/dc' \quad (11)$$

In Figure 2, for  $v = 0.63c$ ,  $\theta_r = \cos^{-1}0.63 = 51^\circ$ . Then  $\gamma = 1/\sin 51^\circ = 1.29$ , which is exactly the same value found for a frame  $S'$  translating at  $0.63c$  when using the standard SR equation for the Lorentz factor,  $\gamma = 1/\beta$ . So it emerges that the functions of the circle - radius, sine and cosine - can be used to represent respectively:  $t$ ,  $\tau$  and  $v$  for the analysis of 'inertial' reference frames as described in SR. The analogous relationship between CFS and Minkowski geometry is further illustrated by the comparisons made in Figures 3A and 3B.

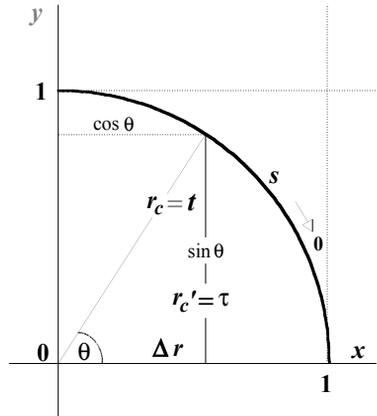
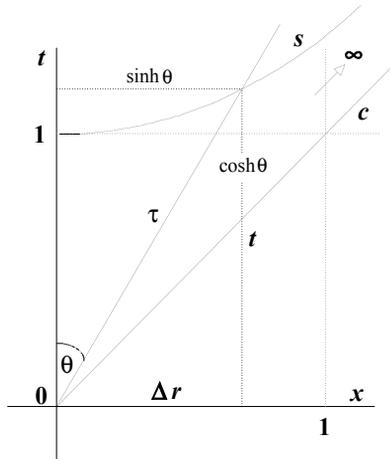


Figure 3A. SR relationships in Minkowski geometry      Figure 3B. SR relationships in CFS geometry

### 3 Simultaneity

The CFS approach can be further illustrated by consideration of the differing views of simultaneity held by observers in the  $S$  and  $S'$  frames of reference, as shown in Figure 4. Following a well-known example, called “Einstein’s Train Paradox”,<sup>7</sup> observers at  $o$  judge themselves to be near coincident with observers at  $o'$ , and mid-way between two events along their  $x$  axis,  $A$  and  $B$ , which appear to be simultaneous light flashes to observers at  $o$ . However, observers at  $o'$ , will judge event  $B$  to have preceded event  $A$  and that when  $o'$  was near coincident to  $o$ , they were closer to place  $B$  than place  $A$ . This can be ascertained using the standard equations (SE) of SR, by comparing the each frame’s measurements of the time interval  $\Delta t$  separating event  $A$  from event  $B$ , where  $L$  is the distance separating them.

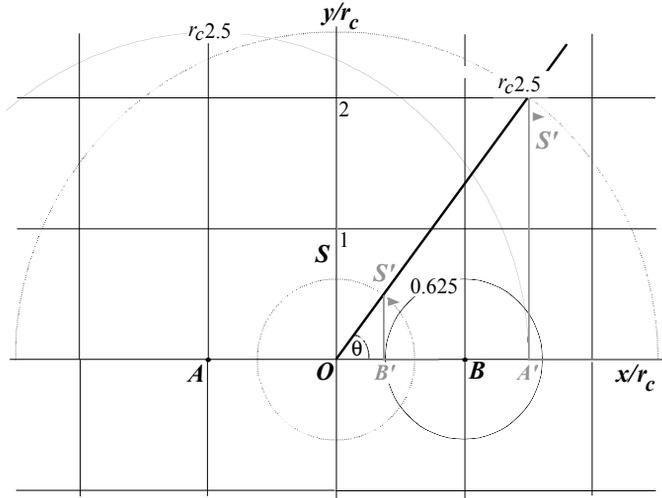


Figure 4. CFS representation of the “Einstein Train Paradox”

If the  $S$  frame judgement of  $L$  is 2 metres of light travel (lm), then observers in that frame may use the above equation to agree that for observers in  $S'$  translating at  $v = 0.6c$ , the value of  $\Delta t' = 1.5$  lm. Figure 4 shows that exactly the same values can be obtained from CFS, by comparison of the  $S$  and  $S'$  spheres of simultaneity, where for  $v = 0.6c, \theta_r = 53.13^\circ$ :

$$\begin{aligned} \text{CFS: } \Delta dc' &= [L(r/dc)] / [1 - (r/dc)^2]^{1/2} \\ &= (L \cos\theta_r) / (\sin\theta_r) = 2/0.8 = 1.5 \text{ lm} \end{aligned} \tag{12}$$

The  $S$  frame calculation of the time  $t_B = dc_B$  at which  $o'$  receives the light signal from B can be determined equally from both the SE and CFS:

$$\text{SE: } t_B = L_B / (L_B v_B + L_B) = 2/(2 \times 0.6c + 2) = 0.625 \text{ lm} \tag{13}$$

$$\text{CFS: } dc_B = L_B / (L_B \cos\theta_r + L_B) = 2/(2 \times 0.6 + 2) = 0.625 \text{ lm} \tag{14}$$

From which  $t_B', r_B$  and  $r_B'$  are easily found, as shown in Figure 4:

$$\text{CFS: } t_B' = t_B \sin\theta_r = 0.625 \times 0.8 = 0.5 \text{ lm} \quad (15)$$

$$\text{CFS: } r_B = dc_B \cos\theta_r = 0.625 \times 0.6 = 0.375 \text{ lm} \quad (16)$$

$$\text{CFS: } r_B' = r_B \sin\theta_r = 0.375 \times 0.8 = 0.3 \text{ lm} \quad (17)$$

To find the distance  $r_A$  at which  $o'$  receives the light signal travelling at  $v = c$  from A, and thereafter, the time  $t_A$ :

$$\text{SE: } r_A = v_S' t_A = v_S' (r_A + L) / c = (0.6r_A + 0.6) / 1 = 1.5 \text{ lm} \quad (18)$$

$$\text{CFS: } r_A = \cos\theta_r (r_A + L) / (r_{\text{signal}} / dc) = (0.6r_A + 0.6) / (1/1) = 1.5 \text{ lm} \quad (19)$$

$$\text{CFS: } r_A' = r_A \sin\theta_r = 1.5 \times 0.8 = 1.2 \text{ lm} \quad (20)$$

$$\text{CFS: } t_A = r_A / \cos\theta_r = 1.5 / 0.6 = 2.5 \text{ lm} \quad (21)$$

$$\text{CFS: } t_A' = t_A \sin\theta_r = 2.5 \times 0.8 = 2 \text{ lm} \quad (22)$$

So it is found both from the usual equations of SR and the rules of CFS that the views of simultaneity differ in  $S$  and  $S'$ , since:

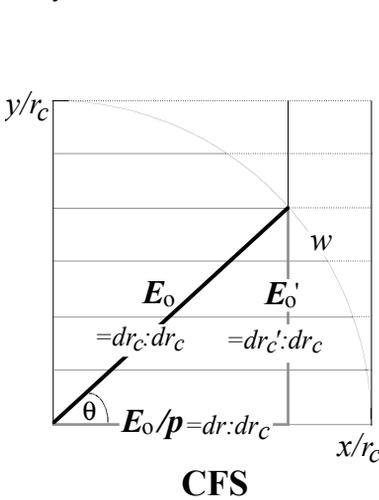
$$t_A - t_B = 2.5 - 0.625 = 1.875 \text{ lm} \quad (23)$$

$$t_A' - t_B' = 2 - 0.5 = 1.5 \text{ lm} \quad (24)$$

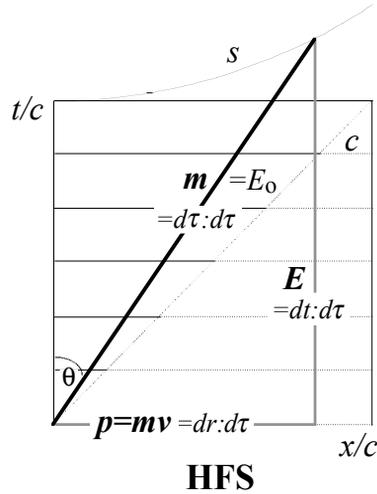
#### 4 Comparison of momentum-energy vectors in CFS and HFS

In Minkowski geometry, illustrated in Figure 5B, the relationships between relativistic mass, energy and momentum,  $m$ ,  $E$  and  $p$ , can be derived by dividing all vector lengths under  $s$  by the frame invariant displacement

$d\tau$ . The analogous operation for vectors in CFS is to divide all vector lengths under  $s$  by the displacement  $dc$ . As illustrated in Figure 5A, energy-momentum relationships appear differently under the circle compared to the hyperbola. The radius  $dc$  gives a fixed measure for the observer's reference frame, with the ratio  $dc/dc$  indicating the total energy  $E_o$ , of objects relatively at rest to that frame.



**CFS**



**HFS**

**Figure 5A Momentum-energy relationships in CFS**

**Figure 5B Momentum-energy relationships in HFS**

In Minkowski vector space,  $m = d\tau$ , is the frame invariant measure and may have values ranging from 1-  $\infty$ . Yet in CFS, since  $dc = dt = E_o = 1$ , it is the ratio  $dc/dc'$  that gives  $E_o'$ , the relative *rest* energy in the accelerated frame, and  $dc/dr$  indicates the relativistic momentum  $p$  of the accelerated reference frame in terms of  $1/p = E_o/p$ . Hence in CFS vector space:

$$\sin^2 dc + \cos^2 dc = dc^2 = (E_o')^2 + (E_o/p)^2 = 1^2 \tag{25}$$

$$E_o = dc = dt = 1 \tag{26}$$

$$E_o' = dc' = d\tau = 1/\gamma = E_o/\gamma \tag{27}$$

$$m = E_o/E_o' = dc/dc' = \gamma E_o \quad (28)$$

As comparison of the models in Figure 5 shows, in CFS, the displacement  $dc' = \sin\theta$  diminishes from 1 at relative rest, to 0 at the limit spatial displacement  $dr = dc$ . Whereas in the Minkowski analogue, the directrix of the hyperbola,  $d\tau = \cosh\theta$ , grows from 1 to infinity at the limit velocity  $v = c$ . Hence in CFS, comparison of the rest value  $^o$ , with the limit velocity value  $^*$  for momentum-energy, reveals the following:

$$E_o/p^o = 0, E_o = dc, p^o = 0 \quad (29)$$

$$E_o/p^* = 1, E_o = dc, p^* = dc \quad (30)$$

$$E_o/E_o'^o = 1, E_o = dc, E_o'^o = dc \quad (31)$$

$$E_o/E_o'^* = 0, E_o = dc, E_o'^* = 0 \quad (32)$$

Equations (29) – (32) agree with Minkowski momentum-energy vectors in one respect: For differing velocities, total momentum-energy remains constant in all frames, but measurements of the ratios between momentum and energy vary from frame to frame. For instance, CFS reflects the observer's judgement that  $E_o'$  relatively diminishes from 1 to 0 at the limit, and  $E_o/p$  grows in proportion from 0 to 1 at the limit. However, the fundamental difference between CFS and HFS revealed by Equations (25) – (32) is that  $1 = dc$ , and  $dc$  is a *finite* physical measure,  $2.998 \times 10^8$  Im. In CFS, combination of Equations (25) to (32) indicates:

$$p^*/E_o = E^*/E_o = dc = 1 \quad (33)$$

These are the correct values required to preserve the principle of equivalence between the two reference frames in question. Yet this is not something obviously evident from the mathematics of the Lorentz factor equation (11) and HFS, which indicate that  $\gamma^* = \infty$  and  $p^* = \infty$ . However, in CFS,  $\gamma^* = dc/dc'^* = 0$ . To derive the result  $\gamma^* = \infty$  from CFS requires reference to the values of  $\text{cosec}\theta = 1/\sin$ , but  $\text{cosec}\theta$  is *not* one of the functions under the circle through which CFS represents the fundamental measures of observer's

time, proper time and velocity. As argued above, these differences between HFS and CFS appear to be ones only of mathematical form, and it should be noted that in its original Einsteinian form, the Lorentz factor equation  $\gamma = 1/\beta$  is an algebraic expression for measures of velocity, *not* mass-energy. While mathematically, there may be an infinity of differing lengths for  $\sin\theta$  under the circle, the reference interval for all CFS judgements is the finite measure  $dc$ , the radius of the observer's light sphere. Bearing this in mind and applying the same procedure as in Equations (31)-(34) to the factor  $\gamma$ , where  $1/\gamma^o = 1$  and  $1/\gamma^* = 0$ , it is found that:

$$1/\gamma^o - 1/\gamma^* = dc = 1 = E_o \quad (34)$$

which leads to:

$$\gamma^o - \gamma^* = 1/dc = 1/E_o \quad (35)$$

Multiplying both  $1/dc$  and  $1/E_o$  by  $dc$  gives:

$$\gamma^o - \gamma^* = dc/E_o = 1 \quad (36)$$

The odd looking conclusion of these equations that both  $\gamma^o = 1$  and  $\gamma^o - \gamma^* = 1$ , accords fully with the principle of relativity. If observers in some rest frame believe their measures to be finite, then they should never encounter a situation where other observers could view other systems as having relatively infinite measures. The traditional approach of relativity theory has been to exclude this possibility by pointing to the infinite dimensions of both the Lorentz factor equation and Minkowski 'length',  $d\tau/d\tau$ , which suggest that it requires infinite energy to accelerate a particle to  $v = c$ . Unfortunately, when this alleged exclusion is applied in the general theory of relativity, it appears to be in conflict with the black hole hypothesis, which proposes a natural mechanism that actually *can* accelerate particles to light velocity at its 'event horizon'!

The revelation of CFS that the proportions of  $\gamma^o$ ,  $E^*$  and  $p^*$  are all equal to  $dc = 1$ , obviates such difficulties by suggesting that that the infinite dimensions of the Lorentz factor and Minkowski length are solely features of a chosen mathematical model, and not of the natural world. The fact that this world can also be modelled by an analogue geometry CFS, which is based on finite dimensions, indicates that the scales of Minkowski geometry are not necessarily true of natural scales. Hence, it is worth considering what

sense CFS makes of situations where  $v=c$ , by proportioning all the values of Equation (36) only in terms of finite natural constants.

Strictly speaking, there are no units for relativistic ratios such as  $E/m$  or  $c/E$ .<sup>8</sup> To find the numerical value of the ratio  $dc/E_0$ , then requires selection of a natural constant for  $E_0$  to proportion it with the natural constant  $c$ . The obvious choice is the quantum of energy, Planck's constant  $h$ , (also appropriate because light propagation  $c$  is a manifestation of quantum physics). In conventional units, the ratio  $c/h$  appears as a momentum of  $4.524 \times 10^{41} \text{kg/m}^1/\text{s}^{-1}$ . Division of this value by the factor  $c$  gives this ratio in units of mass-energy,  $1.509 \times 10^{33} \text{kg}$ . This equates to the numerical factor implied by  $1/E_0$  in Equation (35) of  $(1 \text{ kg/m}^2/\text{sec}^{-2})/h$ . Thus, these proportions put a number,  $1.509 \times 10^{33}$ , to the difference  $\gamma^0 - \gamma^*$ , which will now be denoted as the limit Lorentz factor value,  $\gamma^j$ .

Application of this finite limit value  $\gamma^j$ , permits a consistent account of a hypothetical test object  $\alpha$ , translating at  $v = c$ . To observers in any sub-light reference frame, such an object is indistinguishable from a photon, because in this case,  $m_o/m_o' = dc/dc' = \gamma^* = 0$ . Now imagine the frequency  $f$  of this photon is numerically equivalent to the factor  $\gamma^j$ , then its relativistic energy would be:

$$E_\alpha = hf_\alpha = h\gamma^j = (6.626 \times 10^{-34} \text{kg/m}^2/\text{sec}^{-2})1.509 \times 10^{33} = 1 \text{ kg/m}^2/\text{sec}^{-2} \quad (37)$$

The wavelength  $\lambda$  of this photon would be:

$$\lambda_\alpha = c/f = c/\gamma^j = 1.987 \times 10^{-25} \text{m} \quad (38)$$

Then imagine it were asserted that this same object  $a$ , was a particle with  $v_a = c$ . From equation (37) Observers in sub-light frames would deduce that the rest energy of this object was  $(1 \text{kg/m}^2/\text{sec}^{-2})/\gamma^j = h$ . Consider how they would then judge  $\alpha$ 's de Broglie and Compton wavelengths,  $\lambda_B$  and  $\lambda_C$ . Since for this velocity  $E_\alpha = p_\alpha$ , they would need to divide  $E_\alpha$  by the conversion factor  $c$  to obtain  $p_\alpha$  in conventional units and  $E_\alpha$  by  $c^2$  to obtain its relativistic mass  $m_\alpha$  giving:

$$\lambda_{B\alpha} = h/p_\alpha = h/(E_\alpha/c) = h/(h/c) = 1.987 \times 10^{-25} \text{m} \quad (39)$$

$$\lambda_{C\alpha} = h/m_\alpha c = h/(E_\alpha/c^2)c = h/(h/c^2)c = 1.987 \times 10^{-25} \text{m} \quad (40)$$

From which it would be evident that for any test object in this limit case, there is no distinction to be drawn between its properties as a particle or as a photon, since:

$$\lambda_{\alpha} = \lambda_{B\alpha} = \lambda_{C\alpha} = 1.987 \times 10^{-25} \text{m} \quad (41)$$

Development of the relationships illustrated in Figure 5A then indicates that special relativity and quantum theory appear to meet at a limit Lorentz factor  $\gamma^j$ . This should not be interpreted as an argument for the quantisation of space-time at the scale of  $10^{-25}$ m. It is only to say that for any particle  $\alpha$  accelerated to  $v_{\alpha} \approx c - 10^{-38}$ m/s, where  $\gamma E_{0\alpha} = (1/h)E_{\alpha}$ ,  $E_{\alpha}$  will not rise for any further increase in  $v_{\alpha}$ . In principle, this hypothesis could be the subject of experimental test.

## 5 Time Travel

Time travel is another philosophically difficult idea that finds support in relativity theory, because the construction of the Minkowski light cone gives the time axis a clear “direction” relative to spatial axes, and it partitions the continuum into “past” and “future”. Since this construction also sets  $c$  as the limit rate for all physical signals, efforts to consider faster than light signals (FLT) end in paradox after it is found that some observers of the same events will judge FLT signals to be travelling “back” into the past.

FLT can be analysed without paradox in CFS, because although measurements are made relative to four co-ordinates,  $x, y, z, c$ ,  $c$  is *not* a member of the mutually orthogonal axis set  $x, y, z$ . Since the temporal co-ordinates  $t_p$  of CFS are found on the surfaces of asymmetrically expanding light spheres  $c$ , by definition, no light sphere is going to reverse direction relative to  $x, y, z$ . So although the “travel” of a reference frame may be reversible relative to  $x, y, z$ , it can never be so relative to  $dt = dc$ . Nor is there any subdivision of the CFS continuum into past and future; the mapping starts at any chosen “now” and leads only to later events.

Following a thought experiment summarised by Penrose,<sup>9</sup> it can be shown that events which appear paradoxical when mapped in Minkowski geometry, are not so when mapped in CFS. The situation, illustrated in Figures 6A and 6B, compares the viewpoints of three different observers of FLT signalling,  $P, Q$  and  $R$ .

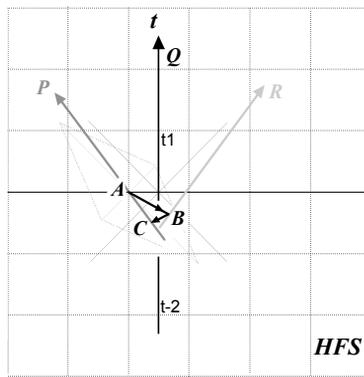
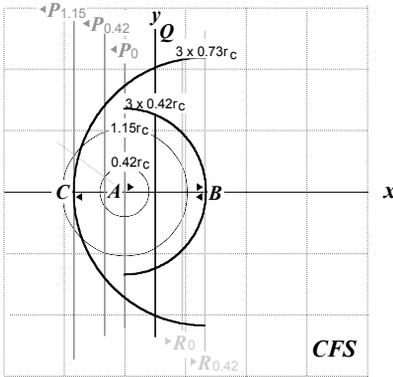


Figure 6A. FLT signalling in CFS

Figure 6B. FLT signalling in HFS

Observers at  $Q$  judge themselves to be in between two reference frames  $P$  and  $R$  each translating away from  $Q$  at  $v=0.8c$  in opposite  $x$  directions. Observers in reference frame  $P$  send an FLT signal at  $3c$  towards  $Q$  and  $R$ . Call the transmission of this signal event  $A$ , which from the  $Q$  viewpoint occurs at time  $t = 0$ sec, and at a place  $A$ ,  $0.5$  lm away on their  $-x$  axis.  $P$  observers judge that their FLT signal arrives at event  $B_p$ ,  $2.1$  lm distant, at a time  $t_p = 0.7$ sec. Now analyse  $Q$ 's view of this situation following the rules of Minkowski geometry, as illustrated in Figure 6B.

For  $Q$  observers to map the  $P$  and  $R$  judgements of the same events in Minkowski geometry, they must plot the slopes  $S_r$  of the  $t$  axes of the translating reference systems, to determine the Poincaré transformations of their co-ordinate systems, relative to the observer's:

$$S_r = \Delta t / \Delta r = \tan\theta_r \tag{42}$$

For all observers, the slope  $S_r$  of an FLT signal is obviously greater than the  $45^\circ$  slope limit for timelike events set by  $c$ , so FLT signals must travel to places in their spacelike zones. For  $Q$  observers, the relative slope  $S_p$  for  $P$ 's velocity of  $0.8c$ , is  $38.7^\circ$ , for which the Poincaré transformation of  $P$ 's co-ordinate system (illustrated in Figure 6B puts  $B$  at a spacelike point in  $P$ , which is coincident with a point in the past timelike zone of  $Q$  that is earlier than  $A$  in  $Q$ 's judgement. Thus  $Q$  observers are forced to conclude that if  $P$  observers are capable of transmitting an FLT signal to  $B$ , it travels from a later to an earlier time. A deeper paradox involving backward causation

arises if it is further imagined that event  $B$  is constituted by the arrival of the FLT signal at a mirror fixed in the  $R$  reference frame, which can reflect the FLT signal back to  $P$ .  $Q$  observers would then conclude by symmetry, that the reflected signal will also travel backwards in time to arrive in  $P$  at an event  $C$ , that all observers will agree is actually earlier than  $A$ !

For Penrose and many others,<sup>10</sup> such paradoxes demonstrate the absurdity of FLT and the validity of the original postulate that  $c$  marks the limit of any signal velocity. However, a better considered view would be that these paradoxes prove only that Minkowski modelling of Einstein's relativity theory cannot model behaviour at variance with the theory's original assumptions about  $c$ . CFS is likewise constructed on assumptions about the nature of light propagation, but these do not involve placing any limits on signal velocity. CFS measures motions by reference *inter-alia* and  $c$  just happens to be the chosen reference constant in situations involving the Einstein principle of relativity. Accordingly, it is not surprising to discover that the above paradoxes do not occur when the same events are modelled in CFS.

In the CFS model shown in Figure 6A,  $Q$  observers set event  $A$  as the starting point  $t = 0$ , for analysis. From the  $Q$  viewpoint, at this time, the position of  $A$  is at  $-x = 0.5$  lm and the position of the  $R$  origin is at  $+x = 0.424$  lm, separated from  $A$  by  $0.924$  lm. Thus, for observers in all three reference frames, any signal sent from  $A$  at  $3 \times dc$  will have to travel across the interval  $AB$  to reach  $R$  at  $B$ , and all three frames will measure the time and distance co-ordinates for this interval differently. From Equations 20 and 22 above, where  $v_P = v_R = 0.8c$ ,  $Q$  observers will determine that the distance  $\Delta r_R$  travelled by  $R$  before the signal of  $v = 3c$  arrives from  $A$  at event  $B$  is:

$$\Delta r_R = \cos 36.9(\Delta r_R + 0.924) / (3/1) = 0.8(\Delta r_R + 0.924)/3 = 0.336 \text{ lm} \quad (43)$$

Since  $Q$  observers judge  $P$  to be coincident at  $A$  at  $t = 0$ , then they must judge that the time taken for the FLT signal to cover the interval  $AB$  is:

$$\Delta t_{AB} = \Delta r_R / \cos 36.9 = 0.42 \text{ lm} \quad (44)$$

Hence the temporal separation between A and B is  $+0.42$  lm, and the spatial separation  $AB_Q$  is:

$$AB_Q = \Delta t_{AB} / (dc/3dc) = 0.42 / (1/3) = 1.26 \text{ lm} \quad (45)$$

From the  $Q$  viewpoint, for observers in  $P$ , the intervals  $AB$  and  $\Delta t_{AB}$  will appear as:

$$AB_P = AB_Q / \sin 36.9 = 2.1 \text{ lm} \quad (46)$$

$$\Delta t_P = \Delta t_Q / \sin 36.9 = 0.7 \text{ lm} \quad (47)$$

These are the same values initially specified for this thought experiment, and none of the time values are negative. Then for the  $Q$  judgement of the same FLT signal reflected back from  $B$  to  $C$ :

$$\Delta r_{BC} = \cos 36.9 (\Delta r + 1.26) / (3/1) = 0.58 \text{ lm} \quad (48)$$

$$\Delta t_{BC} = \Delta r / \cos 36.9 = 0.725 \text{ lm} \quad (49)$$

$$BC_Q = \Delta t_{BC} / (dc/3dc) = 0.725 / (1/3) = 2.176 \text{ lm} \quad (50)$$

And

$$\Delta t_{AB} + \Delta t_{BC} = 0.42 + 0.725 = 1.145 \text{ lm} \quad (51)$$

From the  $Q$  viewpoint, for observers in  $P$ , the intervals  $\Delta r_{BC}$  and  $\Delta t_{BC}$  will appear as:

$$BC_P = BC_Q / \sin 36.9 = 0.966 \text{ lm} \quad (52)$$

$$\Delta t_P = \Delta t_Q / \sin 36.9 = 0.907 \text{ lm} \quad (53)$$

Again, all the time intervals here are positive and the paradox found when using Minkowski modelling disappears. CFS then supports the common sense intuition that backward time travel is no more than an improper extrapolation from the characteristics of the Minkowski light cone.

## 6 Conclusion

The development of a Euclidean analogue for Minkowski geometry from philosophical first principles may then provide useful service. It indicates that Minkowski space is not the only “proper” model of Special Relativity. It warns against literal understandings of the terms “time travel” and “direction” and suggest that “backwards causality” is not an obstacle to the consideration of FLT reference frames. The analogue also indicates that finite, rather than infinite quantities are found at the limits of relativity theory, which proposition may help protect relativity from the philosophical suspicion that the discovery of infinite quantities reveals a limitation, rather than a limit in the theory.

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