The Black Body and the Dulong and Petit Law

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ABSTRACT. The density of thermal energy of the black-body is supposed to be the reflection of the statistical repartition of the energy of the atoms of the solid constituent of the black body. As a result its study starts from that of the Dulong and Petit law. The calculation of the Stefan-Boltzmann constant gives $\sigma = 5.6265 \times 10^{-8}$ W m^{-2} K^{-4}. The difference with the experiment, lower to one per cent, is attributed to the losses inherent to the thermal equilibriums which defines the temperature of the solid, losses including the thermal radiation itself.

1 Introduction

In the study of the black body Planck has used the statistical approach of a physical phenomenon introduced by Boltzmann. As a result he was able to propose a satisfactory fit of the experimental data concerning the distribution of the energy [1]. Using statistics of the gas in the study of the light energy he was introducing the hypothesis of countable particles. There is a difficulty of our physics which is not always clearly realized. Indeed in our scale we have the notion of the continuum that is of measurements taking all possible values along a segment, of energy for example. But the number of the possible values along a segment is much larger that that of the integer numbers, they can always make a discrete series of points along a given segment. As a result the use of atomic statistics in the study of the black body makes the light a phenomenon bound to the atomic scale, thus to the countable. It is by this way that, more or less clearly, the quantum property to the light, soon uses again by Einstein in his interpretation of the photoelectric effect in 1905 [2]. Thus at the door of the twentieth century researchers were slowly becoming aware of the emission processes of the photons by the electrons of the atoms. It is according to this process that Planck interpretation gives satisfactory results. Indeed the photons are emitted by the atoms of the wall of the solid defining the black body. Thence the statistical weights of the energy that they emit must be in connection with those corresponding to the energy of these atoms, hypothesis that this study will allow to verify.

This aspect of the study of the black body is important because the photons moving in vacuum all at the same speed, the statistical distribution of their energy reveals a mechanism essentially different from the one tacking place with the atoms of a gas. The purpose of this study is to extend for the black body the use of the density of probability previously determined for the gas [3], [4]. Now the photons are emitted by the atoms of the solids with which the black body is built. As a result this makes useful to start this study with that of the heat capacity of the solids to high temperature which determines the energy of the emitted photons.

2 The heat capacity of the solids to high temperature

For a solid according to Dulong and Petit law the heat capacity tends to $3k$. As a result the energy stocked by the solid tends to $3kT$. The classical interpretation of this law comes from the law of Hooke, expressing the proportionality between the deformations of a solid and the counter forces. One supposes that there is $kT/2$ for each degree of freedom. The other is supposed to come from the counter forces, in other words from the potential to be overcame by the atom to vibrate [5] [6] and [7]. We do not think that this explanation is sufficient. Indeed the atom of gas has also to overcome a potential when it knocked the wall of the container. As a result it should have the same heat capacity as the solids to high temperature. Thus if we want to explain the Dulong and Petit law we have to understand how the energy of a solid tend to $3kT$ when the temperature increases. We have already underlined that the atoms of a solid at low temperature, according to their cohesion, form a whole [8]. The Dulong and Petit law then can be interpreted supposing that when the temperature increases, the orientation of the atoms becomes progressively disordered. They are the atoms in disordered position which receive the thermal energy $3kT$.

Indeed let us suppose that the stocked thermal energy by one atom stays lower than a given value E_g. The synchronization between the motions of the electrons is responsible of the maximum value of the cohesion energy between the atoms. As a result it tends to equalize the sharing of this thermal energy among the atoms to keep as much as possible the synchronization that is the order between the atoms. It is what is exhibited by the heat capacity of the solids at low temperature [8].

Let us underline that the synchronization is a hypothesis supported by the recent interpretation of Dirac theory based on the trajectory of the electron [9]. Numerous experimental results are corroborating it: among them the lowering of the order disorder temperature with impurity as the fusion temperature in presence of a small amount of another metal. In comparison to the analogous electrons to those of the principal metal those of the impurity have different periods and the synchronization disappears leading to a smaller cohesion and the lowering of the order disorder temperature, in that case the fusion temperature.

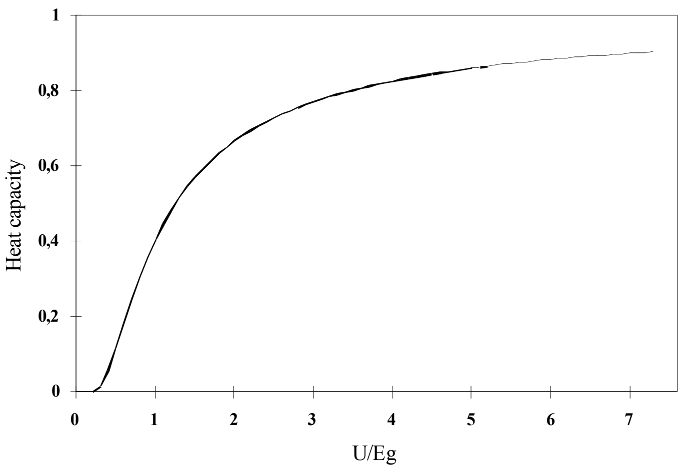

Figure 1. The probability function giving the asymptotic character of the Dulong and Petit law for the solids. In this case the unit is $3k$.

Thus to be in disordered position an atom must break the synchronous bonds developed with its neighbours. The energy need to break these bonds defines a potential barrier E_g. The number of atoms crossing this barrier is given by the probability function $\mathrm{P}(E_\mathrm{g},U)$ already calculated [3], [4]. In this approach one has to keep in mind that E_g decreases when the number of ordered atoms decreases. But the probability function clearly establishes the asymptotic character of the de Dulong et Petit law (figure 1). We have:

$$\mathrm{P}(E_\mathrm{g},U) = \mathrm{A}^{-1} ln[1+\exp -\alpha(E_\mathrm{g}/U -1)] \quad \text{with } \mathrm{A} = 1.7054 \text{ et } \alpha = 1.5049 \qquad (2.1)$$

When $U \rightarrow 0$, $\mathrm{P}(E_g,U) \rightarrow 0$ and when $U \rightarrow \infty$, $\mathrm{P}(E_g,U) \rightarrow 1$ (2.2)

One has now to understand why the disordered atoms keep an amount of thermal energy equal to $3kT$. At low temperature the atoms of a solid are a whole which prevents one atom to vibrate independently from its neighbours. By hypothesis when an atom is in disordered position it can vibrate that is it can receive thermal energy rather independently from its neighbours. But it is not yet an atom of a gas, it cannot vibrate in the same way in all the directions, its neighbours being retained by attraction of coulomb. But the energy is stocked by the electrons moving around the nucleus of their kernel. As a result a same atom can thus receive energy from two opposite directions on the same line. As the energy received in to opposite directions comes necessarily to two different times this amounts to considering that in solid there are six degrees of freedom. On the opposite a gas atom can just receive energy from one direction. Thus if the counter forces are effectively at the origin, for the solids of their specific heat double of that of the monatomic gas, this comes from their possibility to receive energy from to opposite directions.

3 The photon and the disorder in the solids

When the photon is emitted, it propagate with the speed $c(\nu)$. As a hypothesis we suppose that it can be described as a packet of energy, formed of grains like the potential [9] in which gravitate the electrons but with an equivalent mass much smaller than the electron. On the other hand we will see that there are a very small number of photons in the volume of the black body. This aspect of the phenomenon leads to suppose that they are not directly in interaction but that the density to calculate is the reflection of the statistical repartition of the energy of the atoms of the solid constituent of the black body. Since the Dulong and Petit law is independent of the nature of the compound it will be the same for the characteristic laws of the black body. They will just depend of the temperature, an essential property of the thermal radiation of the black body.

Consider an atom which has a thermal energy E higher than E_g. Thus it gets a disordered position allowing it to have a periodic motion asynchronous with those of the neighbour atoms. The period of the motion is defined by the relation $ET = h$ where h is the constant of Planck. When the atom returns to its ordered state, we suppose by hypothesis which it emits a photon having the same period like that of its motion. This point is important because it allows the calculation of the density of energy of the black body radiation. Indeed for the atom its periodic motion leads it to move away or to get closer from its neighbours. When it emits the photon corresponding to E the motions of the homologous neighbour atoms will be disrupted. We suppose that it is such a perturbation which induces the stimulated emission.

4 The density of energy per unit of volume and frequency

The study of the black body is that of the number of photons able to be received upon the surface *ds* during a given time *dt*. The speed of the light being *c*, during the time *dt* the photons cover the distance *cdt*. The number of received photons leads to define the density of energy per unite of volume. The usage and the historical conditions of the study of the black body have led to define the density $u(\nu)$ per unit of volume and per unit of frequency. In summing upon all the frequencies one defines *u* the density of total energy.

The calculation of the density of energy per unite of volume uses several factors associated to the frequency ν of the photon. These factors are: the statistical weight in the vicinity of a segment of energy, the volume V_c associated to the emission of a photon whose the origin is discussed below and the number *N* of photons which can occupy this volume. Following is then the integration on all the frequencies.

The statistical weight. To calculate the density of energy *u* per unite of volume of the black body we have to start from the function of distribution of the thermal energy for a solid. According to the study of the specific heat of the solids just the disordered atoms have a sufficient energy to emit a photon. Their mean thermal energy is them $3kT$. They are in a given way a set of independent atoms thus without any special quantum property able to modify their statistical properties as it is supposed for the photons and the electrons also call bosons or fermions [10]. This aspect shows that the distribution to use is the same as that of the atoms of a gas. The energy of the photons that they emit is that exceeding the potential barrier E_g which they cross to be in a disordered position. It can vary between zero and infinity. Let $D(E,U)$ be the function of the thermal energy of these independent atoms. These atoms being independent they follow the same law that that of the atoms of a perfect gas.

Considering the segment of energy:

$$h\nu \leq E \leq h(\nu+d\nu) \tag{4.1}$$

The probability for an atom of the solid to have an energy included on this segment, in other words the statistical weight on the segment [3,4], is given by:

$$D(h\nu,U)hd\nu = \frac{\alpha}{\mathrm{A}U}\frac{hd\nu}{1+\exp\alpha\left(\frac{h\nu}{U}-1\right)} \tag{4.2}$$

with A = 1.7054 and $\alpha = 1.5049$

Thus the probability to have a photon emitted with this frequency on the width $hd\nu$ is proportional to $D(h\nu,U)hd\nu$. However the photons to participate to the density $u(\nu)$ must be emitted toward volume of the black body. For one direction there is just one out of two. The other is emitted toward the inside of the solid in place to be emitted toward its surface. On the other hand there are in the space three independent directions thus one has to divide the statistical weight $D(h\nu,U)hd\nu$ per height that is 2^3. Then the corresponding probability of emission $P(\nu)$ is:

$$P(\nu) = 2^{-3}\, D(h\nu,U)hd\nu \tag{4.3}$$

The volume of emission. Consider the interferences obtained with monochromatic light after to have crossed Young holes. To get interferences the two holes must be lighted with a point source, the two sources thus obtained are called coherent; this means that phase properties are linked. In the corpuscular approach we will express this property saying that the two sources contain trains of coherent photons, that is having between them an integer multiple of wave length λ of the photons. Furthermore to see the coherency disappears, one has to introduce between the two beams a difference of length of the order of ten meters, giving the order of magnitude of the trains of photons. Moreover the dimensions of the black body are often of the order of the centimeter. Thus we can suppose that each train of photons is continuously reflected or absorbed and emitted again keeping always its properties of coherency. As a result it follows for the photon a volume of occupation V_c associated to the emission, proportional to the sphere of radius $R = \lambda = c\tau = c/\nu$ in which no other photon has been emitted for the considered direction.

That is:

$$\mathrm{Vc} = \frac{4\pi}{3}\, c^3\tau^3 \tag{4.4}$$

The lengths in these kinds of radiation are of the order of the thousand of angstroms to compare to the atomic scale of the order of the angstrom. As a result there are a very small number of photons per unite of volume compared to that of the atoms of the solid. Thus the spectral density $u(\nu)$ is inversely proportional to V_c. Furthermore the emitted photons are the reflection of the density of thermal energy of the atoms in disordered position of the solid defining the black body. As we have seen these disordered atoms have six degrees of freedom, thus from this fact if the total density u of thermal energy is inversely proportional to V_c, on the opposite it is proportional to the six degrees of freedom. Thus it appears a factor 6 which is the number N of photons associated to V_c and to the probability $D(h\nu,U)hd\nu$.

The spectral density. Now let us consider the segment with a width of energy define by the relation (4.1). Along this segment the energy can be taken equal to $h\nu$. The corresponding emitted energy is $h\nu\, D(h\nu,U)hd\nu$. For the frequency ν, tacking into account of the probability of emission $P(\nu)$ given by (4.3), the density of energy $u(\nu)$ for a width in frequency $d\nu$ is given by the relation:

$$u(\nu)d\nu = \frac{6}{\mathrm{V_c}}\,\frac{h\nu}{8}\,D(h\nu,U)hd\nu = \frac{9}{2^4\pi}\frac{\nu^4}{c^3}\,D(h\nu,U)h^2d\nu \qquad (4.5)$$

It comes :
$$u(\nu)d\nu = \frac{\alpha}{\mathrm{A}U}\frac{9}{2^4\pi c^3}\frac{\nu^4h^2d\nu}{1+\exp\alpha\left(\frac{h\nu}{U}-1\right)} \qquad (4.6)$$

With the change of variable $h\nu = xU$ the relation (4.6) can be written:

$$u(\nu)d\nu = \frac{9U^4}{2^4\pi c^3h^3}\,x^4D(x)dx \qquad (4.7)$$

with
$$D(x) = \frac{\alpha}{A}\left[1+\exp\alpha\left(x-1\right)\right]^{-1} \qquad (4.8)$$

Integrating between zero and infinity, the total density of energy u can be written:

$$u = \frac{9U^4}{2^4 \pi c^3 h^3} J_4 \tag{4.9}$$

With: $$J_4 = \int_0^\infty x^4 D(x) dx \tag{4.10}$$

The calculation gives: $J_4 = 11{,}17$

With $U = 3kT$ it comes: $$u = \frac{3^6 J_4}{2^4 \pi} \frac{k^4}{c^3 h^3} T^4 \tag{4.11}$$

5 The Stefan-Boltzmann constant.

The study of the black body leads to measure the total flux of energy W per second. It is given by the relation $W = cu/4$. It comes:

$$W = \frac{3^6}{2^6 \pi} J_4 \frac{k^4}{c^2 h^3} T^4 \tag{5.1}$$

It is the law of Stefan-Boltzmann :

$$\sigma = 40{,}486 \frac{k^4}{c^2 h^3} = 5.6265\text{x}10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \tag{5.2}$$

The approach of Planck gives:

$$\sigma(\text{Planck}) = \frac{2\pi^5}{15} \frac{k^4}{c^2 h^3} = 5.6705\text{x}10^{-8} \text{ W m}^{-2} \text{ K}^{-4} \tag{5.3}$$

The experiment gives: $\sigma(\text{Exp}) = (5.66967 \pm 0.00076) \text{ x } 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$

The value of the Stefan-Boltzmann constant calculated from the relation (5.2) gives a difference lower than 1% in comparison to the experiment [11] and [12]. This difference is small but nevertheless important in comparison to the accuracy of the experiments. It has leads us to revisited our statistical work [3] and [4], that we discuss in the following section.

6 Discussion

The statistical study of the distribution of the thermal energy [3] and [4] starts from the following remark: the sum of the states of energy occupied by the particles of the system divided par their number, must be equal to the mean value U per particle of this energy. The present determinations, to respect this condition, use the method of Boltzmann which supposes that the entropy of the system is proportional to the logarithm of the statistical weight W of the most probable distribution. This hypothesis is introduced in a differential form with the relation: $dS = k \ln W,$ hypothesis that we have replaced by the integral relation:

$$\int_0^{\infty} ED(E,U)dE = U \tag{6.1}$$

To solve the statistical problem we have supposed that the maximum of the statistical weight $W(E)$ associated to a small segment of energy around the value E is obtained for the mean value U. This hypothesis is generally a sufficient approximation, however for very accurate experiment it turn out to be not good enough [13].

Indeed consider a set of particles, here the atoms of the solid of the black body emitting the photons. It is in equilibrium with the thermostat allowing maintaining stable the temperature with the wanted accuracy. Every set of particles is subject to losses of thermal energy. In particular among them the thermal radiation the energy among which is taken that of the solid and which participates to the distribution of that of the solid as it is exhibited by this study. For the small fraction used for the measurement and get out of the black body itself it seems reasonable to consider it negligible or as calculable.

For the rest of the flux it is absorbed again by the black body itself and one could believe that looses, as a result, are equally negligible. It seems not really correct. The photons after have been emitted become source of perturbations for the black body. When they are absorbed again they constitute a new amount of energy which during some time modify the probabilities and do not obey to the same rules of equilibrium and exchanges between the atoms. In some way this energy is ignored during a time, of course short, but sufficient to modify the thermal equilibrium. Thus the photons emitted and absorbed again constitute looses during some time. As a result there is a flux of energy from the thermostat to the set of particles. This flux has for effect to move toward a value M slightly higher to U the maximum of the statistical weight $W(E)$. It is this energy M which replaces U in the calculus leading to the expression of the Stefan-Boltzmann constant.

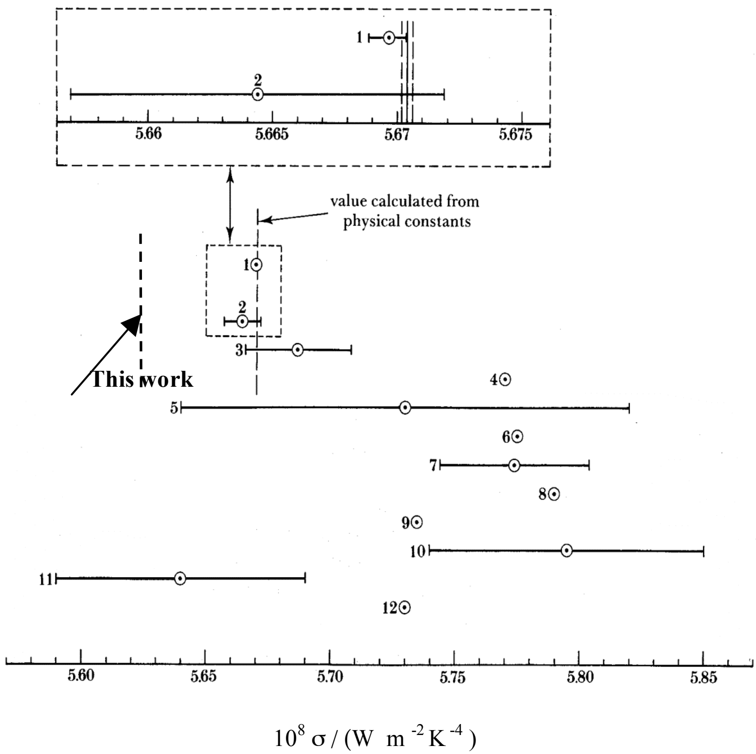

Figure 2. The figure 38 of Quinn et Martin giving experimental values for the Stefan-Boltzmann obtained since 1921.The uncertainties are the authors' own estimates. The arrow indicating: value calculated from physical constants corresponds to the approach of Planck, the other arrow: this work gives the position of the value corresponding to this work. For bibliographic references to all the works before 1971, see Blevin & Brown [15]. (1) Quinn & Martin (1988) (5.66967±0.00076) x 10^{-8} W m^{-2} K^{-4}; (2) Blevin & Brown (1971),

(5.6644±0.0075) x 10^{-8} W m^{-2} K^{-4}; (3) Kendal (1968); (4) Gilham (1968); (5) Faure (1965); (6) Eppley & Karoli (1957); (7) Muller (1933); (8) Mendenhall (1929); (10) Kaussmann (1924).

So ε being a small positive number, it comes $M = U(1+\varepsilon)$ and the relation (5.1) becomes:

$$W = \frac{3^6}{2^6 \pi} J_4 \frac{k^4}{c^2 h^3} T^4 (1+\varepsilon)^4 \tag{6.2}$$

To find again the experimental value of σ one have to take ε = 0.0019.

Let us underline that the published values of σ up to now [12], [14] and [15] are all higher to the theoretical value given by the relation (5.1), see in particular the figure 38 of Quinn et Martin [12] reproduced on the figure 2. This confirms the meaning of the relation (6.3): the experiment can just gives values higher to that of the relation (5.1).

In conclusion we know that the temperature is a macroscopic variable which in accurate experiments stay a well defined variable. We can put $M = 3kT_{\text{eff}}$, the measured or effective temperature is T_{eff}. This measured temperature includes the flux of energy need to balance the losses. It is always higher than the temperature defined from PV = RT the law of perfect gas allowing to define the mean energy of a monatomic gas as $U = 3kT/2$ or that of a solid as $U = 3kT$ value used to determine the statistical distribution of the energy of the corresponding sets.

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