Generalized de Broglie-Bargmann-Wigner Equations,
a Modern Formulation of de Broglie's Fusion Theory

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The photon was the first object of de Broglie's fusion theory. Owing to its universality it is of central importance in physical research. In studying the properties of the photon in theory and experiment, de Broglie's photon model is confronted with modern results of deep inelastic photon-photon scattering experiments. It is argued that these experiments enforce an extension of de Broglie's original theory in order to obtain photon states with (relativistic) spatial extension.The corresponding generalized de Broglie-Bargmann-Wigner equations satisfy this postulat. For the case of photons and electroweak gauge bosons their properties and solutions are discussed in detail.

1 Introduction

To decompose material objects into elementary parts and afterwards to reconstruct them theoretically as well as practically is one of the most important European contributions to physical sciences, namely the idea of atomism.

After the discovery of quantum mechanics it was possible to give atomism a solid mathematical foundation, where theoretically the reconstruction of objects from its constituents is achieved by the calculation of bound states.

All the spectacular progress in applied physical sciences depends on this fact. But in spite of this overwhelming success of quantum mechanics in applied physical sciences, there remains an anoying (theoretical) discrepancy: Based on experiments Voigt,[1], Lorentz,[2] and Poincare,[3] formulated space-time transformations which leave Maxwell's theory invariant, but not quantum mechanics.

How can this discrepancy be resolved?

It was de Broglie,[4], who made the first step to reconcile atomism with relativity theory by formulating his theory of fusion.

And the first object where he demonstrated his theory of fusion was an extremely relativistic system, namely the photon which he considered as a bound state of two spin 1/2-fermions. Later on de Broglie,[5] and Bargmann and Wigner,[6] extended this formalism to the treatment of bound states of n spin 1/2-fermions and the corresponding fusion equations were evaluated by many authors afterwards.

To study the implications of this approach and the need of its extension or modernization it it sufficient to treat the case for n = 2, which mainly is concerned with the photon as a composite relativistic system.

2 Photons as Composite Particles (Quanta)

Already in the twenties in connection with the derivation of the Bose statistic, Eddington and Heisenberg,[7] supposed the photon to be a bound state of positively and negatively charged fermions. But it was de Broglie who argued: For an explanation of electrodynamics by the action of composite photons one is not allowed to use electrodynamics to explain their constitution.

Consequently, de Broglie assumed the photon to be build from a pair of massive neutrinos and he succeeded in evaluating this picture of the photon quantitatively. De Broglie's fusion equations for the photon read

$$\begin{aligned} \partial_\mu \gamma^\mu_{\alpha\beta} \Psi_{\beta\alpha'}(x) - m_0 \Psi_{\alpha\alpha'}(x) &= 0 \\ \partial_\mu \gamma^\mu_{\alpha'\beta} \psi_{\alpha\beta}(x) - m_0 \Psi_{\alpha\alpha'}(x) &= 0 \end{aligned} \tag{1}$$

with the photon wave function $\Psi_{\alpha\alpha'}(x)$. By evaluation of these equations de Broglie was able to derive Maxwell's theory. He was the first physicist to derive an effective relativistic field theory for composite particles. And although this deduction of electrodynamics is mathematically very simple, it was an enormously important and brilliant idea, because it showed how to proceed in relativistic atomism.

This happened as early as in 1932. Now seventy years have passed and what has been discovered in the meantime?

To find an answer to this question, one has to compare the results of de Broglie's theory with modern photon experiments.

De Broglie's wave function $\Psi_{\alpha\alpha'}(x)$ describes the fusion of two spin 1/2-fermions, where the fusion is generated by the spin variables of the fermions, while the fermions themselves are confined to be on the same place, and hence this system has no spatial extension.

Does this correspond to experimental results?

3 Photons with Partonic Structure

Since the midst of the past century an increasingly large number of high energy experiments with photons were performed which provided new insights into the nature of the photon,[8],[9]. Without going into details: Let us consider some schematic diagrams of typical processes in that range, which describe deep inelastic scattering processes of (virtual) photons on target particles. Owing to the inelasticity of the collisions the target particles are transmuted into various fragments and it is important for the interpretation of the photon experiments that the diagrams for protons as target particles and photons as target particles are formally the same.

Figure 1, [10]

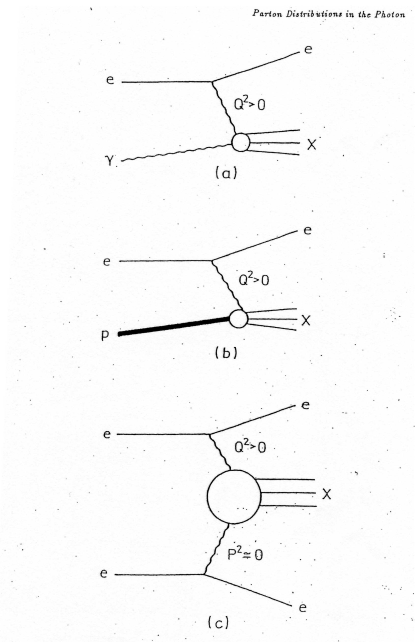

Fig. 3. DIS diagrams: (a) $e\gamma$ scattering, (b) ep scattering, (c) two-photon exchange in e^+e^- scattering.

Among the fragments of this inelastic collision processes with photons as target particles are electron-positron- and quark-antiquark pairs as well as vector mesons. This shows that apart from quantum electrodynamic effects also quantum chromodynamic effects are involved. In the conventional treatment, where the photons are assumed to be elementary, these effects are qualitatively explained by the fluctuations of the target photon into various other states,[11]. But this hypothesis leads to difficulties which were frequently discussed in literature and in addition the fluctuation of the target photon into vector mesons rises the question: Is it so easy to transmute a structureless point particle into particles with internal structure? Thus from an atomistic point of view another hypothesis is likely, namely that the photon possesses a permanent partonic substructure which is revealed in such experiments.

4 Photons in experiment

The elastic and inelastic photon-proton scattering processes were thoroughly studied in high energy physics. Owing to the similarity between deep inelastic photon-photon processes and photon-proton processes, the latter can be used to find an appropriate interpretation of the photon-photon scattering. Hence we start with the discussion of photon-proton processes.

As far as the photon-proton processes are concerned one can visualize them by considering the proton as a fixed target of a high energy electron microscope, where the scattered electrons lead to a diffraction spectrum which contains the information about the structure of the proton.

This interpretation is essential as it shows that the information about the structure of the proton has to be extracted from the diffraction picture of the scattered electron and not from the scattered proton or its fragments.

In this picture the laboratory system, where the proton is at rest, is a distinguished frame of reference. In this frame the differential cross-section for the scattered electron is defined by the exchange of a virtual photon between electron and proton. To formulate this the following definitions are required

$$\begin{array}{lcll} p & := & (m_N, 0) & \text{initial proton momentum} \\ k & := & (E, \mathbf{k}) & \text{initial electron momentum} \\ k' & := & (E', \mathbf{k}') & \text{final electron momentum} \end{array}$$

$q \quad := \quad (k - k')$ momentum transfer by γ^*
$d\Omega \quad := \quad 2\pi dcos\Theta$ Θ angle between k and k'

Then the cross-section for the scattered electron reads,[12]:

$$\frac{d^2\sigma}{dE'd\Omega} = \frac{4\alpha^4(E')^2}{q^4}(2W_1 sin^2\frac{1}{2}\Theta + 2W_2 cos^2\frac{1}{2}\Theta) \tag{2}$$

where in general the structure functions $W_i, i = 1, 2$ should depend on $\nu = E - E'$ and $Q^2 = -q^2$, as independent variables.

But experiments show that at least in a certain energy range not ν or Q^2, but rather

$$x = \frac{Q^2}{2m_N\nu} \tag{3}$$

is the independent variable of W_1 and W_2.

This "scaling" behavior can be explained if the nucleon is assumed to be composed of point-like spin half constituents (partons) and if the structure functions for deep inelastic reactions can be viewed as to be built up from an incoherent sum of elastic scatterings of the virtual photon on these constituents. In this case one finds a dependence of W_1 and W_2 upon only the variable x as desired.

However: if one wishes to elaborate this concept in more detail, by theoretical considerations one is forced to change the reference system in order to give an appropriate description of such reactions,[12].

Therefore, clearly, the first step in such a theoretical discussion must be a reformulation of this cross-section in terms of covariant variables. Without going into details we give the result of such a reformulation. The invariant variables are given by $q^2 = -Q^2$ and by $s = (p + k)^2$ and

$$x = -\frac{q^2}{2(pq)}; \qquad y = \frac{(pq)}{(pk)} \tag{4}$$

Then the covariant cross-section reads

$$\frac{d^2\sigma}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}[xy^2F_1 + (1 - y - \frac{xym_N^2}{s})F_2] \tag{5}$$

where $F_i, i = 1, 2$ depends on x and Q^2. By means of this formulation one is free to choose any frame of reference for the description of the scattering process which seems to be appropriate from the theoretical point of view.

Astonishingly this appropriate system tuns out to be the infinite momentum frame S^∞. In this frame the nucleon's four momentum is given by,[12]:

$$p = ((P^2 + m_N^2)^{1/2}, 0, 0, -P) \tag{6}$$

where the speed of the reference frame as seen in the laboratory frame is

$$\beta = \frac{P}{(P^2 + m_N^2)^{1/2}} \approx 1 \tag{7}$$

for sufficiently large P , i.e., in the limit of infinite P. In this case it is possible to perform quantitative calculations using the parton model. It is not my intention to describe these calculations.

But we observe: S^∞ is a natural frame of reference for photons and this brings us to the relationship between the proton, partons and the photon.

For the deep inelastic photon-photon process in S^∞ the standard scaling variables are the same as in the proton case and the cross-section can be formulated in the following way,[13]

$$\frac{d^2\sigma^\gamma}{dxdy} = \frac{4\pi\alpha^2 s}{Q^4}(xy^2 F_1^\gamma + (1-y)F_2^\gamma] \tag{8}$$

and, as the target photon is (quasi) real, the mass m_N of the proton has to be replaced by $m_\gamma \approx 0$. A comparison of both the formulas shows: the similarity is complete.

Hence one can use the diffraction pattern of the outgoing electron as a high energy microscope for the target photon and the structure functions $F_i^\gamma, i = 1, 2$ can be treated as the structure functions of this photon.

In this process the virtual probing photon is assumed to be a point-like object. Although the virtual photon may also develop a nontrivial structure, viewing it as a point-like object is a very good approximation at large Q^2,[14].

This similarity has an immediate consequence: Like the proton, the photon is assumed to be composed of partons. In the case that the partons are identified with quarks, from theoretical considerations one has to expect the following results:

Figure 2, [10]

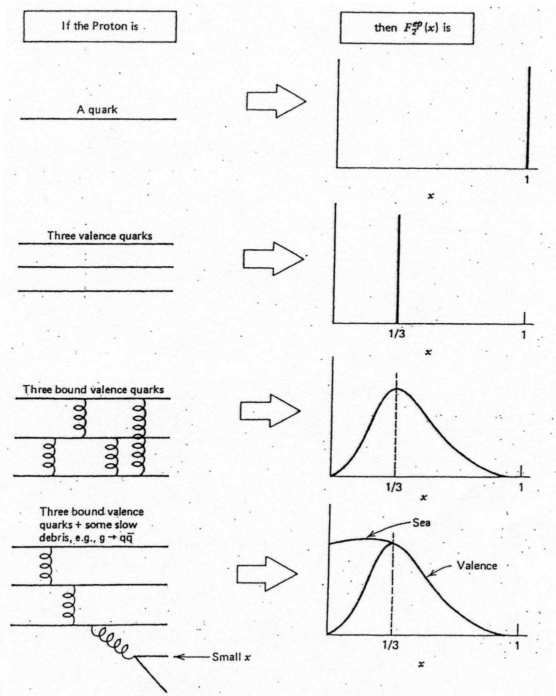

Fig. 9.7 The structure function pictured corresponding to different compositions assumed for the proton.

But it should be emphasized that such partons are not necessarily to be identified as quarks. Such an identification of the partons rather depends on the field theoretic model which is used for the description of high energy processes.

Some experimental results which demonstrate our statements are given by the following figures:

Figure 3, [10]

Extracting structure functions from experiment

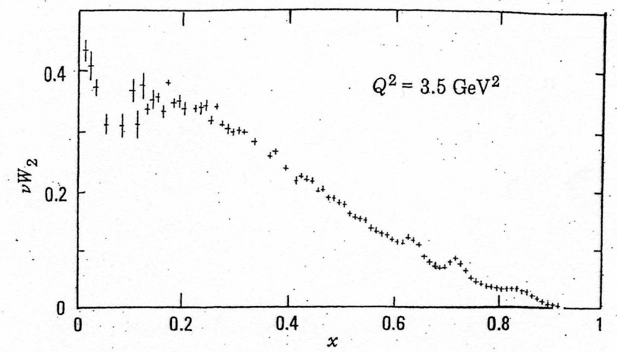

Fig. 2.2 νW_2 structure function of the proton from SLAC electroproduction experiments at $Q^2 = 3.5$ GeV^2. As well as data from E87, E49B, E89-2, E49-A at SLAC, muonproduction data from Fermilab E-98 (small x data with $R = 0.52$) are included. See Poucher *et al.* (1974), Bodek *et al.* (1979), Gordon *et al.* (1979) and Mestayer *et al.* (1983).

Figure 4, [10]

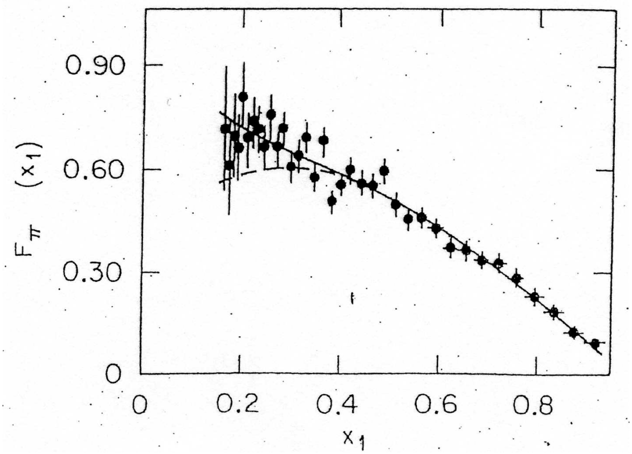

Figure 5, [10]

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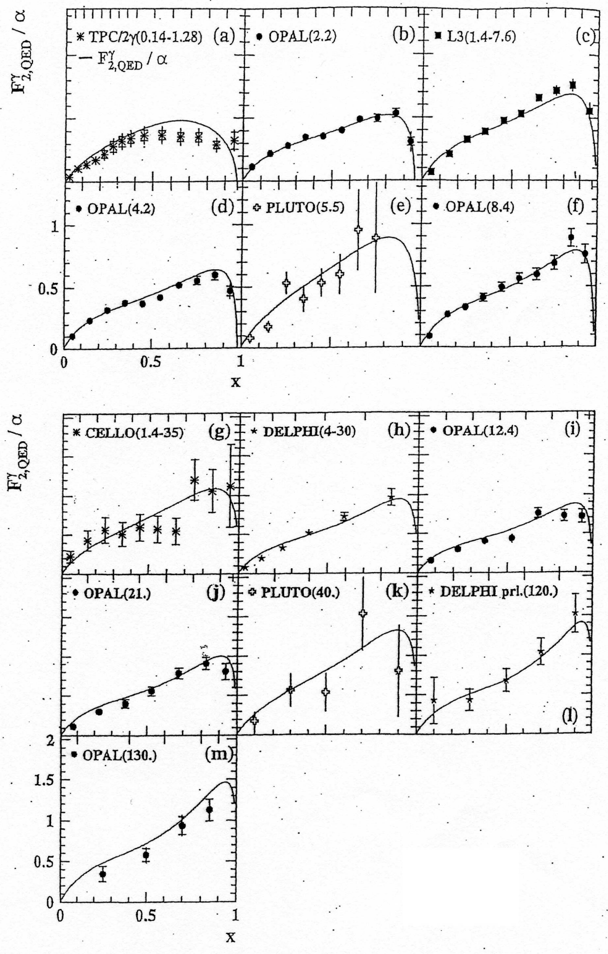

Fig. 36. The world summary of $F^{\gamma}_{2,\mathrm{QED}}$ measurements. The data are compared to $F^{\gamma}_{2,\mathrm{QED}}(x,\langle Q^2\rangle,\langle P^2\rangle)$, or $\langle F^{\gamma}_{2,\mathrm{QED}}(x,Q^2,\langle P^2\rangle)\rangle$, with numbers as given in the text. The points represent the data with their statistical (inner error bars) and total errors (outer error bars). The quoted errors for (h) are statistical only. The tic marks at the top of the figures indicate the bin boundaries.

5 The theoretical consequence of experiments

Figure 2 shows: If the partons are assumed without mutual interaction, it is not possible to obtain theoretical cross-sections which reproduce the results of the high energy experiments. Hence a parton interaction is needed.

If this idea is consequently pursued it necessarily leads to a quantum field theoretic formulation of de Broglie's fusion principle. But historically with the further development of quantum field theory for electromagnetic and strong forces the de Broglie-Bargmann-Wigner formalism was superseded by genuine quantum field theoretic methods in order to obtain appropriate descriptions of relativistic bound states. The most preferred version of such a quantum field theoretic formalism for bound quark and lepton states was the theory of Bethe-Salpeter equations which was started fifty years ago, [19] and was elaborated by numerous authors, [20], [21], [22].

With respect to the Bethe-Salpeter equations their closer inspection revealed serious drawbacks which became apparent by the existence of abnormal solutions and negative norms of the corresponding wave functions, [20][23]. And inspite of many efforts it was not possible to cure these difficulties and all attempts to ease the situation of the exact theory by introducing quasipotential approximation, etc., mutilated in one way or another the necessary quantum field theoretic assumptions and generally introduced spurious singularities, [23]. So the constructions of relativistic two-body (and many body) equations and the comparison between different methods are active areas of current research, [24].

The latter remark implies: Without sacrifacing the quantum field theoretic background one should look for relativistic many particle equations which avoid the difficulties of the Bethe-Salpeter formalism. Indeed such an approach is possible. It is based on the idea that de Broglie's spin fusion should be caused by direct interaction of fermions without the assistance of bosons which in this (in de Broglie's) picture are fusioned objects and not elementary entities of the fermion dynamics. If this picture and model is consistently developed it not only gives consistent dynamical equations for bound states but leads also to an improved understanding of the structure of matter.

The corresponding theory which exclusively deals with spinorial interactions is based on a nonperturbatively regularized nonlinear spinor field with canonical quantization and probability interpretation. It can be considered as the quantum field theoretic generalization of de Broglie's fusion theory and as a mathematical realization and physical modification of Heisenberg's approach, [25], and is expounded in [26], [27]. Owing to this generalization the basic ingredients of this theory are assumed to be unobservable subfermions and not neutrinos as mentioned above. And only after having derived the conventional gauge theories as effective theories it is possible to introduce the corresponding (composite) gauge bosons into the dynamical interplay of matter.

6 Relativistic photon equations

By means of the field theoretic formalism wave equations for single composite particles with partonic substructure can be derived,[26], [27]. Here I cannot describe this extensive and comprehensive formalism into which these equations are incorporated. Rather I concentrate on these equations themselves and their interpretation.

But before beginning a detailed discussion it should be emphasized that in the framework of such a theory the existence of composite photon states is not in contradiction to "no go" theorems, because these theorems do not take into account the consequences of regularization.

We start with the hard core equations for composite photon states, I use a highly symbolic notation in order to obtain clearly organized expressions. In particular the following definitions are used: $\mathbf{r} \in R^3$, $x \in M^4$, and $Z = (i, \kappa, \alpha)$ where κ means superspin-isospin index, $\alpha =$ Dirac spinor index, $i =$ auxiliary field index. The latter index characterizes the subfermion fields which are needed for the regularization procedure.

Let $\varphi_{Z_1 Z_2}(x_1, x_2)$ be the covariant, antisymmetric state amplitude of the composite particle (quantum). Then the following set of covariant photon equations can be derived

$$[D^{\mu}_{Z_1 X_1} \partial_{\mu}(x_1) - m_{Z_1 X_1}] \varphi_{X_1 Z_2}(x_1, x_2) = 3 U_{Z_1 X_2 X_3 X_4} F_{X_4 Z_2}(x_1 - x_2) \varphi_{X_2 X_3}(x_1, x_1) \tag{9}$$

and owing to antisymmetry simultaneously with equation (9) also the equation

$$[D^{\mu}_{Z_2 X_2} \partial_{\mu}(x_2) - m_{Z_2 X_2}] \varphi_{Z_1 X_2}(x_1, x_2) = -3 U_{Z_2 X_2 X_3 X_4} F_{X_4 Z_1}(x_2 - x_1) \varphi_{X_2 X_3}(x_2, x_2) \tag{10}$$

has to hold. In this representation the following definitions are used:

$$D^{\mu}_{Z_1 Z_2} := i \gamma^{\mu}_{\alpha_1 \alpha_2} \delta_{\kappa_1 \kappa_2} \delta_{i_1 i_2} \tag{11}$$

and

$$m_{Z_1 Z_2} := m_{i_1} \delta_{\alpha_1 \alpha_2} \delta_{\kappa_1 \kappa_2} \delta_{i_1 i_2} \tag{12}$$

and

$$F_{Z_1 Z_2}(x_1 - x_2) := -i\lambda_{i_1} \delta_{i_1 i_2} \gamma^5_{\kappa_1 \kappa_2} [(i\gamma^\mu \partial_\mu(x_1) + m_{i_1})C]_{\alpha_1 \alpha_2} \Delta(x_1 - x_2, m_{i_1}) \tag{13}$$

where $\Delta(x_1 - x_2, m_{i_1})$ is the scalar Feynman propagator.

The meaning of the index κ can be explained by decomposing it into two parts $\kappa := (\Lambda, A)$ with $\Lambda = 1, 2$ as the index of spinors and charge conjugated spinors and $A =$ 1,2 as the isospin index, which can be equivalently expressed by $\kappa = 1, 2, 3, 4$.

The vertex terms in equations (9) and (10) are fixed by the following definitions:

$$U_{Z_1 Z_2 Z_3 Z_4} := \lambda_{i_1} B_{i_2 i_3 i_4} V^{\kappa_1 \kappa_2 \kappa_3 \kappa_4}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} \tag{14}$$

where $B_{i_2 i_3 i_4}$ indicates the summation over the auxiliary field indices and where the vertex is given by a scalar and a pseudoscalar coupling of the subfermion fields:

$$V^{\kappa_1 \kappa_2 \kappa_3 \kappa_4}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4} := \frac{g}{2} \{[\delta_{\alpha_1 \alpha_2} C_{\alpha_3 \alpha_4} - \gamma^5_{\alpha_1 \alpha_2} (\gamma^5 C)_{\alpha_3 \alpha_4}] \delta_{\kappa_1 \kappa_2} [\gamma^5 (1 - \gamma^0)]_{\kappa_3 \kappa_4} \}_{as[2,3,4]} \tag{15}$$

The parameters λ_i originate from the regularization procedure and fulfill the conditions $\sum_i \lambda_i = 0$ and $\sum_i \lambda_i m_i = 0$ which guarantee the finiteness of the regularized expressions.

The properties of these equations can be summarized as follows:

i) these equations are relativistically invariant quantum mechanical many body equations

ii) they contain nontrivial interactions between their constituents

iii) the interactions are mediated by causal functions

iv) the solutions of the secular equations for the mass values are finite by inherent selfregularization,[28]

v) these equations admit for the corresponding wave functions a probability interpretation with finite norms,[29],[28]

vi) for vanishing coupling constant $g = 0$ de Broglie's original fusion equations for local photons are obtained

vii) a Schroedinger representation for equal times $t_1 = t_2 = t$ exists exactly,[26],[27]

viii) the statements i) - vii) hold also for any n

Therefore these equations can be considered to be generalized de Broglie-(Bargmann-Wigner) equations.

Although Bargmann-Wigner published their analysis of de Broglie's original equations sixteen years after de Broglie's first paper, in the anglo-american literature these equations are frequently named Bargmann-Wigner equations. So to do justice to their inventor and to facilitate identification I chose the combinations used above.

Equation (9) admits exact solutions which are antisymmetric and for a solution of the set (9) and (10) only equation (9) has to be used if the wave functions are antisymmetric. We only give the result of such calculations and refer for details to [30], [26], [27].

Proposition : Let φ be a solution of equation (9). Then φ describes a vector boson with definite momentum k , if it is given by the following expression:

$$\varphi_{Z_1 Z_2}(x_1, x_2) = T^a_{\kappa_1 \kappa_2} exp[-i\frac{k}{2}(x_1 + x_2)] A^\mu \chi^\mu_{\substack{i_1 i_2 \\ \alpha_1 \alpha_2}}(x_1 - x_2|k) \tag{16}$$

with the internal wave function

$$\chi^\mu_{\substack{i_1 i_2 \\ \alpha_1 \alpha_2}}(x) := \frac{2ig}{(2\pi)^4}\lambda_{i_1}\lambda_{i_2}\int d^4p e^{-ipx}[S_F(p+\frac{k}{2}, m_{i_1})\gamma^\mu S_F(p-\frac{k}{2}, m_{i_2})C]_{\alpha_1\alpha_2} \tag{17}$$

(no summation over i_1, i_2) and $S_F(p, m) := (i\gamma^\mu p_\mu - m)^{-1}$.

The above given representation of the boson wave function holds for all wave vectors k, as the calculation was performed in a strictly relativistic invariant way. The integral in (17) can be evaluatued by standard methods and leads, of course, to a singular behavior on the light cone. Therefore for physical interpretation a regularization is needed. This will be discussed in Section 8.

7 Symmetries and quantum numbers

We start with the discussion of the symmetry connected with the superspin-isospin indices. The properties of the states (13) with respect to these indices are expressed by the matrices T^a . For vector boson states these matrices must be antisymmetric, and as four-dimensional matrices they can be represented by the antisymmetric elements of the Dirac algebra. From the general field theoretic formalism quantum conditions for the corresponding quantum numbers f = fermion number, t = isospin, t_3 = third component of isospin can be derived, see [26], [27].

Without decomposing κ into superspin and isospin indices (Λ, A) we directly formulate this problem in terms of Dirac matrices. In particular we consider representations with fermion number $f = 0$. They are given by [31],[26],[27]:

$$T^0_{\kappa_1\kappa_2} = \begin{pmatrix} 0 & \mathbf{1} \\ -\mathbf{1} & 0 \end{pmatrix}_{\kappa_1\kappa_2} \tag{18}$$

and

$$T^k_{\kappa_1\kappa_2} = \begin{pmatrix} 0 & \sigma^k \\ (-1)^k\sigma^k & 0 \end{pmatrix}_{\kappa_1\kappa_2} \qquad k = 1,2,3 \tag{19}$$

and can be used for the characterization of the electro weak gauge bosons in the case of the unbroken original symmetry.

Proposition :The $U(1)$ singlet state: T^0, and the $SU(2)$ triplet states: T^k, k = 1,2,3 , are characterized by the quantum numbers of the following table

	T^0	$\hat{T}^1$	$\hat{T}^2$	T^3
t	0	1	1	1
t_3	0	1	-1	0
f	0	0	0	0

with $\hat{T}^1 := (1/2)[T^1 + iT^2]$ and $\hat{T}^2 := (1/2)[T^1 - iT^2]$.

The Dirac algebra also contains antisymmetric elements which lead to eigenstates with fermion number $f = +2$ or -2 . These states are unphysical ones and the reason for their dropping out of electro weak calculations is the superselection rule for fermion numbers of the underlying spinor field dynamics.

Finally it is well known that in the context of the Standard model the original input of the $U(1)$ gauge group does not immediately lead to the photon field. Only after symmetry breaking, etc., one obtains the electromagnetic field. This phenomenological treatment has its counterpart in the microscopic theory of gauge bosons, see [32], [27]. But such considerations are irrelevant for the quantum theory of single vector boson states. Hence in the following we do not refer to these aspects of the general theory.

Next we turn to the reaction of the system under transformations of the space-time groups. From (16) and (17) it follows that they transform as second order spin tensors. The group theoretical classification of such spin tensors can be performed by the application of the Pauli-Lubanski spin vector and owing to its covariant formulation, calculations can be done in special frames of reference without loss of generality. In particular we choose the rest frame. In this case the Pauli-Lubanski spin vector reads

$$W_\mu = \frac{1}{2p_0}\varepsilon_{\mu\nu\varrho 0}M^{\mu\nu}P^0 \tag{20}$$

and in this expression the representation of the generators P^μ and $M^{\mu\nu}$ depends on the particle number or the dimension of the coordinate space respectively. With $\varepsilon_{ijk0} = \varepsilon^{0ijk} = -\varepsilon_{ijk}$ we obtain $W_0 = 0$ and

$$W_i = -\frac{1}{2}\varepsilon_{ijk}M^{jk} = -J_i, \qquad i = 1,2,3 \tag{21}$$

where J_i are the rotational angular momentum operators of Dirac spinors.

By means of the Pauli-Lubanski spin vector one can formulate two eigenvalue conditions which characterize the representation:

$$\begin{aligned} W_3\varphi(u,z) = & \frac{1}{2}\Sigma^3\varphi(u,z) + \frac{1}{2}\varphi(u,z)(\Sigma^3)^T \\ & + i[u_1\partial_2(u) - u_2\partial_1(u)]\varphi(u,z) = s_3\varphi(u,z) \end{aligned} \tag{22}$$

and

$$W^2\varphi(u,z) = (J_1^2 + J_2^2 + J_3^2)\varphi(u,z) = s(s+1)\varphi(u,z) \tag{23}$$

and it can be demonstrated that these conditions are compatible with the dynamical equations (9) and (10)

8 Probability interpretation and conservation

So far nothing was said about the role of the auxiliary fields (indices) which appear in the generalized de Broglie-Bargmann-Wigner equations (9) and (10) and in their solutions (16),(17) In this context the corresponding energy equation is needed. The energy equation can be directly derived from the set of covariant equations (9) and (10), [26],[27]

$$
\begin{aligned}
i\partial_t \varphi_{Z_1Z_2}(x_1,x_2) = & iD^0_{Z_1X_1}[D^k_{X_1X_2}\partial_k(x_1) - m_{X_1X_2}]\varphi_{X_2Z_2}(x_1,x_2) \\
& iD^0_{Z_2X_1}[D^k_{X_1X_2}\partial_k(x_2) - m_{X_1X_2}]\varphi_{Z_1X_2}(x_1,x_2) \\
& -3i[D^0_{Z_1X_1}U_{X_1X_2X_3X_4}F_{X_4Z_2}(x_1-x_2)\varphi_{X_2X_3}(x_1,x_1) \\
& -D^0_{Z_2X_1}U_{X_1X_2X_3X_4}F_{X_4Z_1}(x_2-x_1)\varphi_{X_2X_3}(x_2,x_2)
\end{aligned}
\tag{24}
$$

Now the auxiliary fields and their role in regularization can be discussed. For free fermion field propagators the summation over i_1, i_2 corresponds to an ordinary Pauli-Villars regularization of the propagators. And although this regularization is exemplified for the special case of a free auxiliary field propagator, in the general theory one has no other means to perform a regularization as to apply this definition and in addition the secular equation for the calculation of the mass eigenvalue of the vector bosons automatically enforces this kind of regularization.

The essential point of this definition is that one achieves regularization without destroying the relativistic transformation properties of the corresponding expressions.

Hence in the case of the photon state amplitudes we define the physical, i.e., regularized state amplitudes $\hat{\varphi}$ by:

$$
\hat{\varphi}_{\substack{\alpha_1\alpha_2\\ \kappa_1\kappa_2}}(x_1,x_2) := \sum_{i_1 i_2} \varphi_{Z_1Z_2}(x_1,x_2) \tag{25}
$$

One immediately realizes that the physical $\hat{\varphi}$ has the same transformation properties as the original φ as the general spin tensor transformations do not depend upon the auxiliary field numbers of the states under consideration

In order to derive a probability interpretation for the bosonic wave functions the single time formulation of the energy equation (24) is needed.

As a subsidiary condition the result of such a transition must coincide with the (single time) algebraic Schroedinger representation following from the Hamiltonian dynamics. This coincidence is guaranteed if by the transition the antisymmetry of the wave function is conserved.

Corollary: By the definition:

$$\varphi_{Z_1 Z_2}(\mathbf{r}_1, \mathbf{r}_2, t) := \frac{1}{2}\left(\lim_{t_1 - t_2 \to t+0} + \lim_{t_1 - t_2 \to t-0}\right)\varphi_{Z_1 Z_2}(x_1, x_2) \tag{26}$$

the antisymmetry of the single time wave function is conserved if the covariant wave function is antisymmetric.

If this limit is performed in (24) and if we decompose the index $Z := (\alpha, \kappa, i)$ into $Z = (z, i)$ and sum over i_1, i_2 in the energy equation then from (24) one can derive a current conservation law, [29]:

Proposition: In the limit of equal auxiliary field masses $m_i = m$ the density

$$\hat{\varphi}^\dagger \hat{\varphi} := \sum_{z_1 z_2} \hat{\varphi}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2, t)^* \hat{\varphi}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2, t) \tag{27}$$

is a conserved positive quantity which results from current conservation:

$$\partial_t(\hat{\varphi}^\dagger \hat{\varphi}) + \sum_{l=1,2} \partial_k^l[\hat{\varphi}^\dagger \alpha^k(l) \hat{\varphi}] = 0 \tag{28}$$

This limit can be performed in the regularized wave functions without any difficulty after all calculations were done.

Owing to current conservation the densities (27) are conserved positive quantities ,i.e., the physical state amplitudes (25) are elements of a corresponding Hilbert space with the norm expression

$$\langle \hat{\varphi} | \hat{\varphi} \rangle = \sum_{z_1 z_2} \int d^3 r_1 d^3 r_2 \hat{\varphi}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2, t)^* \hat{\varphi}_{z_1 z_2}(\mathbf{r}_1, \mathbf{r}_2, t) \tag{29}$$

Hence one is able to extract all quantum mechanically meaningful information about this system from its given state space. It should, however, be emphasized that the density (27) is only the probability density for finding two partons at time t in the system, but it is not the probability density for finding *free* partons at the positions $\mathbf{r}_1, \mathbf{r}_2$ at time t in the system, because in the bound state these partons are not on their mass shell.

9 Effective photon dynamics

The regularized amplitudes (25) which according to (29) are elements of a corresponding Hilbert space contain the information about the partonic substructure of photons. But, of course, this information about photons has to be supplied and completed by statements about the macroscopic properties of these quanta. Obviously the latter information is contained in the vector potentials A^μ in (16)

The vector potentials A^μ in (16) are not at free disposal. On the contrary they are fixed to be solutions of the secular equation associated to equations (9) and (01), [28]

The secular equation associated with equations (9) and (10) can be directly derived from the corresponding integral equation for bound states.

Proposition: The secular equation is exclusively referred to the vector potential A_μ

$$A_\mu = \frac{2ig}{(2\pi)^4}\{[J_0 + J_1 k^2 + 2J_2]A_\mu - 2J_1 k_\mu k^\nu A_\nu\} \tag{30}$$

while the field strength $F_{\mu\nu}$ results from the vector potential by

$$F_{\mu\nu} = \frac{2g}{(2\pi)^4}[J_\nu^+ A_\mu + J_\mu^- A_\nu]_{as(\mu\nu)} \tag{31}$$

with

$$J_0 := \int d^4p S(p+\frac{k}{2})S(p-\frac{k}{2}) \tag{32}$$

$$J_1 k_\mu k^\nu + J_2 \eta_\mu^\nu := -\int d^4p R_\mu(p+\frac{k}{2})R^\nu(p-\frac{k}{2})$$

and

$$J_\nu^+ := \int d^4p S(p+\frac{k}{2})R_\nu(p-\frac{k}{2}) \tag{33}$$

$$J_\mu^- := \int d^4p R_\mu(p+\frac{k}{2})S(p-\frac{k}{2})$$

with

$$R_\nu(p) := \sum_i \frac{p_\nu \lambda_i}{p^2 - m_i^2 + i\varepsilon} =: p_\nu R(p) \tag{34}$$

$$S(p) := \sum_i \frac{m_i \lambda_i}{p^2 - m_i^2 + i\varepsilon}$$

Defining

$$A'_\mu := c(k^2) A_\mu \tag{35}$$

owing to the homogeneity of equations (30) these equations hold equally well for A'_μ . Hence we have

$$A'_\mu = \frac{2ig}{(2\pi)^4} \{[J_0 + J_1 k^2 + 2J_2] A'_\mu - 2J_1 k_\mu k^\nu A'_\nu\} \tag{36}$$

and

$$F_{\mu\nu} = -i\{A'_\mu k_\nu\}_{as(\mu\nu)} \tag{37}$$

The latter equations are identical with the phenomenological definition of the field strength tensor in terms of the vector potential. Thus it remains to study the equations (30) for the vector potential itself. These equations are a homogeneous system which can only be solved if the corresponding determinant vanishes. But owing to the special form of equations (30) one can derive their secular condition by means of the gauge conditions for massless photons.

Corollary: For massless photons with $k^2 = 0$ the secular condition is gauge invariant for Lorentz gauge and temporal gauge and the vector potential satisfies the corresponding wave equation.

And if the secular equation (36) has a nontrivial solution $k^2 = \kappa^2$, then (36) gives clearly the wave equation

$$\{k^2 - \kappa^2\} A'_\mu = 0 \tag{38}$$

which after Fourier transformation gives the wave equation for the vector potential.

Photons are characterized by $k^2 = 0$ or a very tiny mass value which in the present investigation can be neglected. Hence equation (36) has to be satisfied for vanishing photon mass. This gives a condition for the yet unknown coupling constant g which by this condition is fixed for all other calculations of the theory.

In this limit all integrals can be exactly calculated and give the following values, [28]:

$$J_0 = \frac{6i\pi^2}{5m^2} \qquad J_1 = \frac{2i\pi^2}{7m^4} \qquad J_2 = \frac{i\pi^2}{5m^2} \tag{39}$$

i.e. one obtains finite values for these integrals and for massless photons the coupling constant takes the value $g = -\pi^2 5m^2$. Furthermore the solutions show that the eigenvalues $k^2 = 0$ lead to an energy spectrum of positive and negative energies $E(\mathbf{k}) = (\mathbf{k}^2)^{1/2}, -(\mathbf{k}^2)^{1/2}$. These energy values are necessary for the completeness of the weak mapping procedure and have to be treated in analogy to the energy spectrum of the Dirac equation, i.e., these solutions make only physical sense in a quantum field theoretic treatment with renormalization of the vacuum.

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