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The integer-fractional quantum hall predicament can be resolved by returning to primary quantization

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ABSTRACT. Assessments of the quantum Hall effect by wave function procedures are compared with a global assessment using Aharonov-Bohm and Ampère-Gauss period-integrals for flux and charge quanta.

The contemporary assessments of the quantum Hall effect (QHE) is dichotomous in nature in the sense that an inherently statistical tool (Schroedinger's equation) is being used on one of the most highly ordered manifestations in modern experimentation. Extremely low temperatures and very high magnetic fields create the order. Normally one might not use a statistical tool in trying to cast light on a non-statistical situation. So why are not more people upset about such contradictory behavior? The answer is that Copenhagen made the statistics nonclassical, which meant its rules are bent.

The fact is that all of us have been exposed to a measure of brainwashing by the successes of the Copenhagen school of quantum mechanics. The Copenhagen school teaches that its Ψ function statistics is nonclassical, which in practice means no real *universe of discourse* has been established. Its views of quantum mechanics see Ψ as probability amplitude relating to (point) particle presence. This picture just falls short of a universe of discourse. Hence, the nonclassical touch remains; if you can't avoid it, learn to live with it. That is how we have been intimidated to tolerate nonclassical ways of thinking.

The Copenhagen methodology as here depicted is believed to be fair. They have some unnecessary *absolutes* that have never been proven and can never be proven. In fact, a first counter-example to the thesis of a nonclassical statistics was inadvertently produced by Planck [1]. His example goes back to 1912; hence he could hardly have been accused of prejudiced against the Copenhagen views emerging decades later.

Copenhagen *absolutes* range from a rather silent proposition of not having a universe of discourse for the Ψ statistics, all the way to a dictum of a priori positional uncertainty. They, in turn, have given birth to related concepts of an everywhere infinite zero-point energy of vacuum and last but not least a doctrine of taking the Schroedinger equation to be applicable in all quantum situations. These unproven features, ordained as doctrines of physics, reveal a religious sentiment in the Copenhagen interpretation that has to be identified if we are to become aware of the contradictory consequences thereof.

Let the quantum Hall situation be taken as experimentum crucis to serve as test case. The QHE has been almost exclusively assessed using Schroedinger-based procedures. In this approach Ψ function solutions have been sought to account for the observed onset of its plateau states of order. In three interconnected papers R B Laughlin [2,3,4] has taken a leading initiative in these procedures. An inspection of Laughlin's papers shows attempts at understanding the condensed QH states through Ψ function behavior using the many body procedures of quantum mechanics. A delta function behavior of the statistics could be an onset of order, which means looking for regular sequence of generalized functions [5]. Yet, the analytic hurdles of many body theory, *i.e.,* three bodies and upwards, are so formidable that one has to settle for exotic abstractions. As it stands QHE theory in refs.{2,3,4] emerges from a necessarily contrived narrow basis of probing by means of many body theory, which is based on a premise of a ground state of orbital flux.

Independent of these specifics one may expect a formation of plateau states, as they manifest themselves in the QHE, to exhibit some form of Bose-Einstein condensation of cyclotron systems in a two dimensional ensemble. Assuming the transitions between normal- and plateau states as fermion-boson transitions is indeed supportive of extensive reduction features observed in the ratio of sample resistance over Hall resistance in the transition range between normal- and plateau states [6]. The inverse chain line hereby observed can be satisfactorily assessed as a fermion-boson transition.

The next phase in quantum Hall observations made it necessary to accommodate a so-called Fractional QHE [7]. The subsequent Laughlin [4] step was one of allowing rational fractions instead of even boson-paired orbital charge fillings. While this formally accounts for FQHE observations, this step remains a mathematical artifact unless higher flux states are proven to be impossible. A conceivable argument in support of this somewhat silent assumption would be the experimentally observed superconducting state of the sample current when the sample enters the plateau state.

If the ground state flux is accepted, the physical consequence of Laughlin's step would either be a suggestion of real fractionally charged entities or contrived lattices consisting of sub-lattices of cyclotron orbits of different orbital filling.

Even so, while other realms of physics, *e.g.,* quark theory, have experimented with fractional charge, thus far they have remained unconfirmed. There is an obligation here to check whether fractional charge is compelling. This confronts us with *the alternative of fractional charge versus superconductivity at higher flux states.*

Here we hold out for superconductivity at higher flux states *nh/e*. For plateau states of a uniform cyclotron lattice moving in the 2-dimensional interaction space, the Hall voltage over current V/I equals VT/IT, which is the flux/charge ratio per cyclotron where T is the cyclotron period. This flux/charge ratio is a global feature relating to a ratio of two favorite integrals of quantum interferometry. They are known as the Aharonov-Bohm (AB) integral and the Gaus-Ampère(GA) integral.

Assuming, for the time being, a ground state flux h/e, the quantum Hall impedance becomes

$$Z = (1/s)\,(h/e^2), \tag{1}$$

in which s is an integer for what has become known as the integer quantum Hall effect; *h* and *e* are the quanta of action and charge.

According to Laughlin [4] s can become a fraction, either pertaining to real fractional charge or a result of a compound lattice structure with sublattice mixtures of different orbital charges. The literature speaks here of composite fermions creating distinct quantum liquids.

Laughlin may well be in the best position to comment on whether a Schroedinger many particle approach without a presumed ground state of flux is doable. Yet without a proof one way or the other the fractional charge remains too much of an adaptation after the facts. The ground state of flux, as cited in ref.[4], is not a sine qua non for superconductivity. If instead we accept the possibility of flux states *nh/e*, with n an integer, the Hall impedance formula assumes the more attractive form Eq.2, which gives a rather sensible and unified account of integer and fractional effects both:

$$Z = (n/s)\,h/e^2\ ;\ n \text{ integer, } s \text{ even.} \tag{2}$$

In Eq.2 the Quantum Hall effect is governed by two quantum numbers, the flux integer n and the ***even*** number s counting total charge of boson paired orbital electrons. Since the observed s/n in experiments is naturally cited as a fully reduced fraction, the *observed results exhibit a preponderance of odd-values n; as indeed reported by Bell Lab. teams* [7]. ***The flux ground state proposition cannot account for this phenomenon! Hence Bell Labs' observation clearly pleads in favor of including higher flux states.***

Neither directly nor indirectly does Schroedinger's process yield conclusive evidence whether just ground- or also higher orbital flux states are relevant. The Landau-like energy states, cited by Laughlin still retain zero-point energy components, which raises awkward questions whether they should also have an associated flux. Unsolved repercussions thereof have been recently reviewed [8], without generating much concern about the current Hall effect situation. The predicament can be best highlighted by the simple fact that Schroedinger gives energy *eigen*values, whereas the QHE needs flux and charge *eigen*values, such as given by the AB and GA integrals. In other words the quantum Hall effect is obviously not quantum-mechanical- but ***quantum-electrodynamical*** in nature.

Since Eq.2 is contingent on states of orbitally linked flux *nh/e*, the question is: do we have an appropriate justification compatible with fundamental law? The Aharonov-Bohm integral [9] really provides the best argument by far. According to current insights, the AB integral derives from Ψ single valuedness. Unlike the related particle-based Bohr-Sommerfeld in configuration space, flux quantization *nh/e* in *external* magnetic fields as envisioned by London [10] is a particle independent spacetime statement, which qualifies it to be used in conjunction with the particle counting GA integral. So the quantum-electrodynamical process avoids here the analytic trials of many body procedures.

Linked flux *nh/e* as evaluated by Aharonov-Bohm can, similarly as the Gaus-Ampère integral, become a mathematically exact statement, if the integration loop c resides where the exterior derivative of the integrand vanishes. Since in the present case c is the cyclotron orbit, the AB integral really equals *nh/e* ***if and only if electrons are taken to have a field-free interior.*** Compare [14] how this leads to electron moment anomaly.

The unparalleled precision of validity of Eq.2 makes the field-free electron interior, as here postulated, a viable proposition. Just for comfort, classical mechanics plus Planck's energy states verifies the flux states *nh/e.* Under the stipulated conditions, ***AB and GA integrals both give generally invariant exact spacetime quantization statements.***

Ironically, the period-integral status of the Gauss integral of electrostatics is already an item of undergraduate instruction; an extension to the space-time domain is no problem. Summarizing, the global approach to the quantum Hall effect, using the period integrals AB and GA, seems a well-suited and legitimate way to defend the unified quantum Hall formula Eq.2.

Quantization aspects of period integration were pioneered by R M Kiehn [11]. Its potential for describing the quantum Hall effect was briefly outlined in the conclusion of ref.12 and then explicitly stated in ref.13.

Schroedinger and period integral approaches to QHE both emerged in the Eighties. Fears of conflict with Copenhagen views may have prevented the period integral view from catching more attention. Yet, period integrals [12, 13] may be said to predate Copenhagen-based methodology. More extensive discussions in ref. [14].

Recently C A Mead [15] favored Eq.2 as common sense description of integer and fractional Hall effect, using the equivalent of independent flux- and charge quantization, yet no explicit comparisons with Schroedinger or period integral approaches. Please let me know what I might have been doing wrong using those period integrals?

Recognizing the ratio of two quantum numbers as governing the QHE is an inescapable mathematical necessity; the question is what seems the more reasonable physical justification? Let the reader choose between fractional charge versus flux/charge ratio.

When efforts at using existing theory for assessing new manifestations of nature are becoming too contrived, there comes a time for experiment to have its say. Not fractional charge, but a revision of Copenhagen views should here be in the making. There is a vast volume of thousands of papers on Schroedinger-based approaches to the quantum Hall effect. Yet, except Mead's book and my own, I do not know of one that considers a flux/charge ratio as primary input for the Hall impedance. I wonder why has that probability been so low? It seems people are trying to prove (quantum) laws of electrodynamics by a reducing them to (quantum) mechanics.

References

[1] M. Planck, *Theory of Heat Radiation,* see page 141, (Dover reprint 1959)

[2] P. B. Laughlin, Phys. Rev. B23. 5632 (1981)

[3] P. B. Laughlin, Phys. Rev. B27, 3383 (1982)

[4] P. B. Laughlin, Phys. Rev. Lett. 50, 1395 (1983)

[5] M. J. Lighthill, *Fourier An. and Generalized Functions* (Cambr. Univer. Press 1958)

[6] B. Huckestein, Revs. Mod. Phys.67, 357(1995
[7] D. C. Tsui et al, Phys. Rev. Lett.48. 1559(1982)
[8] R. Willet et al, Phys. Rev. Lett 59, 1776(1987)
[9] E. J. Post, Ann. der Physik 8 (5), 405(1999)
[10] Y. Aharonov and D. Bohm, Phys. Rev. 115, 485(1959]
[11] F. London, *Superfluids* I (Dover reprint 1950) p.152
[12] R. M Kiehn, J. Math,. Phys.18, 614 (1977)
[13] E. J. Post, Foundations of Physics 12, 169 (1982)
[14] E. J. Post, Phys. Lett. 94 A, 343 (1983)
[15] E. J. Post, *Quantum Reprogramming* (Kluwer, Dordrecht-Boston (1995) related material in Ann., Fond. Louis de Broglie 27, 217(2002) and also Ann. der Phys.12, 300 (2003) For mathematical info on period integration and de Rham theory :
[16] G. de Rham, *Variétés Differentiables* (Herman, Paris 1956)
[17] W. V. D Hodge, *Harmonic Integrals* (Cambridge 1939)
[18] C. A. Mead, *Collective Electrodynamics* (MIT Press, 2000) p.71

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